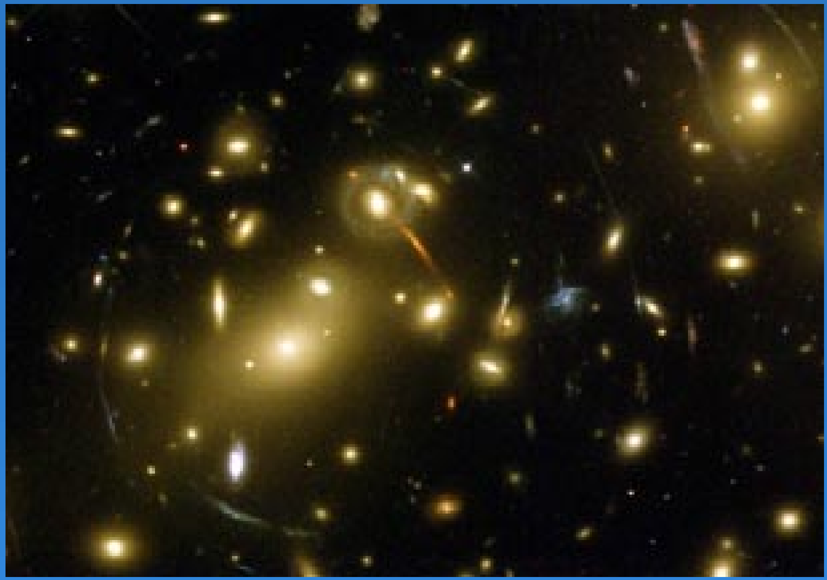


Astronomy 80 B: Light

Lecture 9: curved mirrors, lenses, aberrations

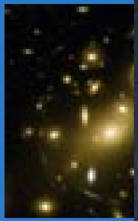
29 April 2003

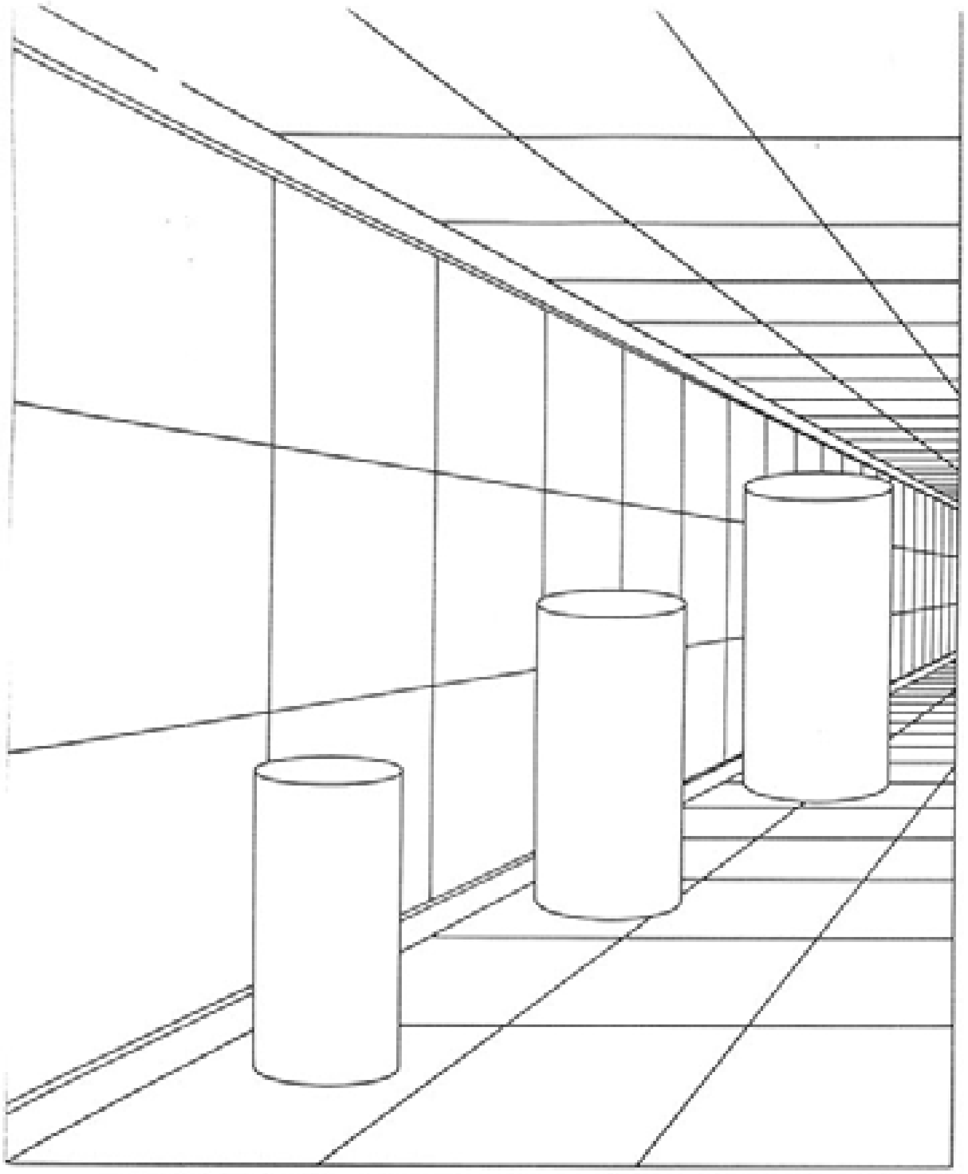
Jerry Nelson

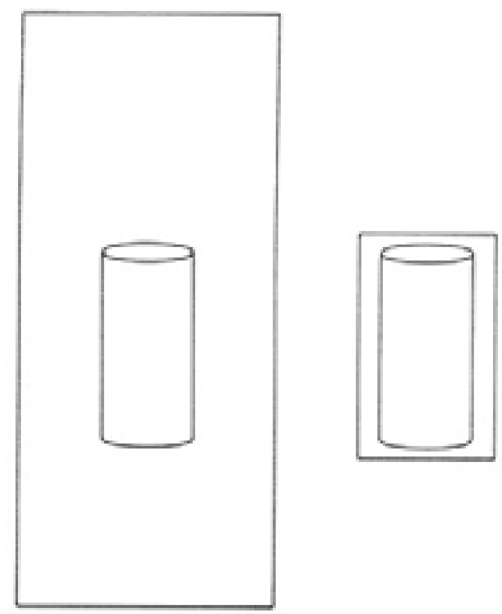
Sensitive Countries LLNL field trip

Topics for Today

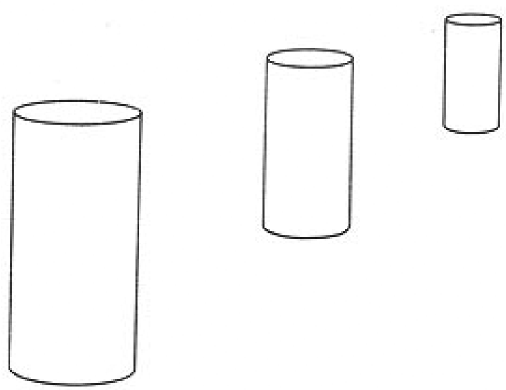
- **Optical illusion**
- **Reflections from curved mirrors**
  - Convex mirrors
  - anamorphic systems
  - Concave mirrors
- **Refraction from curved surfaces**
  - Entering and exiting curved surfaces
  - Converging lenses
  - Diverging lenses
- **Aberrations**

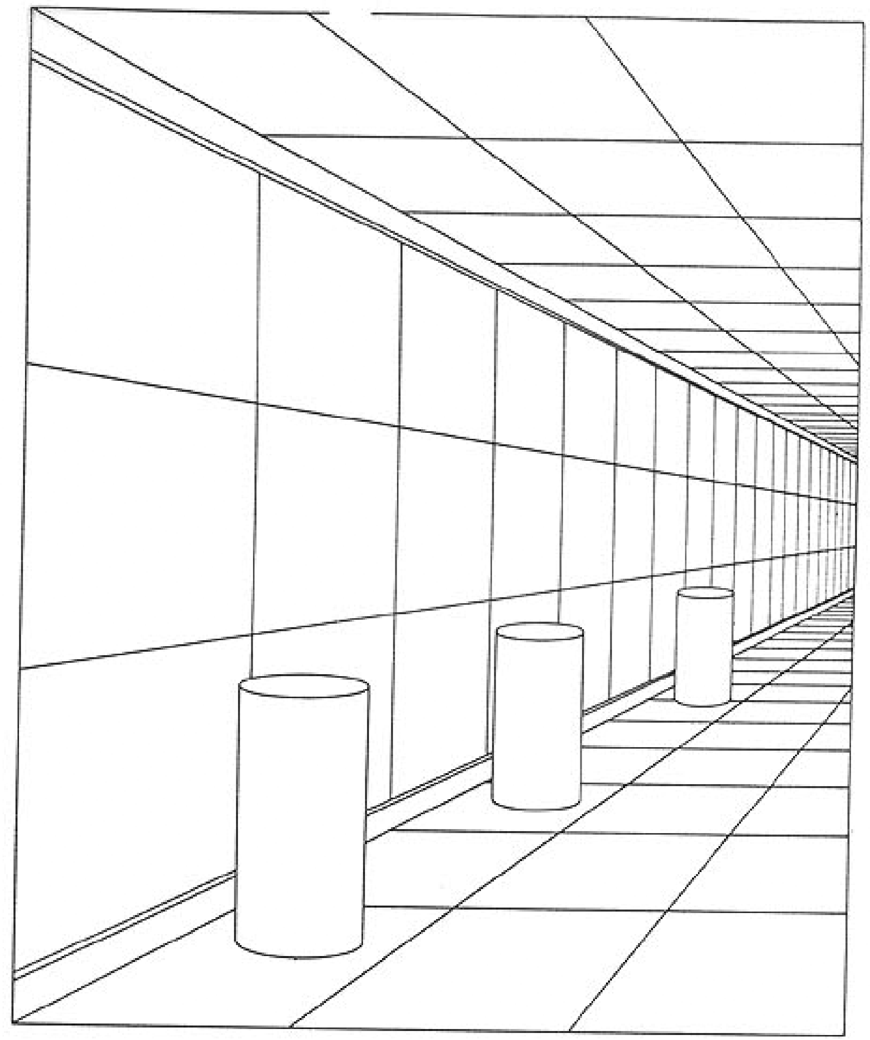




In the corridor illusion (above, left), the three cylinders look unequal in size. In the test of the contrast and assimilation theory of the corridor illusion (directly above), the cylinders appear to be only slightly different in size.





By equalizing assimilation and contrast for the three cylinders in the figure above, right, the effect of depth processing can be isolated. The cylinders there should appear to be equal or less different in size than they are. Whether or not they do can be seen by comparing the way they appear there with the way the cylinders appear in the figure directly above, where they are shown without any background.

House of Mirrors

at indicated locations one sees no images of ones self

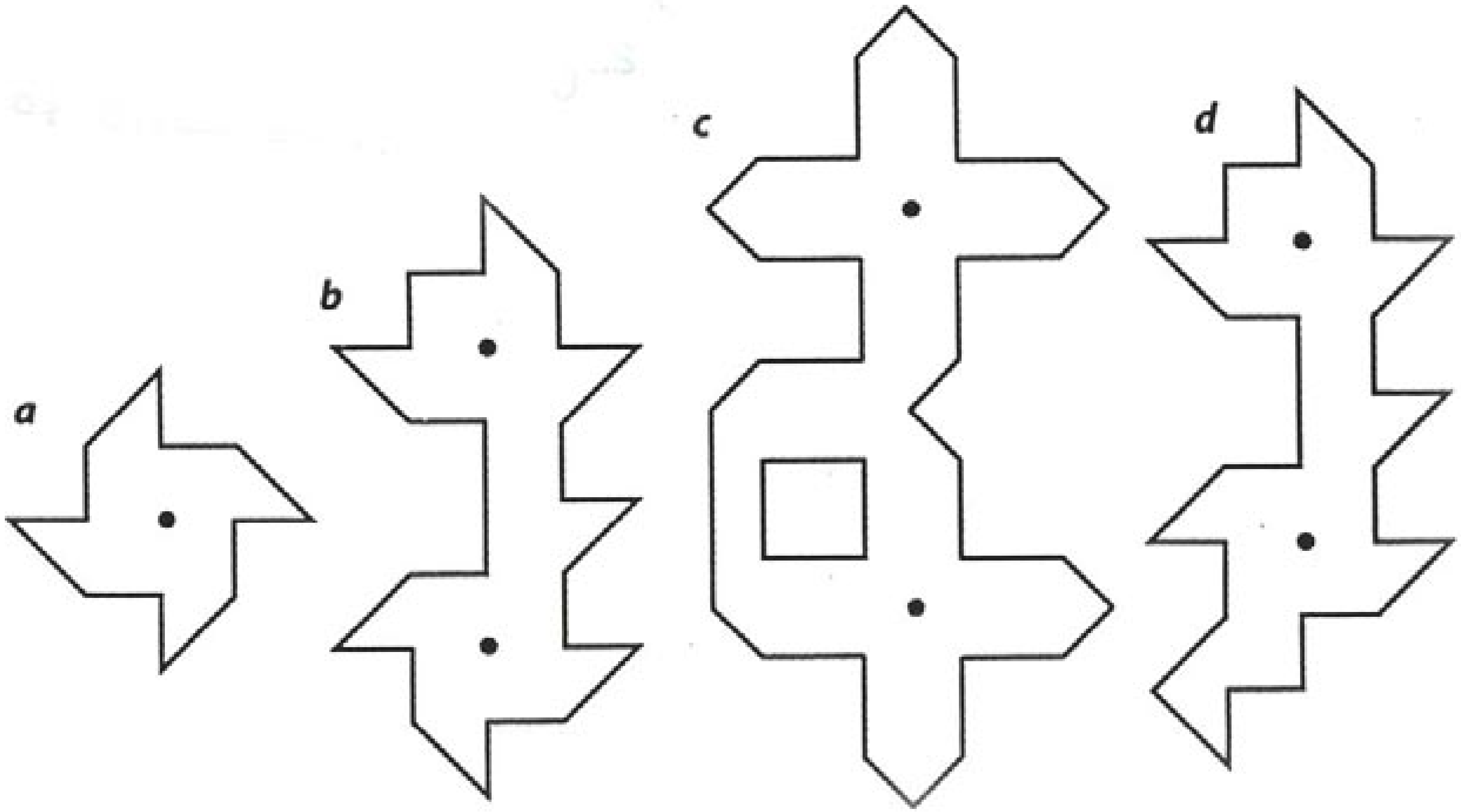
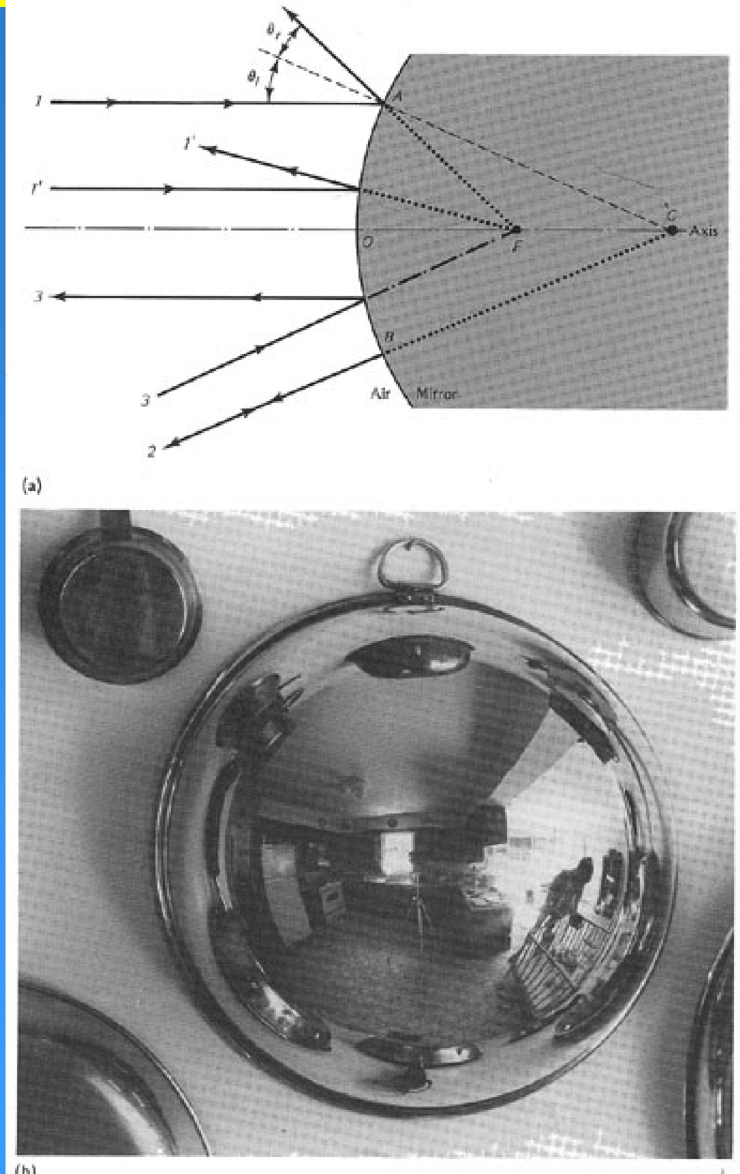


FIGURE 3.5

(a) Light rays illustrating the three rules, incident on a convex spherical mirror. Note direction of the rays—they are *not* all incident parallel to the axis! In this two-dimensional diagram and throughout the rest of the book, the three-dimensional spherical mirror appears as a part of a circle. **(b)** A shiny copper bowl makes a fine convex spherical mirror.



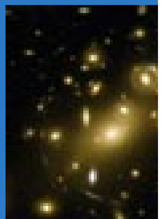
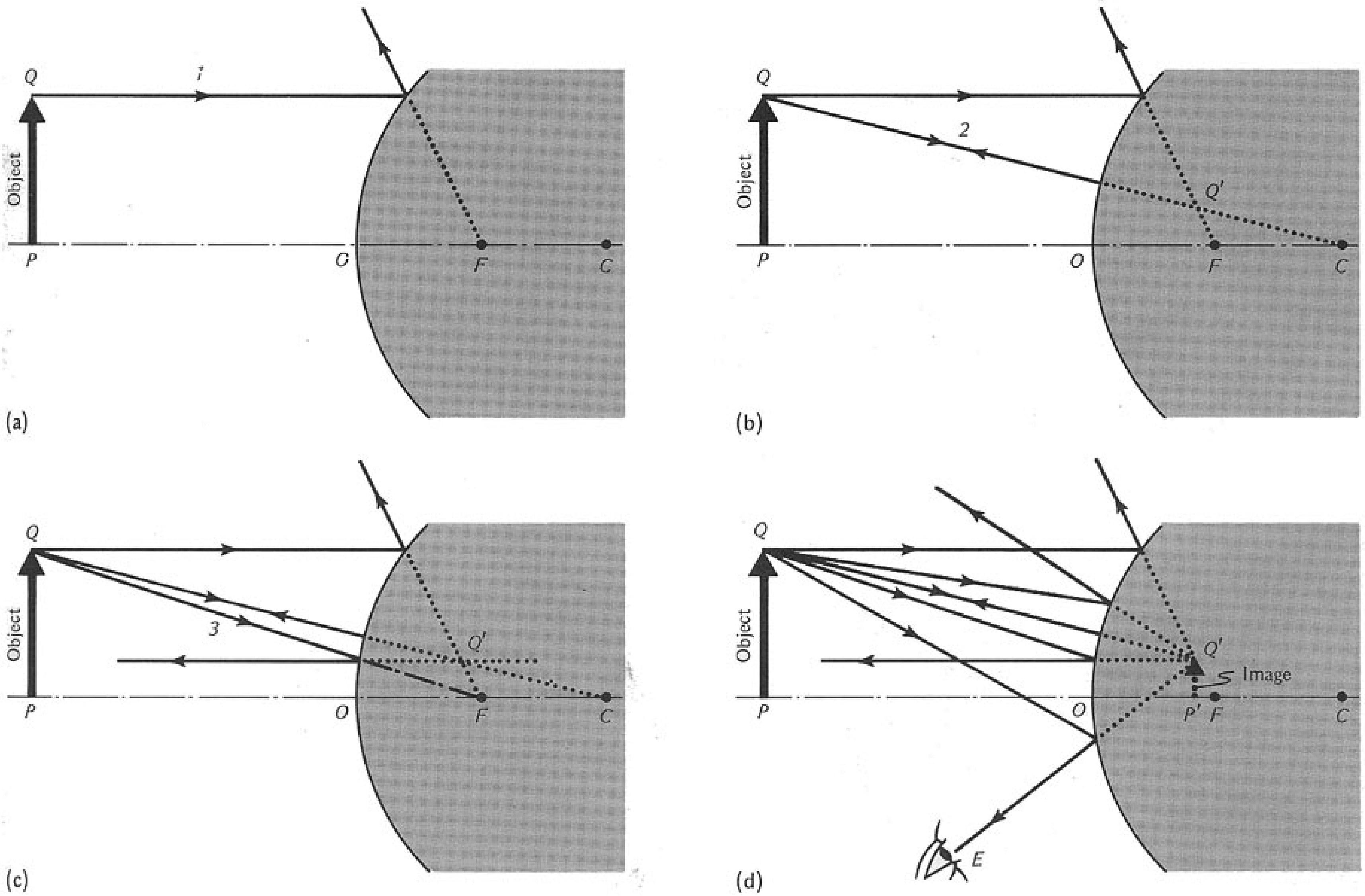
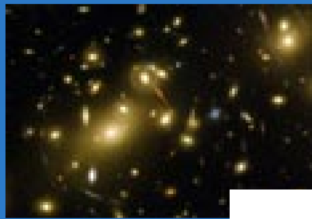


FIGURE 3.6

Construction of the image in a convex mirror: **(a)**, **(b)**, and **(c)** show the use of ray *1*, ray *2*, and ray *3* rules respectively; **(d)** shows how you can mass produce other rays—for example, one going to eye *E*—once you have found the image.





Images from convex mirror

FIGURE 3.7

Photograph of a convex mirror used to give a wide-angle view. The plane mirror on which it is mounted shows the normal view. **(a)** The camera is focused on the image in the convex mirror, just behind the mirror (see Fig. 3.6). **(b)** The camera is focused on the image in the plane mirror, as far behind the mirror as the object is in front. From a distance, your eye can focus on both images simultaneously.

(a)

(b)

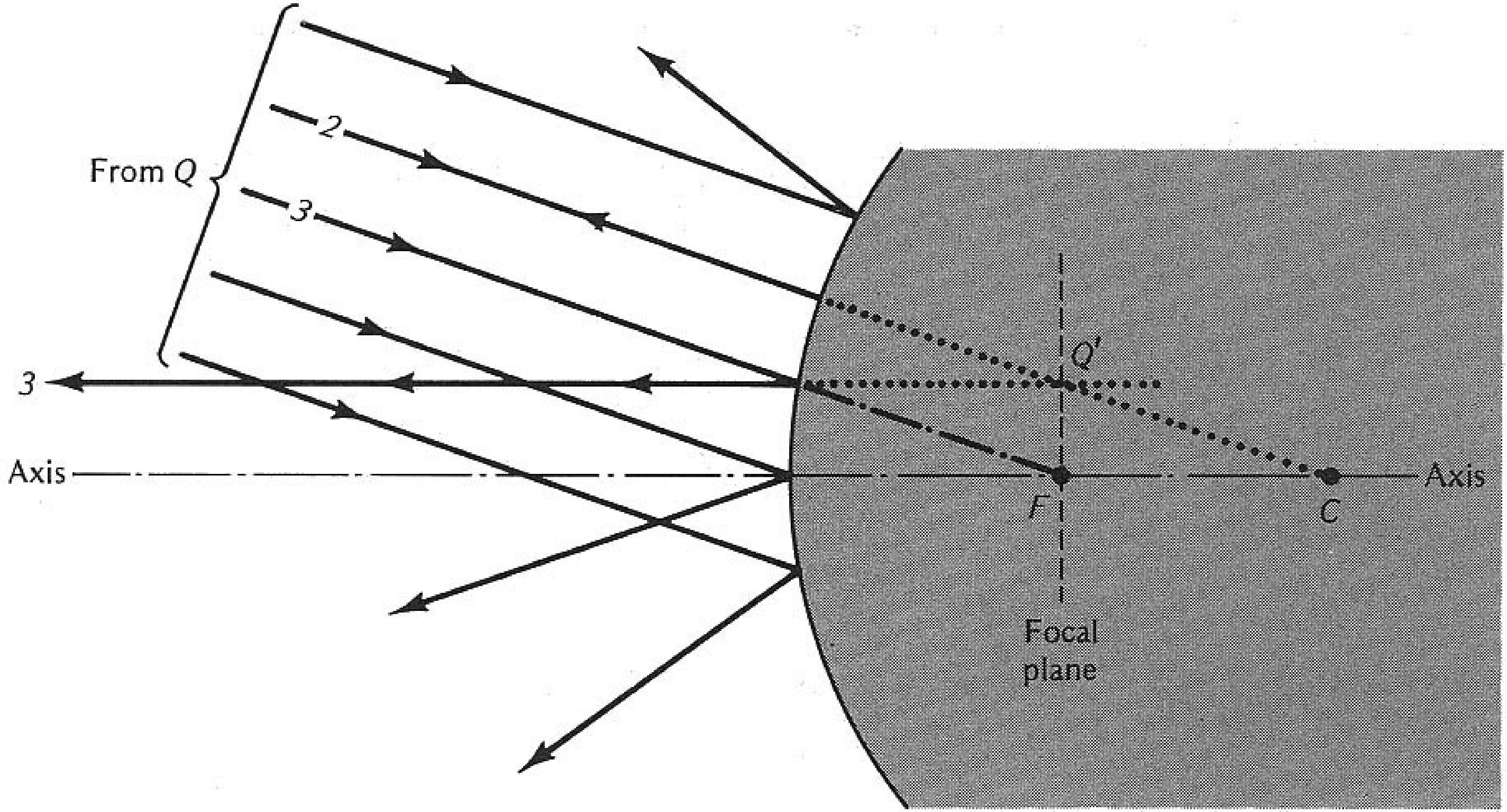


FIGURE 3.8

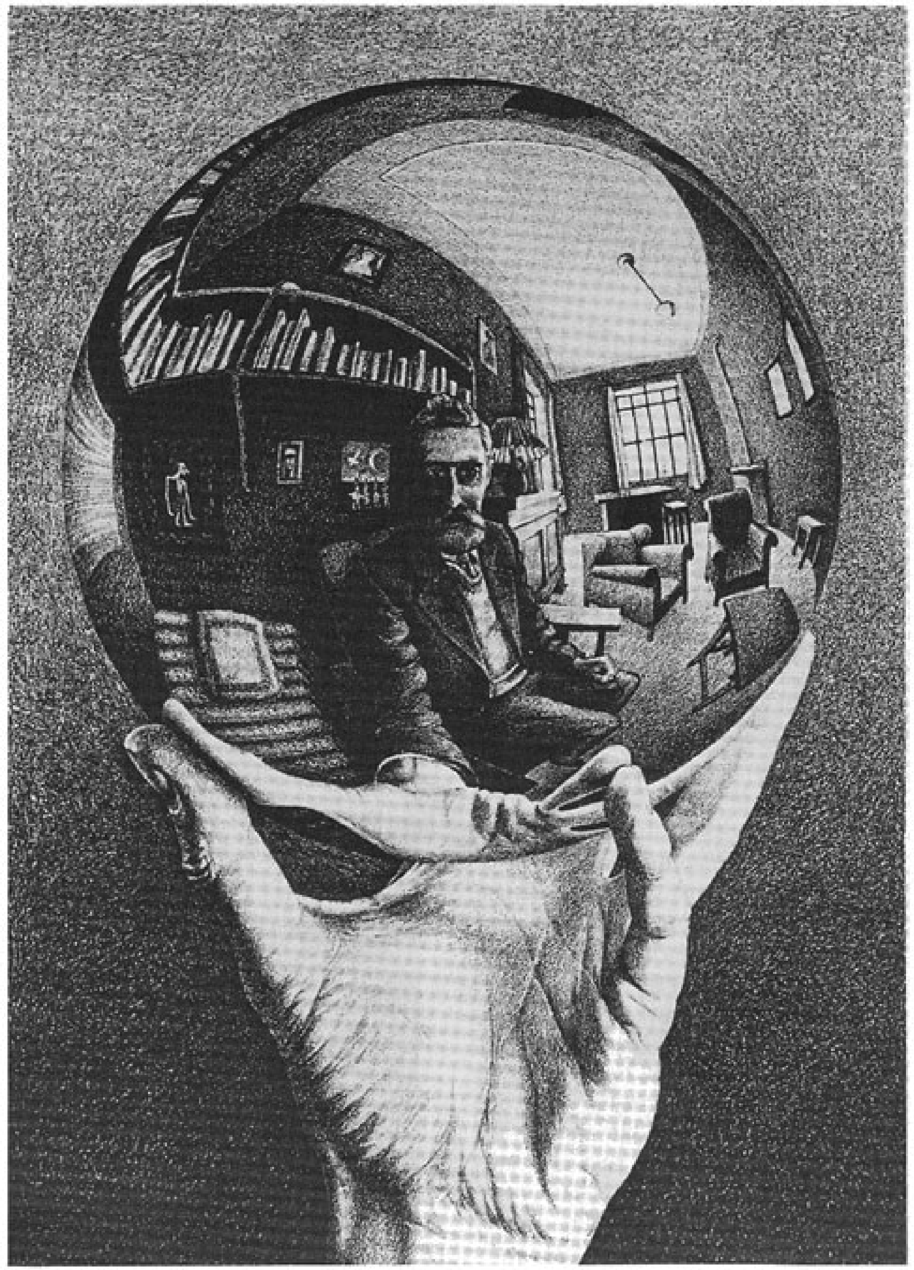
An object Q, too distant to be shown, sends parallel rays to a convex mirror. Its image, Q′, lies in the mirror's focal plane.

Reflection from sphere

- **Escher drawing of images from convex sphere**

FIGURE 3.9

M. C. Escher, "Hand with Reflecting Globe."



- **Anamorphic mirror and image**

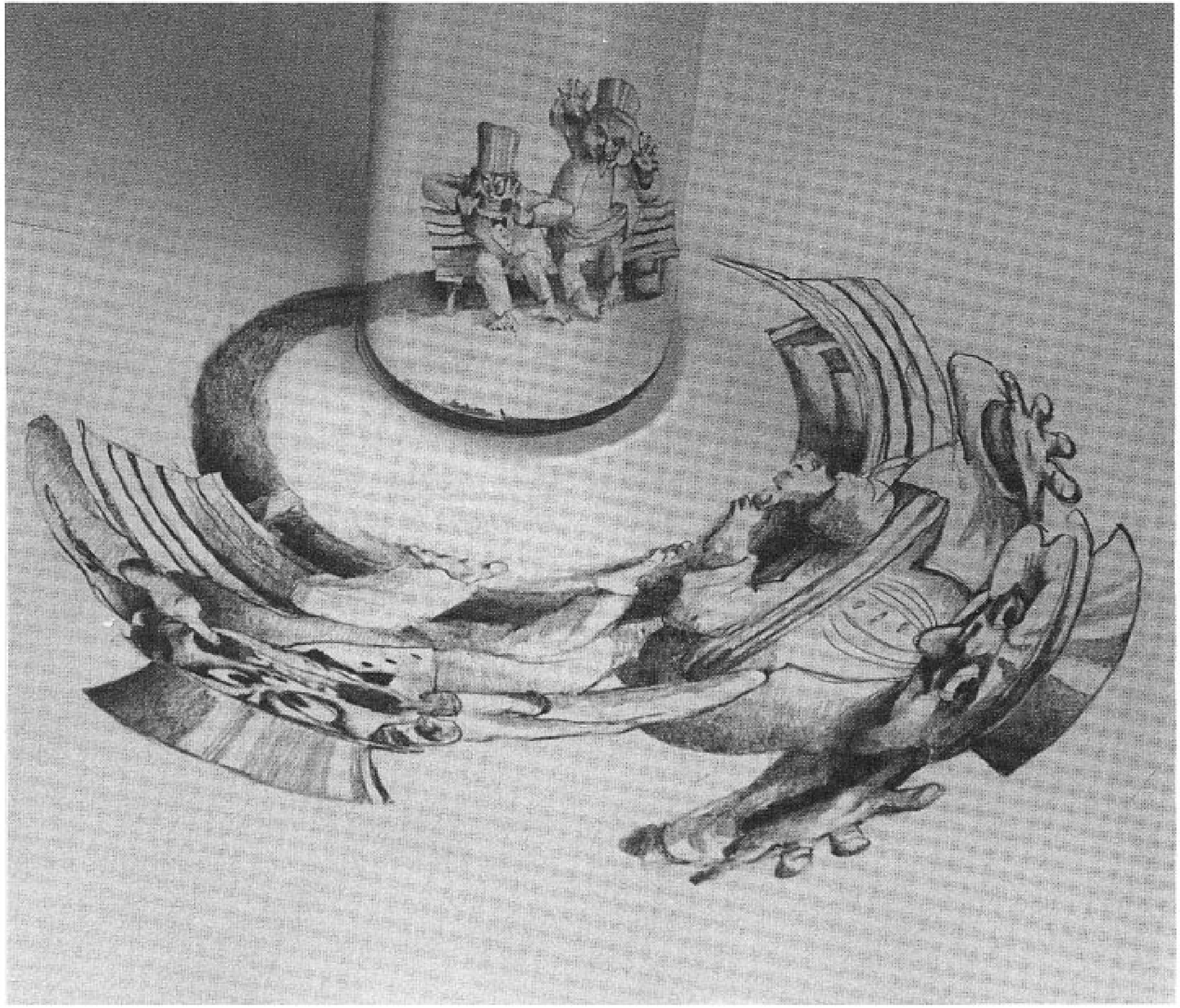


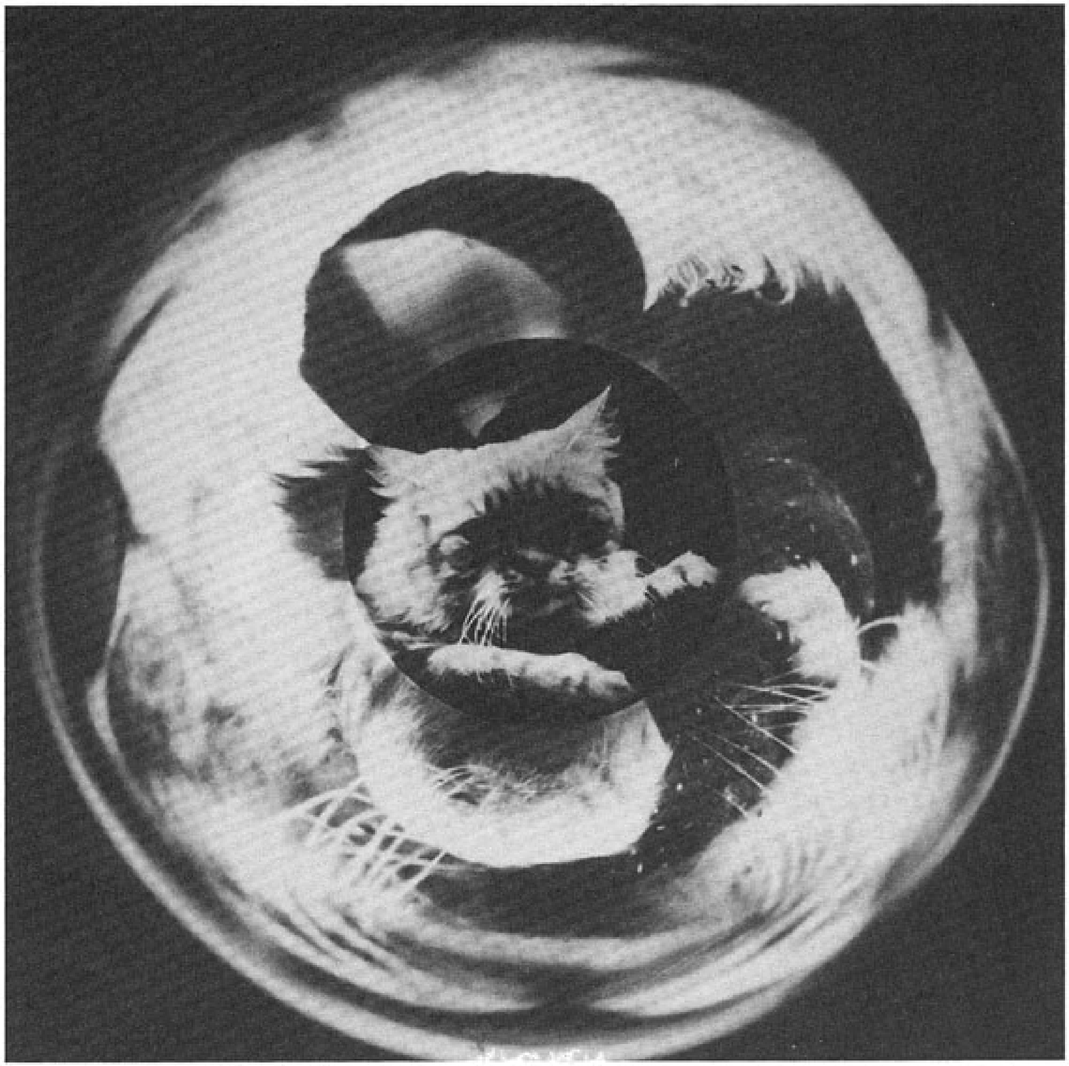
FIGURE 3.10

Cylindrical anamorph, with mirror that reconstructs the image.

- **Anamorphic mirror (conical)**

FIGURE 3.11

A conical anamorphic photograph, made by the method of the TRY IT. The conical mirror in the center reconstructs the undistorted image from the anamorph surrounding it, so you see the cat's head in the central circle surrounded by the deformed image.



- **The artist Hans Holbein made anamorphic paintings**

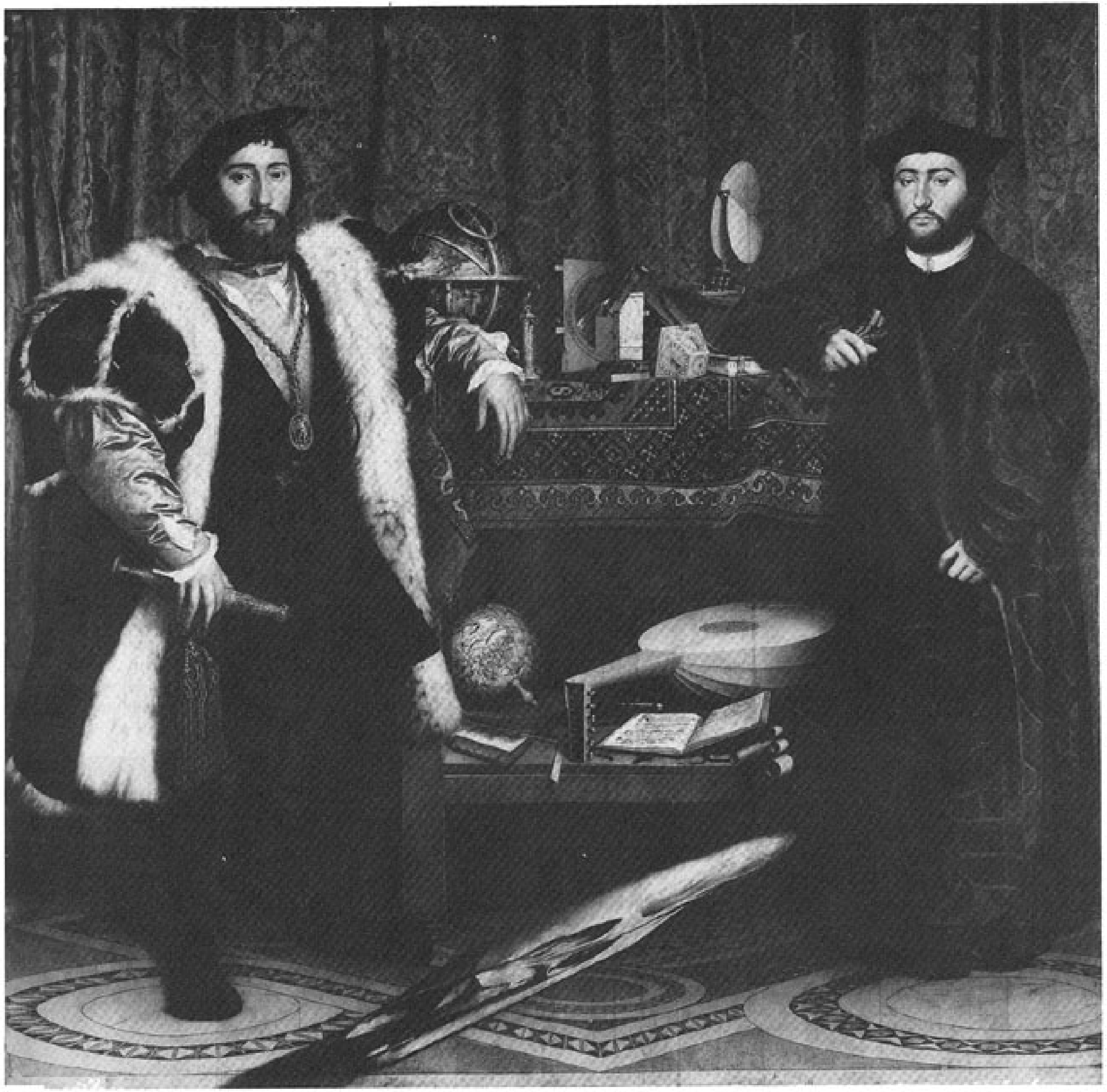


FIGURE 3.12

Hans Holbein's "The Ambassadors." View the streak across the foreground of the picture from the upper right. (Reproduced by courtesy of the Trustees, The National Gallery, London.)

Ray rules for concave mirrors

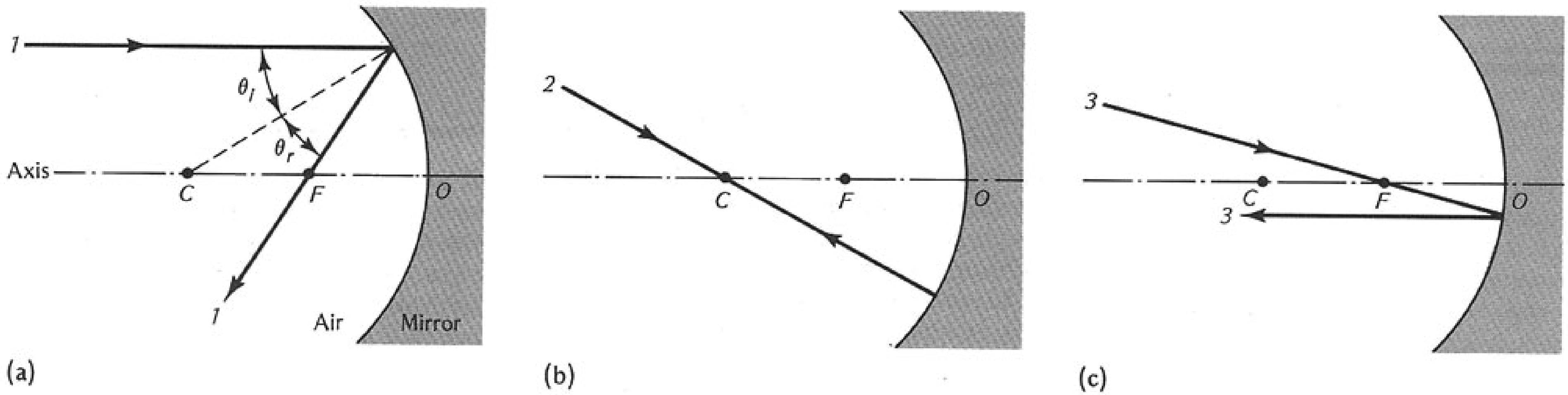


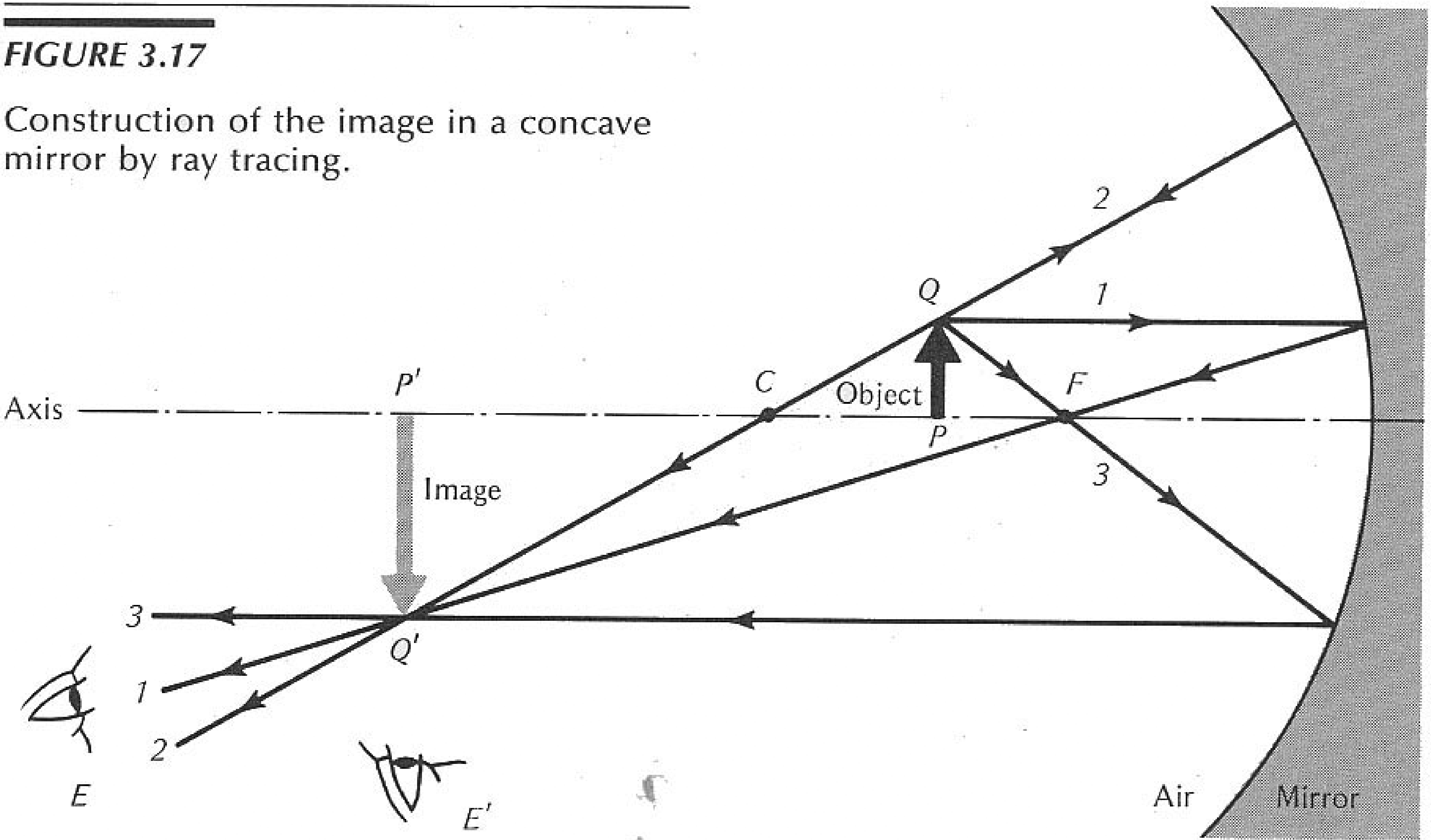
FIGURE 3.16

Three rays obeying their respective rules, for a concave mirror.

Image from concave mirror

FIGURE 3.17

Construction of the image in a concave mirror by ray tracing.



Reflections get complex

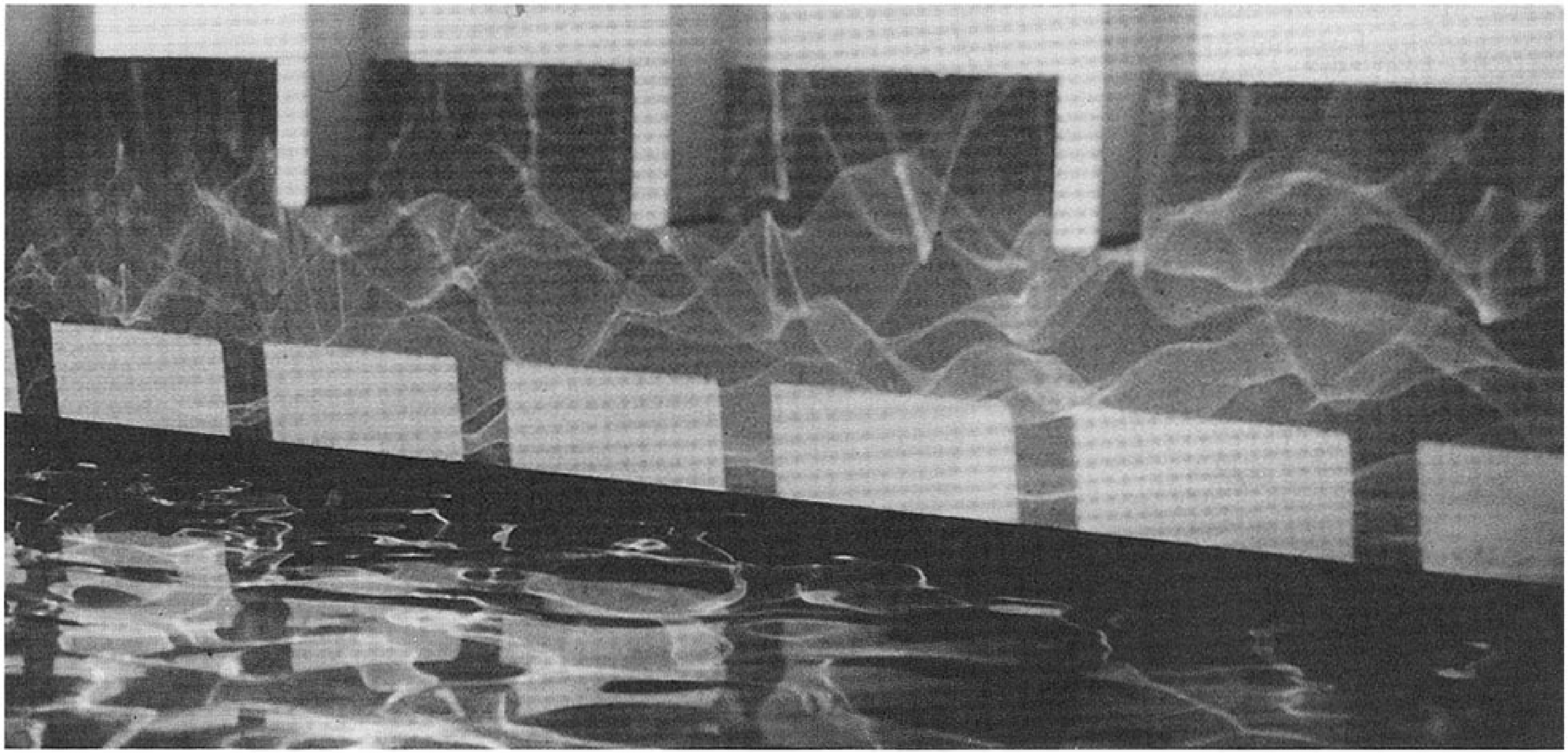


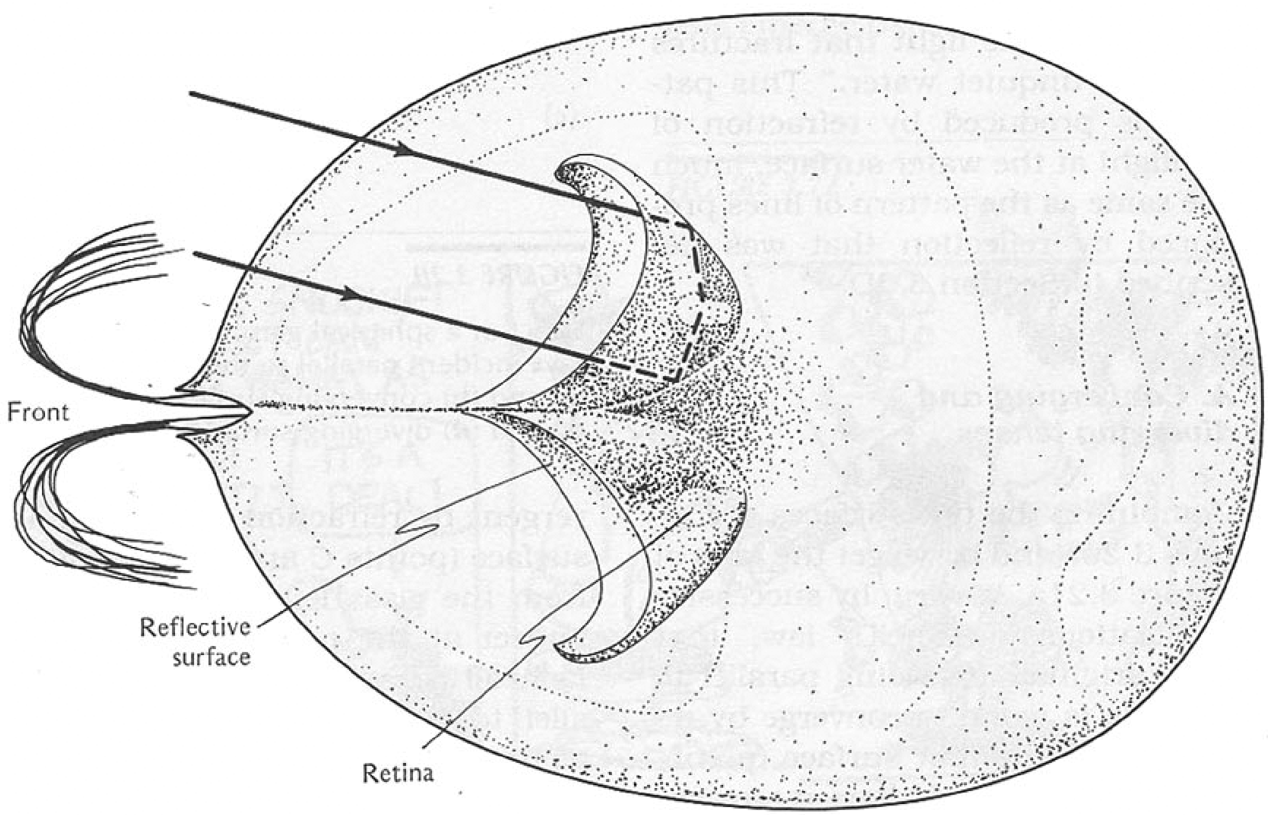
FIGURE 3.18

Rippling lines of light reflected and focused by the uneven surface of the water. Lines can be seen near the surface of the water as well as on the wall, in the shadow of the fence.

Mirror eyes in a plankton

FIGURE 3.19

Drawing of *Gigantocypris*. The dark curves are the reflecting eyes, with the retina in front of the reflector. They look through transparent windows in the carapace.



Constructing images with rays and mirrors

- **Paraxial rays are used**
  - These rays may only yield approximate results
  - The focal point for a spherical mirror is half way to the center of the sphere.

  - **Rule 1**: All rays incident parallel to the axis are reflected so that they appear to be coming from the focal point F.
  - **Rule 2**: All rays that (when extended) pass through C (the center of the sphere) are reflected back on themselves.
  - **Rule 3**: All rays that (when extended) pass through F are reflected back parallel to the axis.

  - **Parallel Rays Rule**: Rays parallel to each other are imaged to the same place on the focal plane

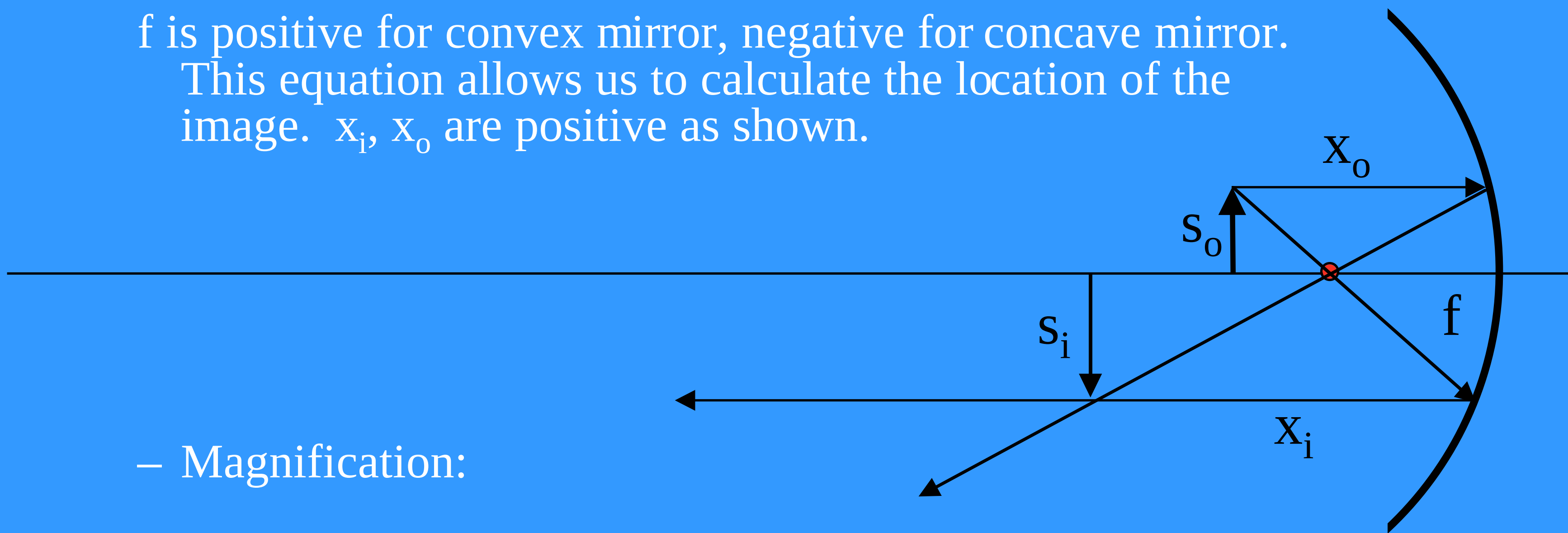
Spherical mirror images

- **Convex spherical mirrors**
  - image is virtual
  - focal length is half the radius of the mirror
  - image is closer to mirror
  - image is erect
  - makes a wide angle mirror
- **Concave spherical mirrors**
  - focal length is half the radius of the mirror
  - If object is further than the center of curvature:
    - image is real
    - image is closer to mirror
    - image is inverted
    - image is de magnified
  - If object is between the center of curvature and the focus:
    - image is real
    - image is further from mirror
    - image is inverted
    - image is magnified
  - If object is between the focus and mirror:
    - image is virtual
    - image is erect
    - image is magnified

Mirror Equation

$$\frac{1}{x_o} + \frac{1}{x_i} = -\frac{1}{f}$$

f is positive for convex mirror, negative for concave mirror. This equation allows us to calculate the location of the image. x_i, x_o are positive as shown.



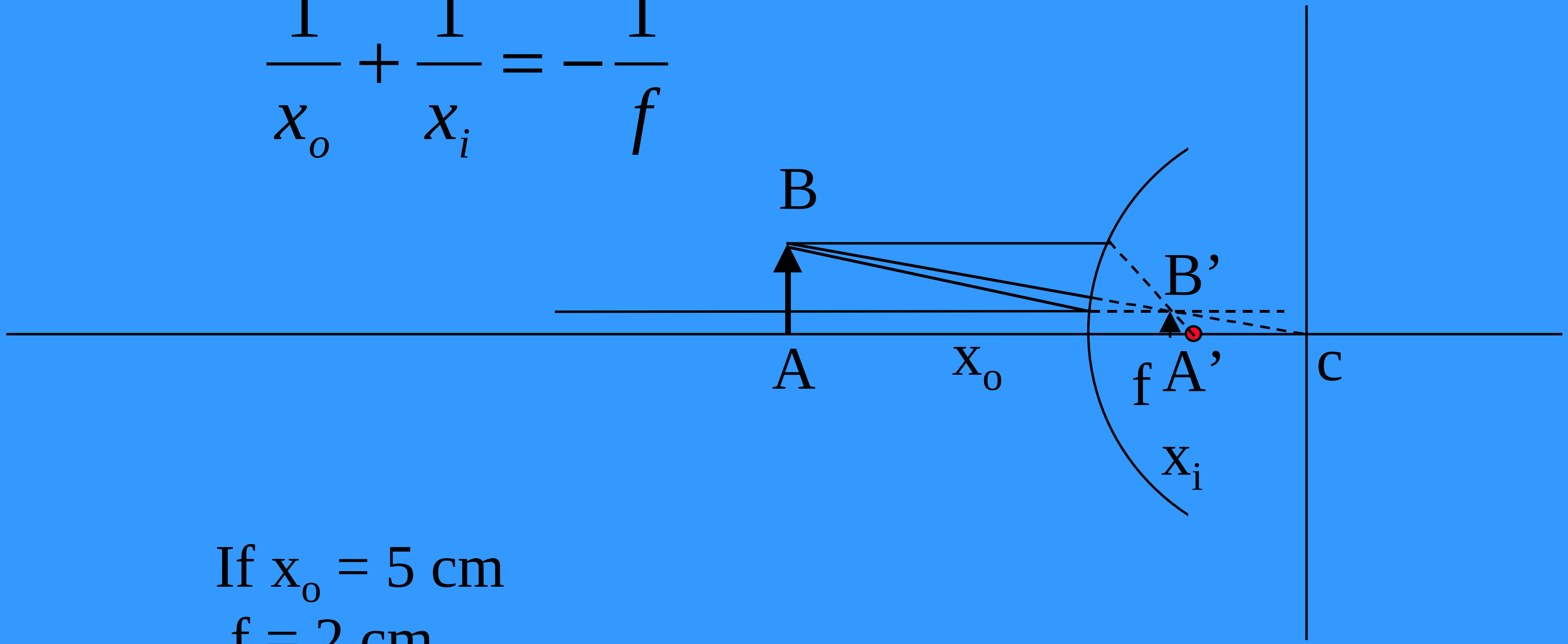
– Magnification:

$$\frac{s_i}{s_o} = -\frac{x_i}{x_o}$$

– This equation allows us to calculate the size of the image

Example: convex mirror

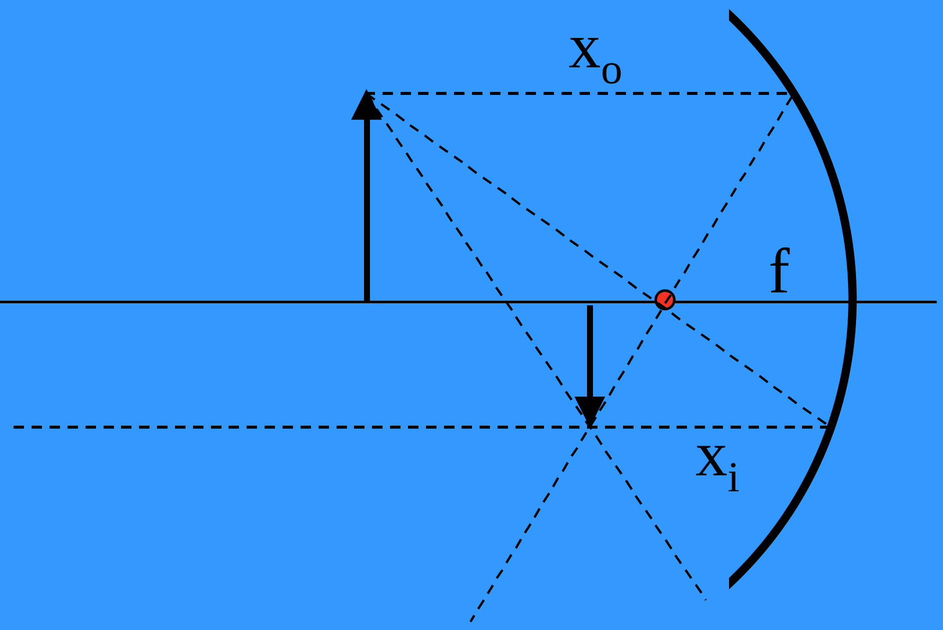
$$\frac{1}{x_o} + \frac{1}{x_i} = -\frac{1}{f}$$



If x_o = 5 cm
f = 2 cm
x_i = –1.43 cm

Example: Concave mirror

x_o = 10 cm
f = –3 cm
x_i = +4.29 cm



$$\frac{1}{x_o} + \frac{1}{x_i} = -\frac{1}{f}$$

Mirror Equation Examples

- **Given f and x_o find x_i**

  $1/x_i = -(1/f) - (1/x_o) = [(x_o+f)/x_of]$

  $x_i = -(x_of)/(x_o + f)$

- **Given x_o and x_i find f**

  $f = -(x_ox_i)/(x_o+x_i)$

- **Given f and x_i find x_o**

  $x_o = -(x_if)/(x_i + f)$

- **Example**

  x_o = 49 mm, f = – 30 mm (neg for concave mirror)

  x_i = 78 mm

  s_o = 20 mm

  s_i = – 32 mm (negative for inverted image)

Refraction at spherical surface

- Refracting properties of spherical lens surfaces
- (remember, light is “pulled” towards normal as it enters higher index material)
- Light is “pushed” away from normal as it enters lower index material

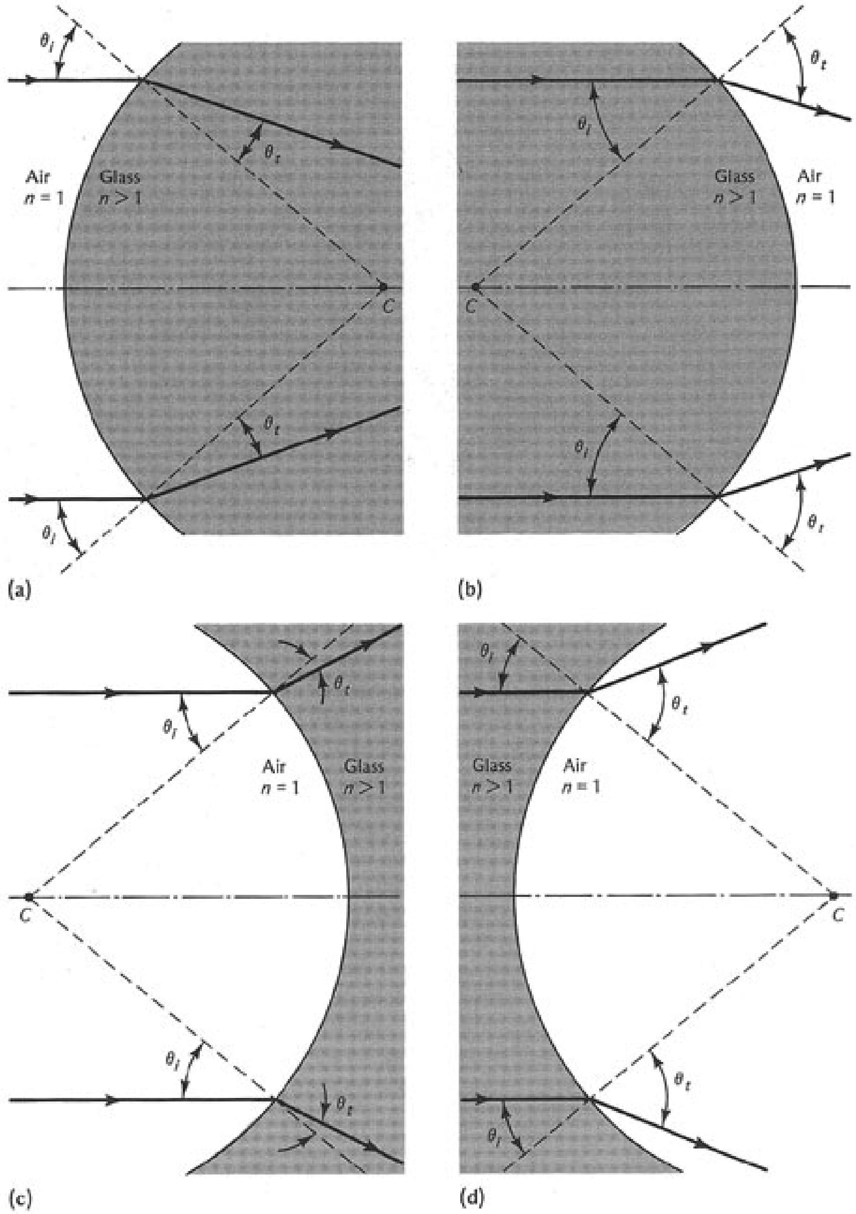


FIGURE 3.20

Effect of a spherical glass surface on light rays incident parallel to the axis:
(a) and **(b)** converging surfaces,
(c) and **(d)** diverging surfaces.

Converging Lenses

- Converging lenses can focus parallel light
- Converging lenses generate parallel light from a point source placed at the lens focal point
- Focal length f related to index n and surface radii r

$$\frac{1}{f} = 2\left(\frac{n-1}{r}\right)$$

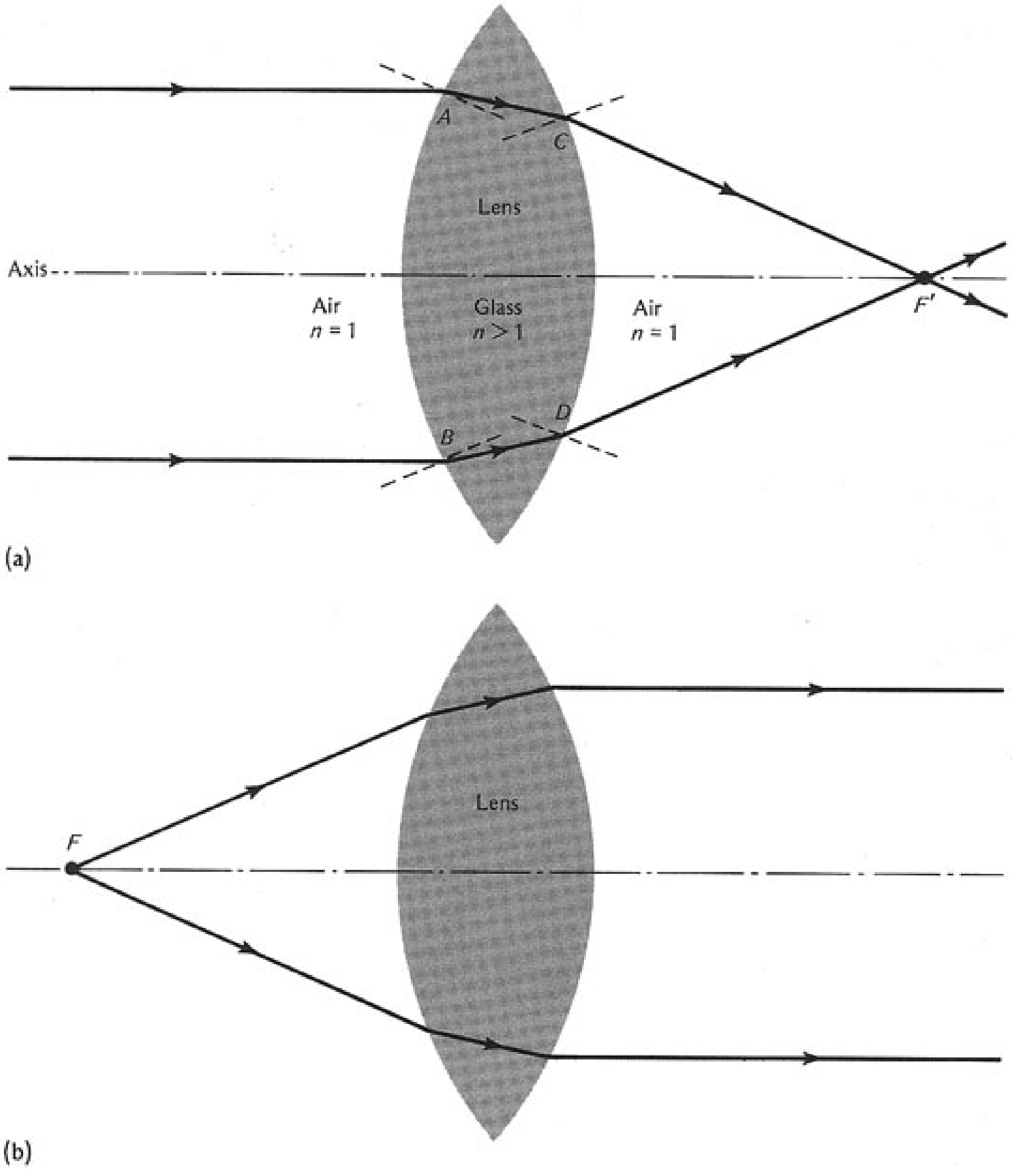


FIGURE 3.21

A converging lens, consisting of two converging surfaces. **(a)** Rays parallel to the axis are focused at F'. **(b)** Rays originating at F are also made to converge and emerge parallel to the axis.

Diverging lenses

- Often, doubly concave lenses
- Make parallel light diverge

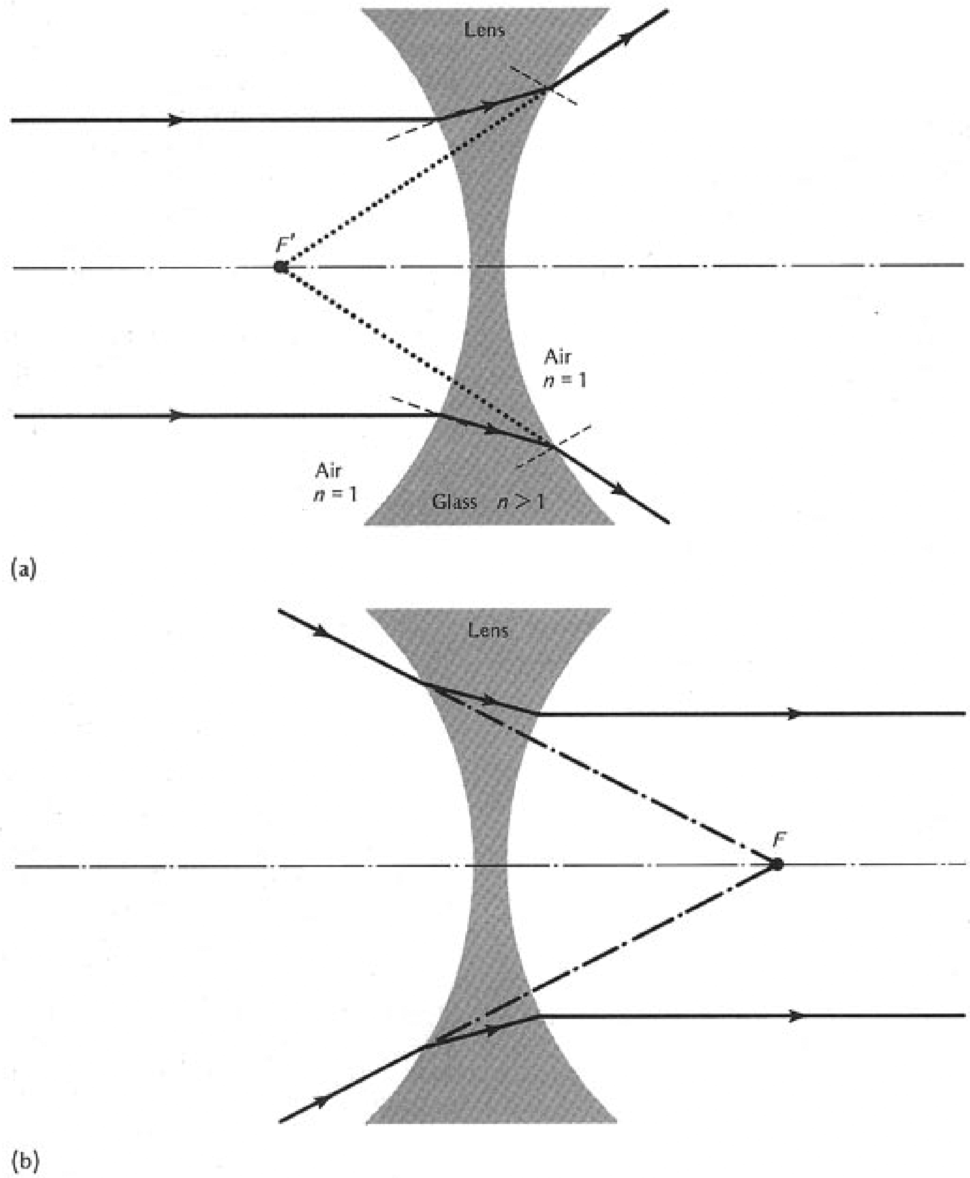


FIGURE 3.23

A diverging lens, consisting of two diverging surfaces. **(a)** Rays parallel to the axis seem to come from F' after they pass through the lens. **(b)** Rays converging toward F are also made to diverge and emerge parallel to the axis.

Pinholes and eye

FIGURE 3.24

Optics of an eye viewing through multiple pinholes. The lens of the eye creates one image from rays that passed through different pinholes.

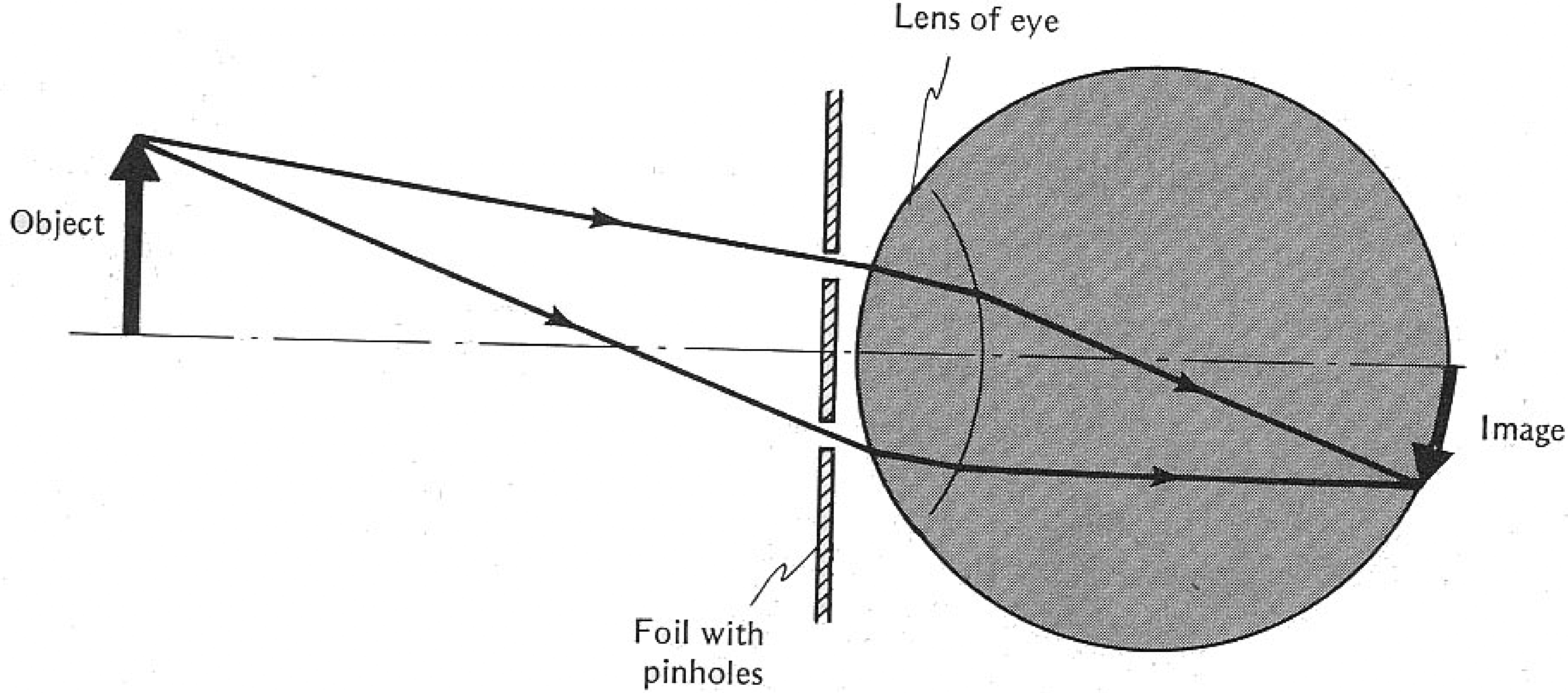
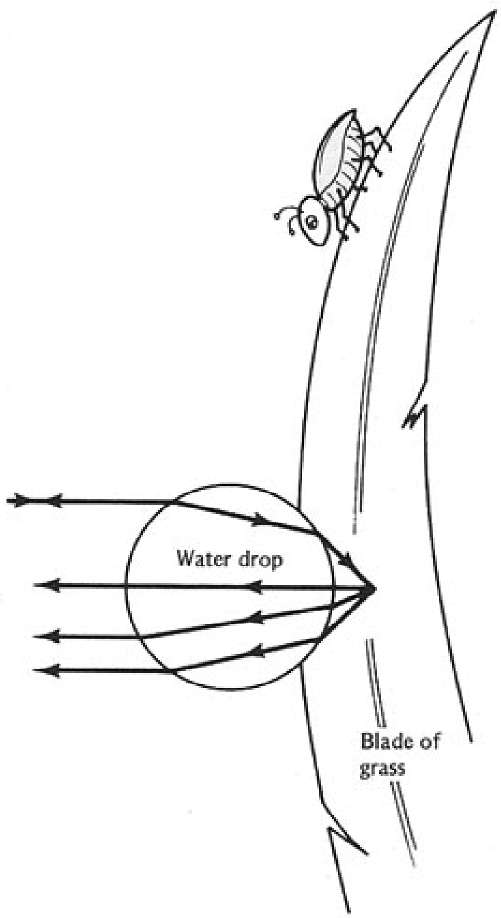


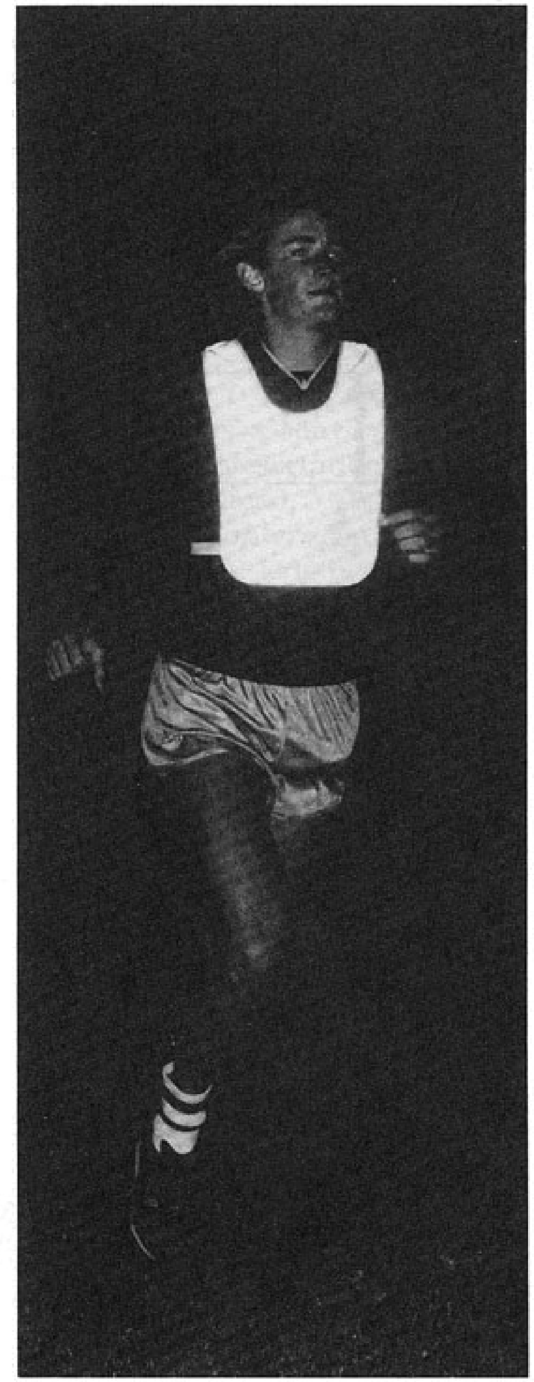
FIGURE 3.25

(**a**) Water droplet and blade of grass acting as a retroreflector. Only *one* incident ray is shown, and only a few of the many, diffusely reflected rays due to this one incident ray. (**b**) Dew heiligenschein around the shadow of the photographer's head. (**c**) Glass beads used as retroreflectors make this jogger's vest visible in car headlights.



(a)

(b)



(c)

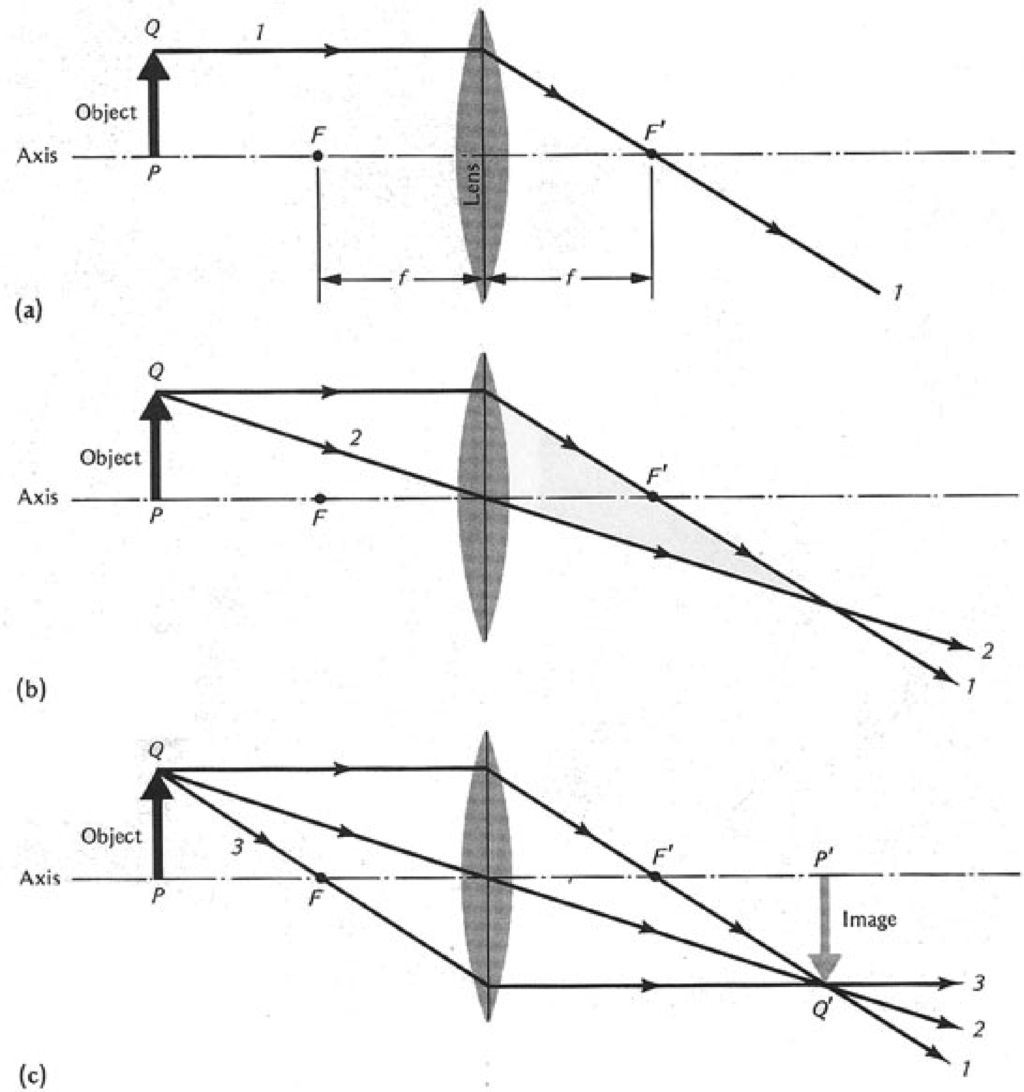
2003 A

lenses

- The three ray rules for constructing an image from a lens

FIGURE 3.26

Three stages of construction by ray tracing of the image formed by a converging lens. The lens is shown relatively thick so that you can see it but treated as if it were just the vertical plane through its center.



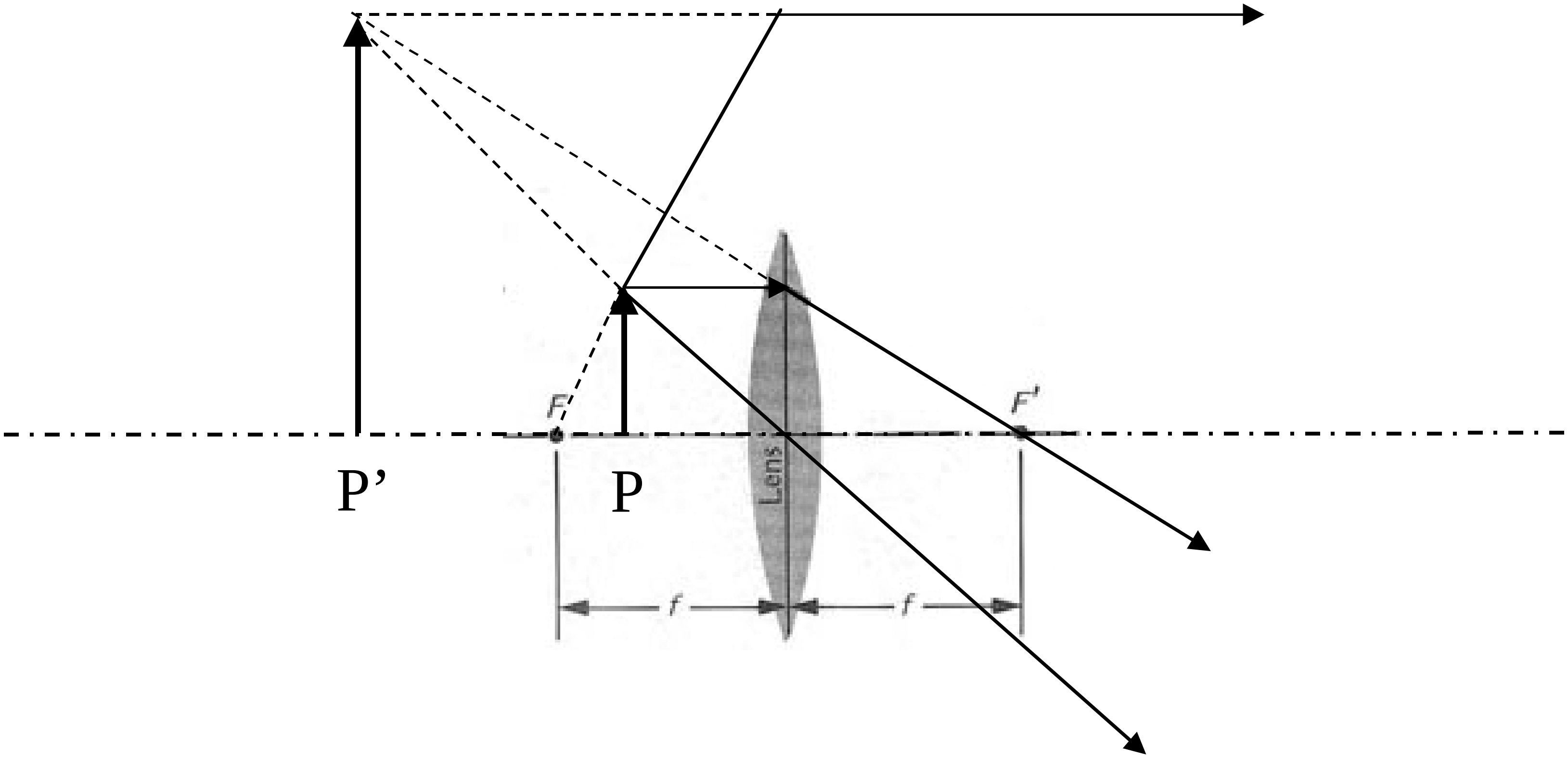
Constructing images with rays and lenses

- **Paraxial rays are used**
  – These rays may only yield approximate results
  – Thin lenses only
- **focal length is positive for converging lens**
- **focal length is negative for diverging lens**

  – **Rule 1:** A ray parallel to the axis is deflected through F' (or as if it came from F')
  – **Rule 2:** A ray through the center of the lens continues undeviated
  – **Rule 3:** A ray to the lens that (when extended, if necessary) passes through F is deflected parallel to the axis

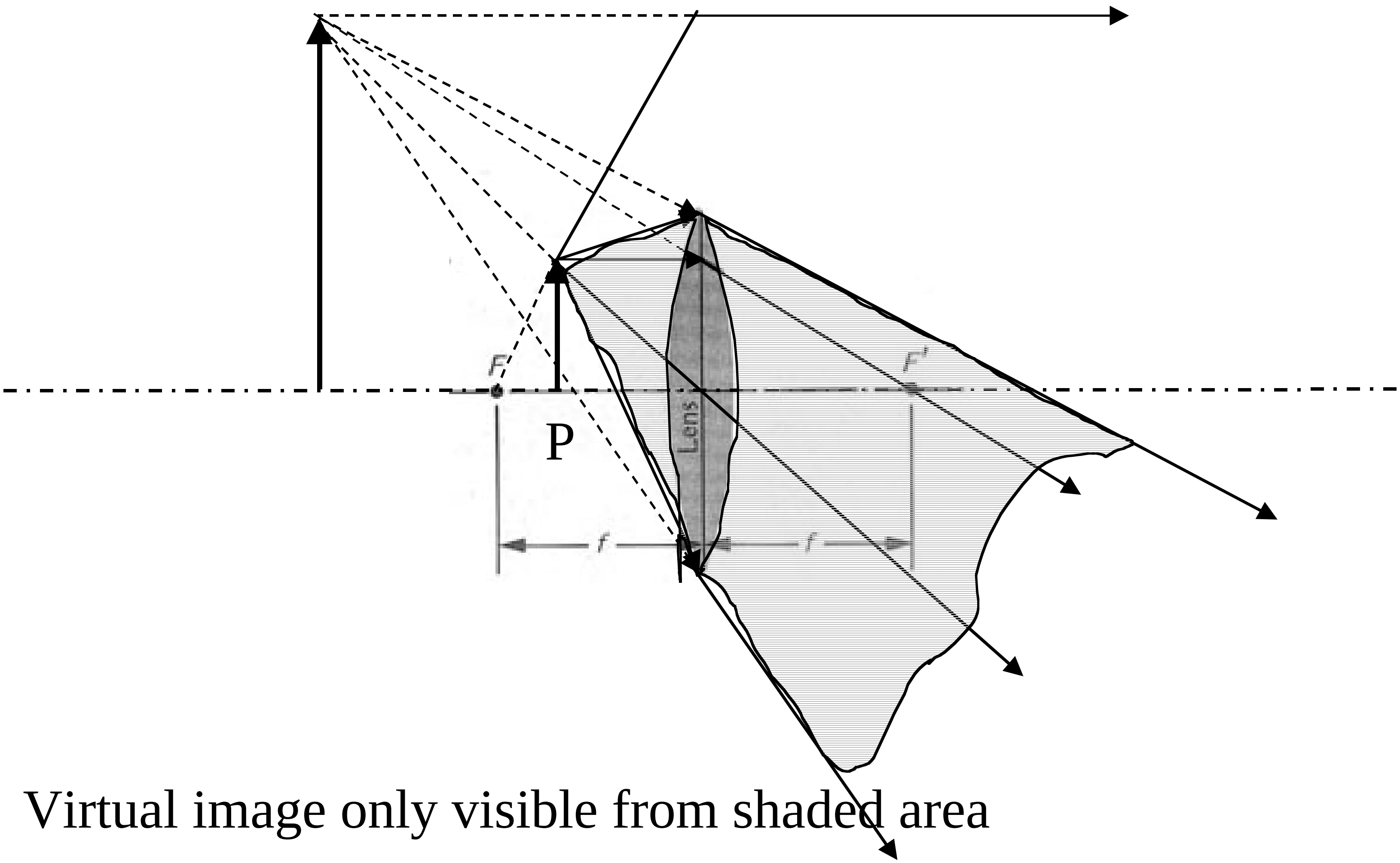
  – **Parallel Rays Rule:** Rays parallel to each other are imaged onto the same point in the focal plane

Ray tracing in a converging lens



Here, image P’ is virtual, erect and larger than the object P

Visibility in a converging lens



Virtual image only visible from shaded area

visibility

- a) showing how lens is extended for construction and region of visibility

- b) parallel light focussed onto focal plane and region of visibility

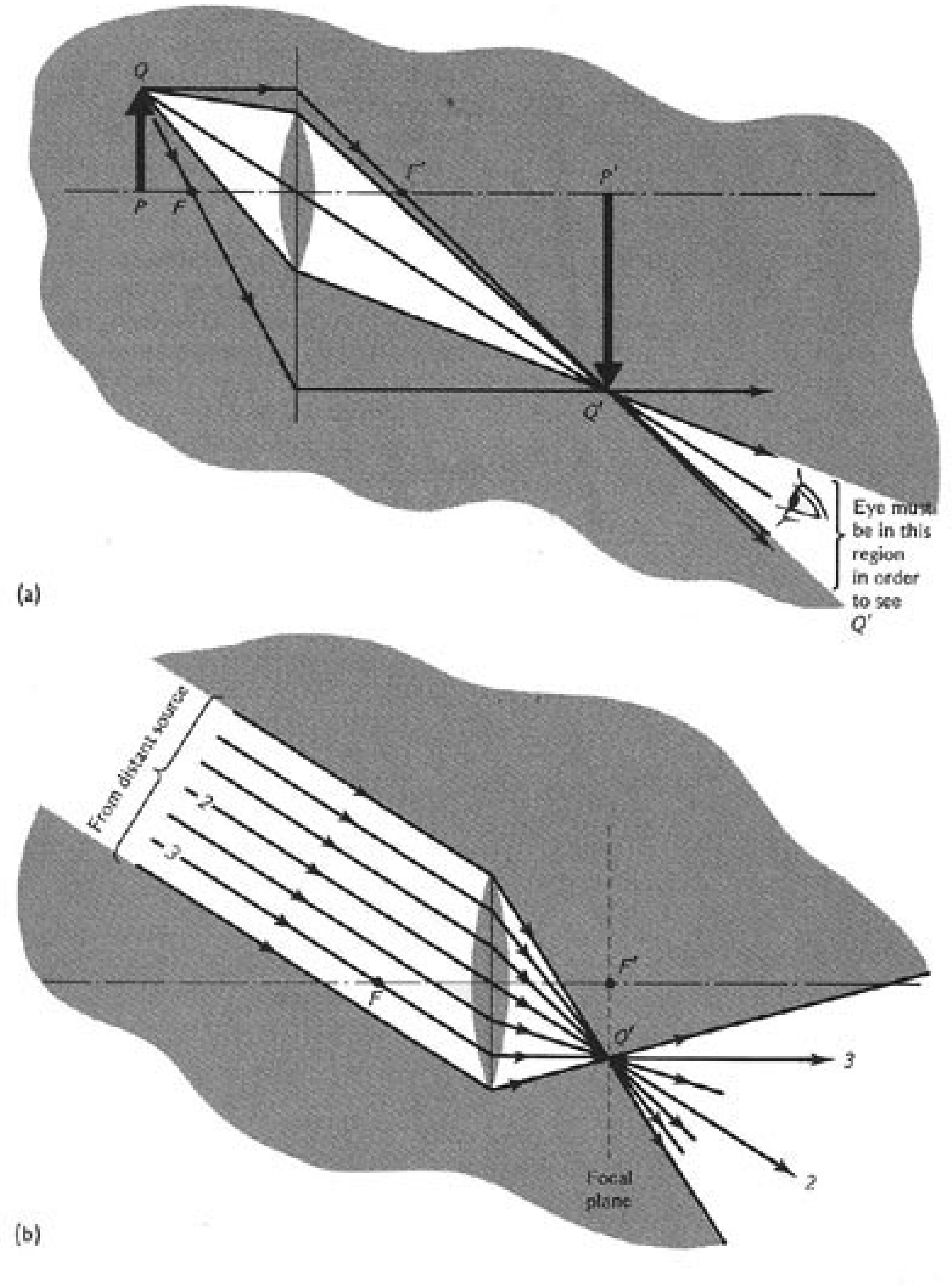


FIGURE 3.27

Ray tracing a diverging lens

- Draw three “standard” rays

- These will intersect at image

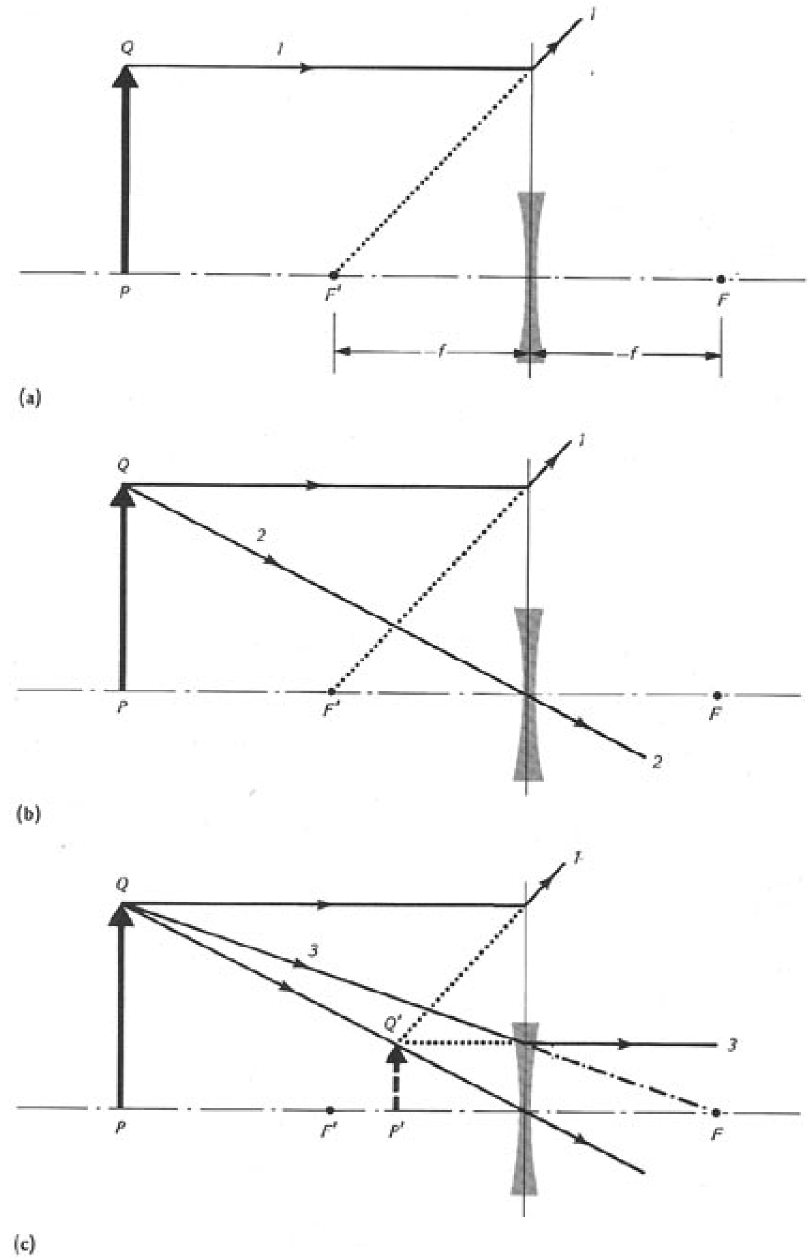
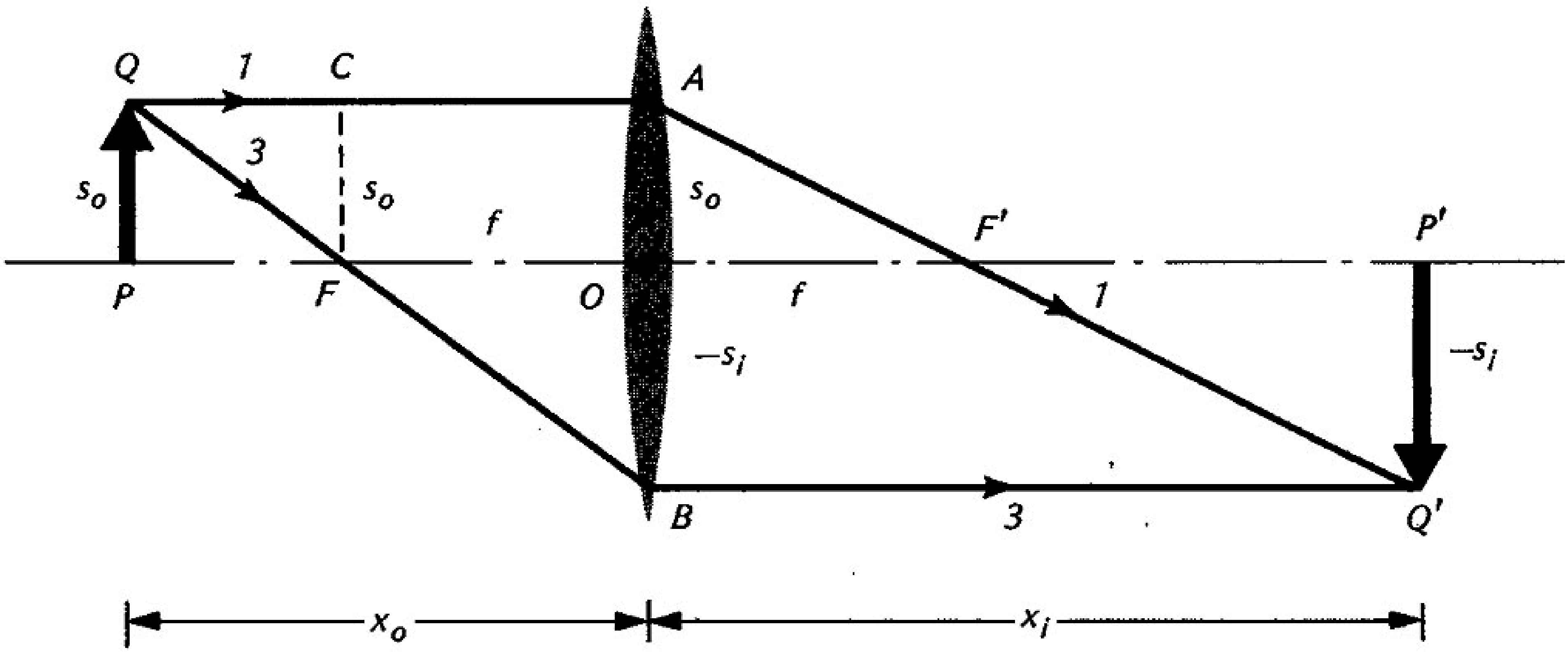


FIGURE 3.28

Three stages of construction, by ray tracing, of the image formed by a diverging lens.

Converging lens and sign conventions



Lens equation

$$\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f}$$

- f is positive for converging lens, negative for diverging lens. This allows us to calculate location of the image

$$\frac{s_i}{s_o} = -\frac{x_i}{x_o}$$

- This equation allows us to calculate the size of the image (magnification)

Lens Equation Examples

- **Given f and x_o find x_i**

  $1/x_i = (1/f) - (1/x_o) = [(x_o - f)/x_o f]$

  $x_i = (x_o f)/(x_o - f)$

- **Given x_o and x_i find f**

  $f = (x_o x_i)/(x_o + x_i)$

- **Given f and x_i find x_o**

  $x_o = (x_i f)/(x_i - f)$

- **Example**

  $x_o = 49$ mm, $f = 30$ mm (positive for converging lens)

  $x_i = 77$ mm

  $s_o = 20$ mm

  $s_i = -32$ mm (negative for inverted image)

- **Constructing imaging from multiple lenses**
  - Construct image from 1st lens
  - Add rays that will be useful for the 2nd lens construction (the 3 rays)
  - Complete ray tracing with these 3 rays through the 2nd lens to find final image

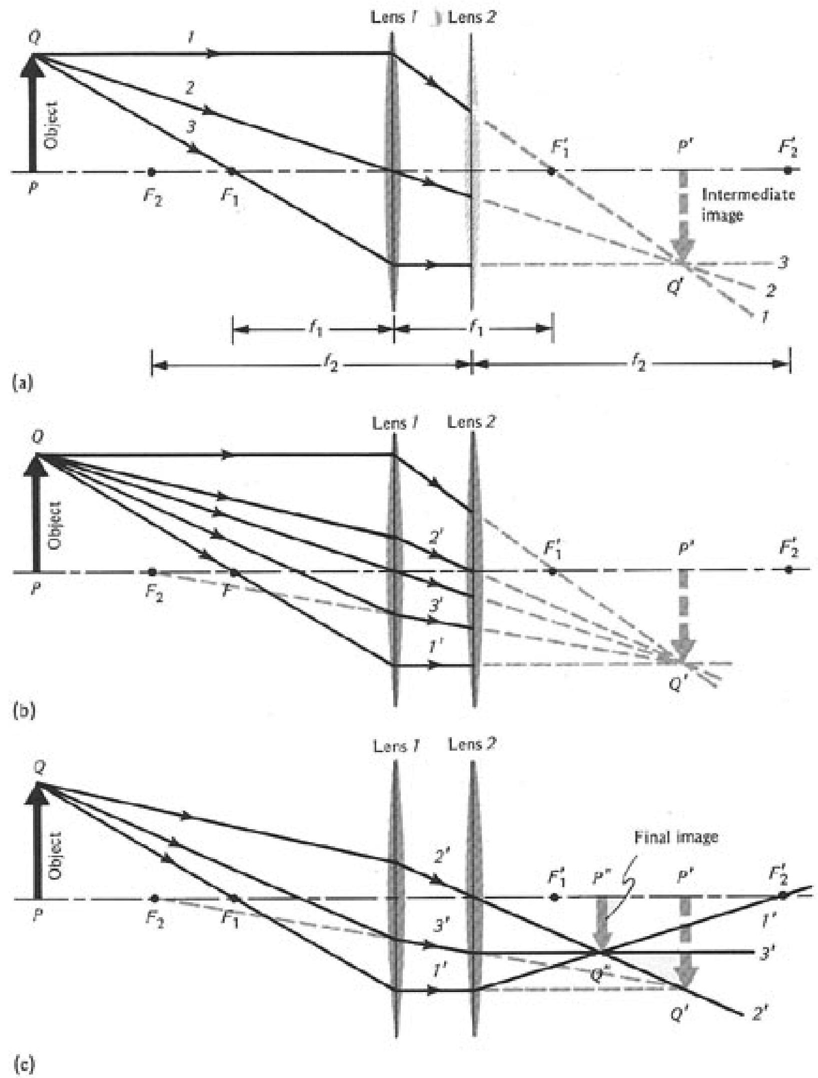


FIGURE 3.32

Construction by ray tracing of the final image of a compound lens consisting of two converging lenses. We have indicated all the focal points and the focal length of each lens. **(a)** Construct the intermediate image, due to the first lens alone. **(b)** Find the three rays incident on lens *2* that are needed for constructing the image *it* forms. **(c)** Use ray tracing rules on lens *2* to find the final image.

Power of lenses

- **The power of a lens is its inverse focal length (how strongly it can focus parallel light)**
  – P = 1/f
- **By convention, the units of power are measured in diopters**
  – It is numerically equal to the inverse of the focal length of the lens, measured in meters.
  – Examples:
    - A converging lens with a 1m focal length has a power of 1D
    - A lens with a power of -5D is a diverging lens with a focal length of -0.2m
    - A doubly concave lens (diverging) with a focal length of - 2m has a power of -0.5D
- **The powers of adjacent lenses add to form net power**

Fresnel lens principle

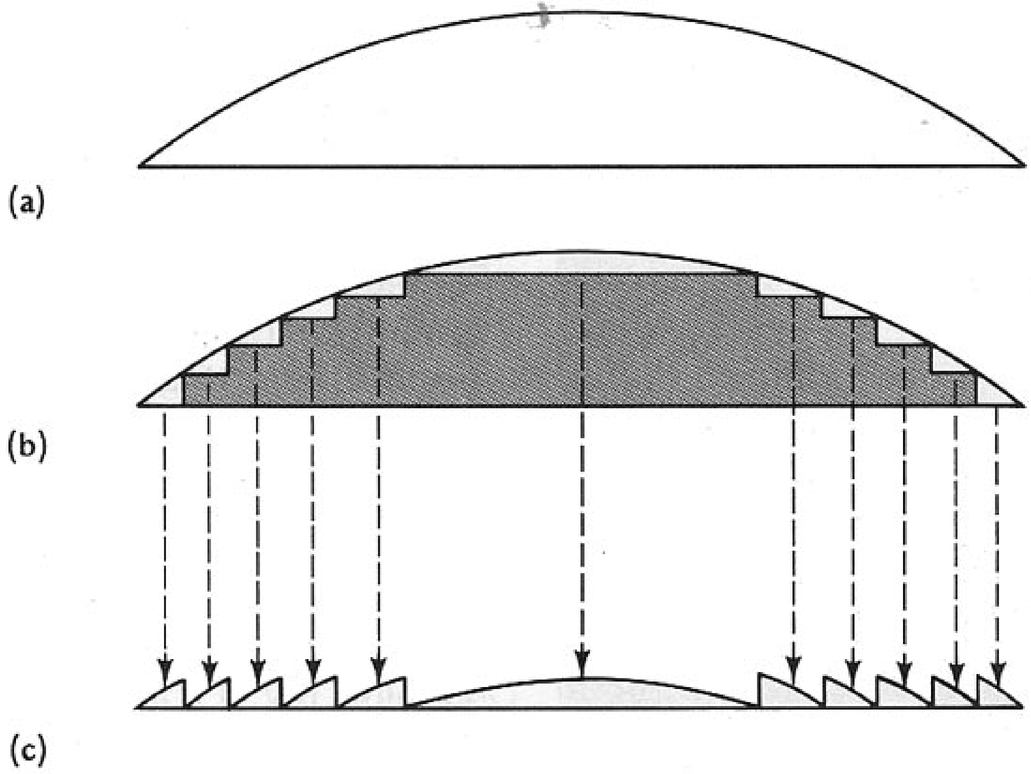
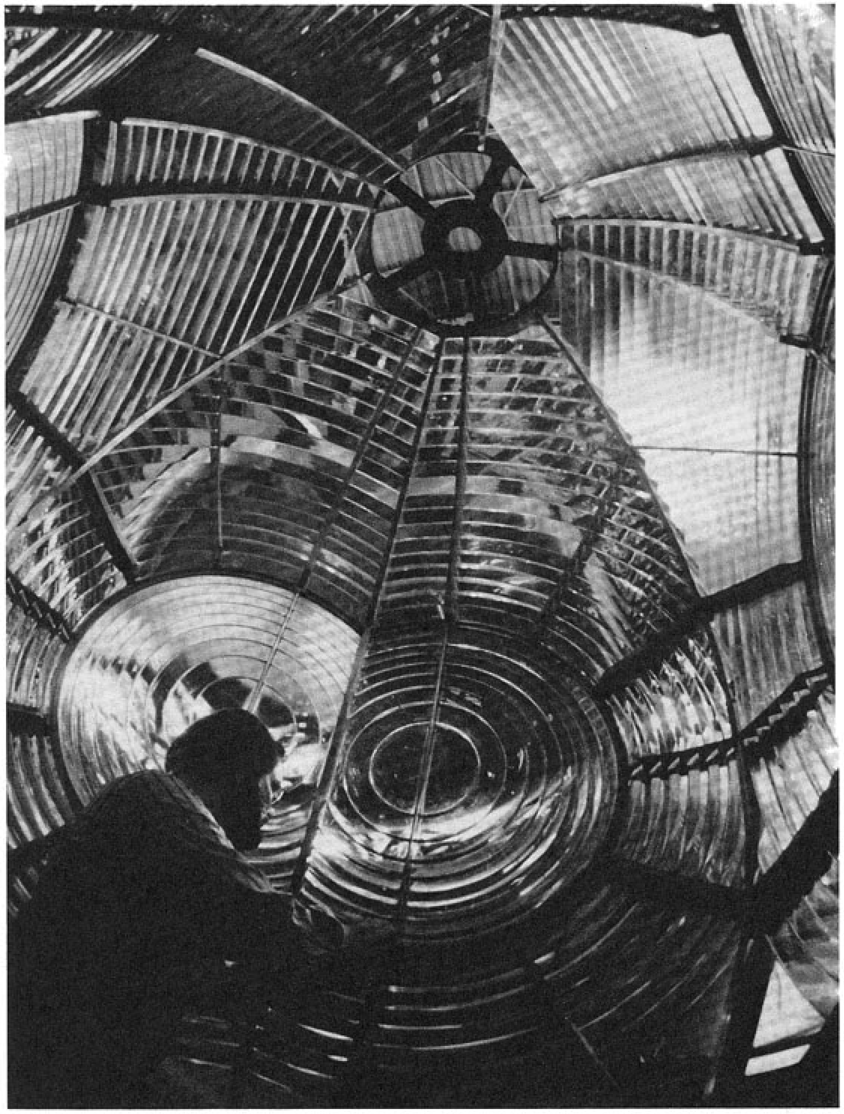


FIGURE 3.29

(a) A thick, converging lens (with one flat side). **(b)** Parts of the glass of this lens that have no essential effect on the bending of light (shown shaded). Remember that these sections are really rings oriented perpendicular to the paper. **(c)** After removing the nonessential glass and rearranging the lens, you get a Fresnel lens.

Fresnel lens applications



(a)

FIGURE 3.30

(a) Photograph of a Fresnel lens designed for use where Fresnel first intended: in a lighthouse. **(b)** Photograph of a Fresnel spotlight.

(b)

Traffic light

- **Fresnel lens images scene onto ground glass screen at ~ focal distance**
- **Light source illuminates ground glass**
- **Mask the screen to block light that would go to undesired locations**

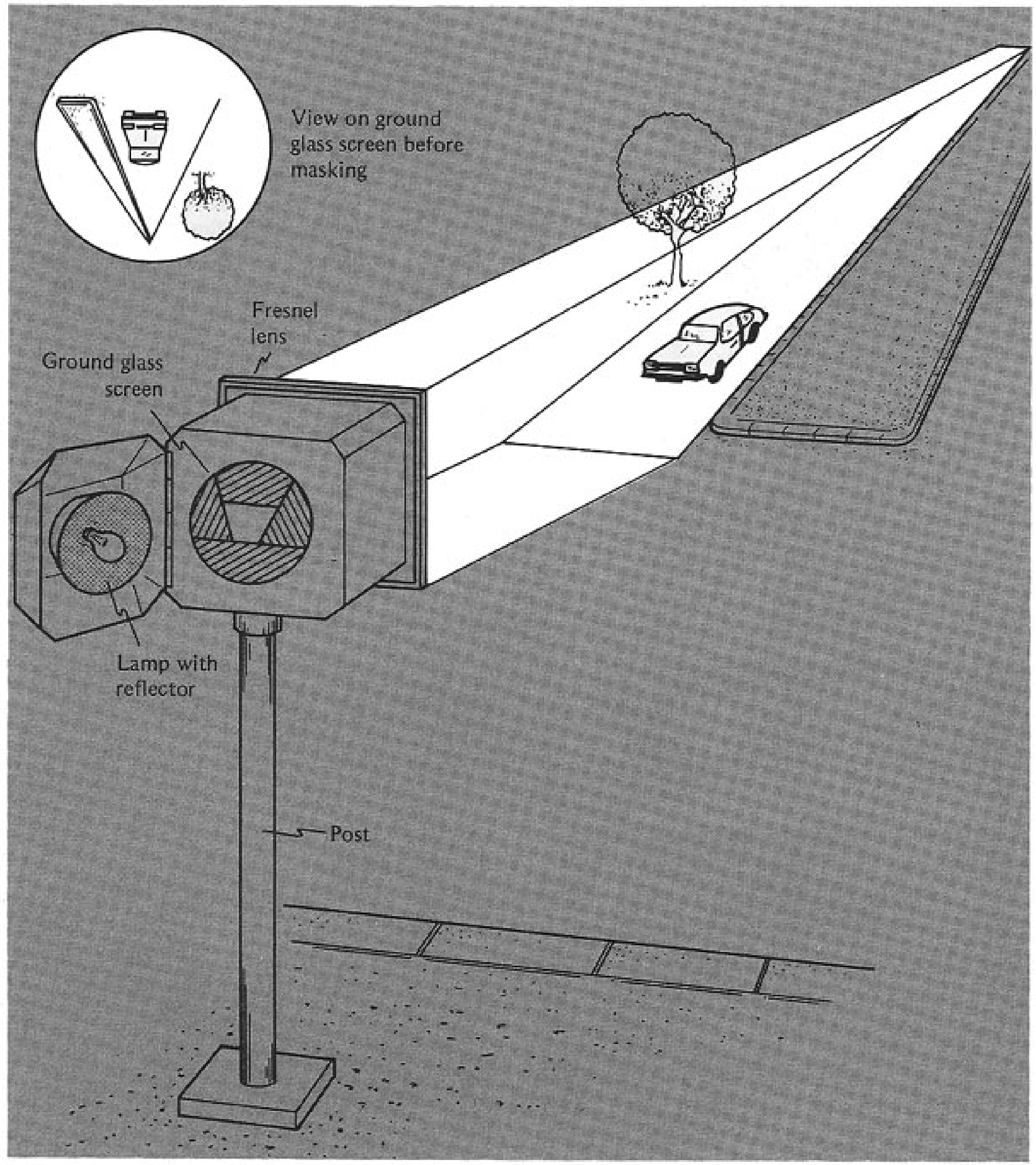


FIGURE 3.31

An "optically programmed" traffic light.

Aberrations

- **Chromatic Aberration**
- **Spherical Aberration**
- **Field angle effects (off-axis aberrations)**
  - Field curvature
  - Coma
  - Astigmatism
  - Distortion

Chromatic aberration and doublets

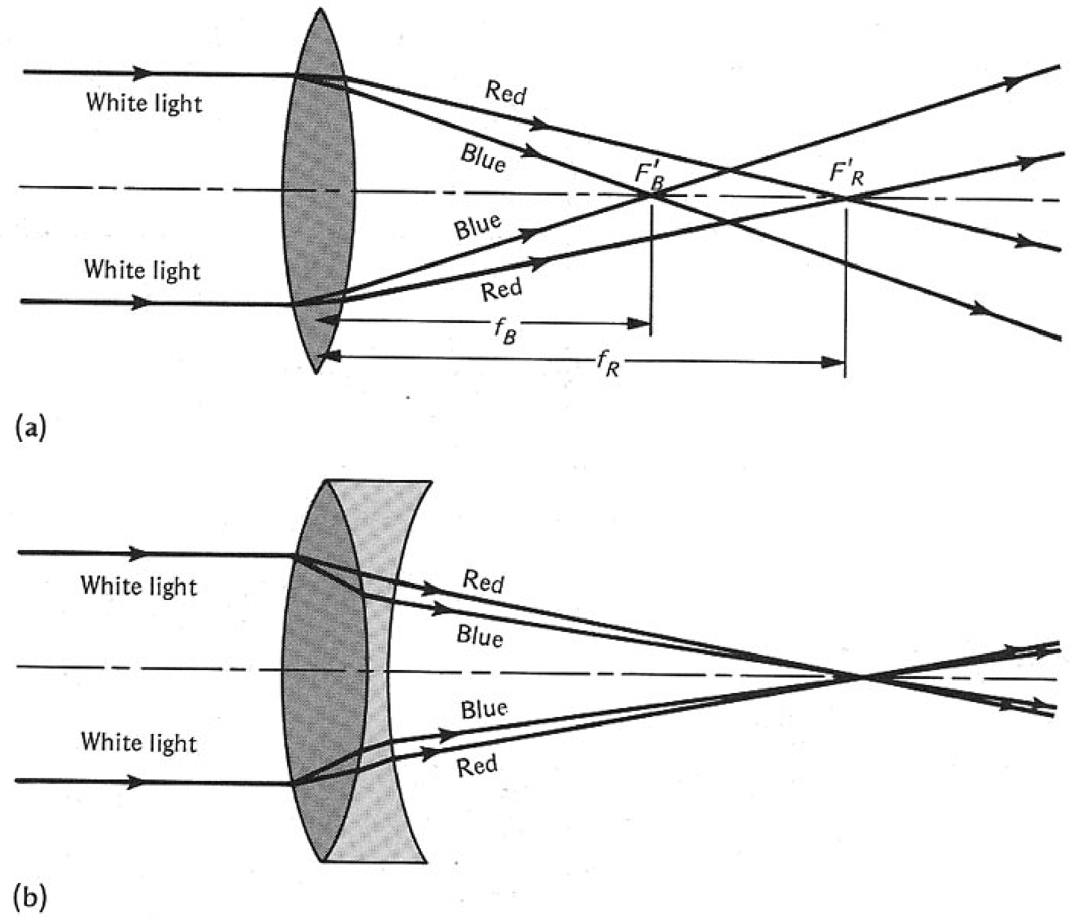


FIGURE 3.33

(a) Chromatic aberration of a converging lens. **(b)** Elimination of this aberration by an achromatic doublet.

Spherical aberration from a lens

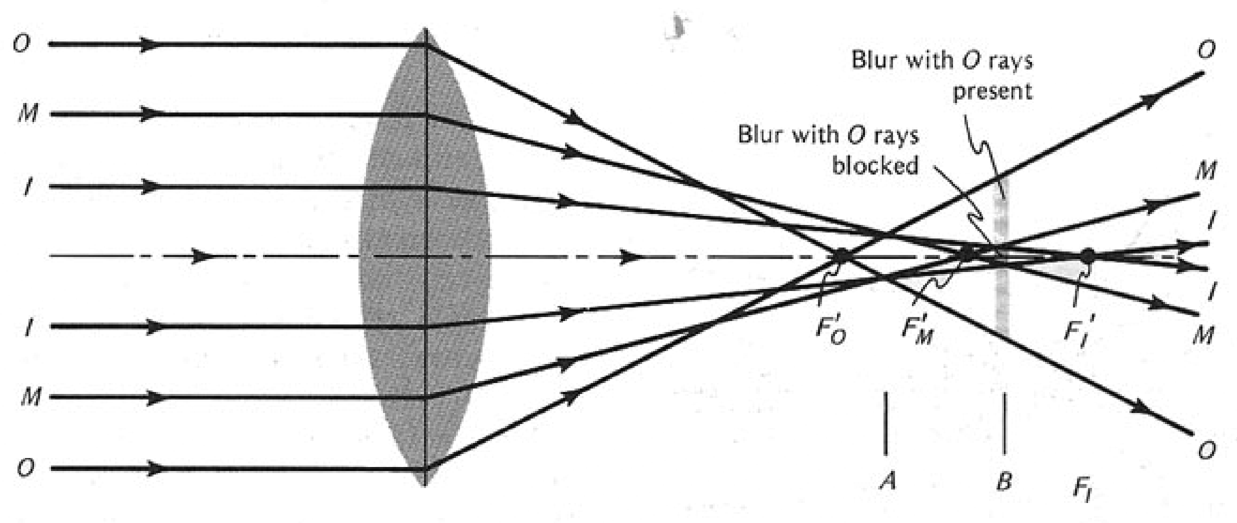


FIGURE 3.34

Spherical aberration in a converging lens. The inner rays, *I*, closest to the axis, have the farthest focal point, F_I'. The outer rays, *O*, are bent most, so they have the nearest focal point, F_O'. The rays in the middle, *M*, are focused between these extremes, at F_M'.

Spherical aberration from a concave mirror

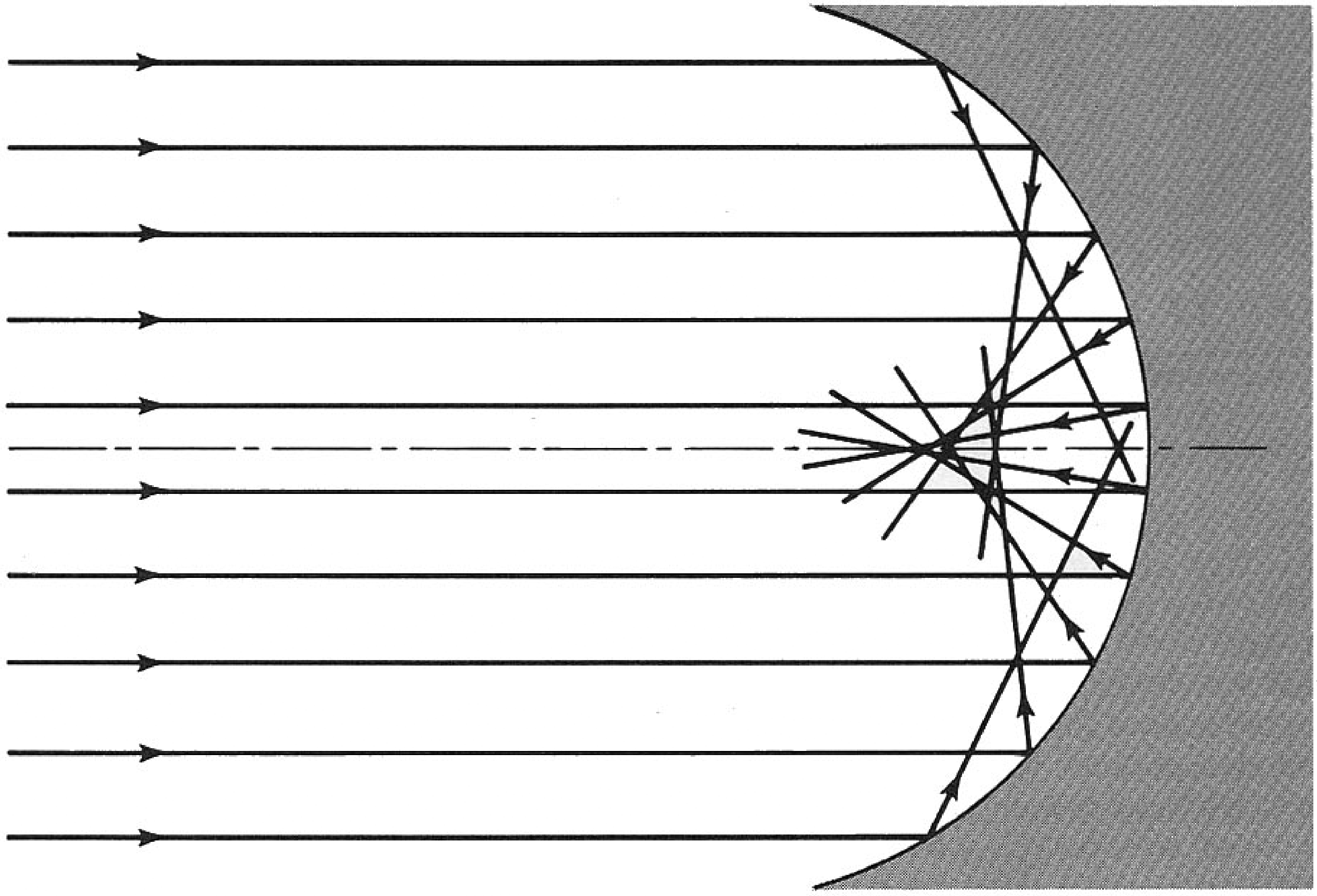


FIGURE 3.35

Spherical aberration in a concave mirror.

Parabolic mirrors have no spherical aberration (on axis)

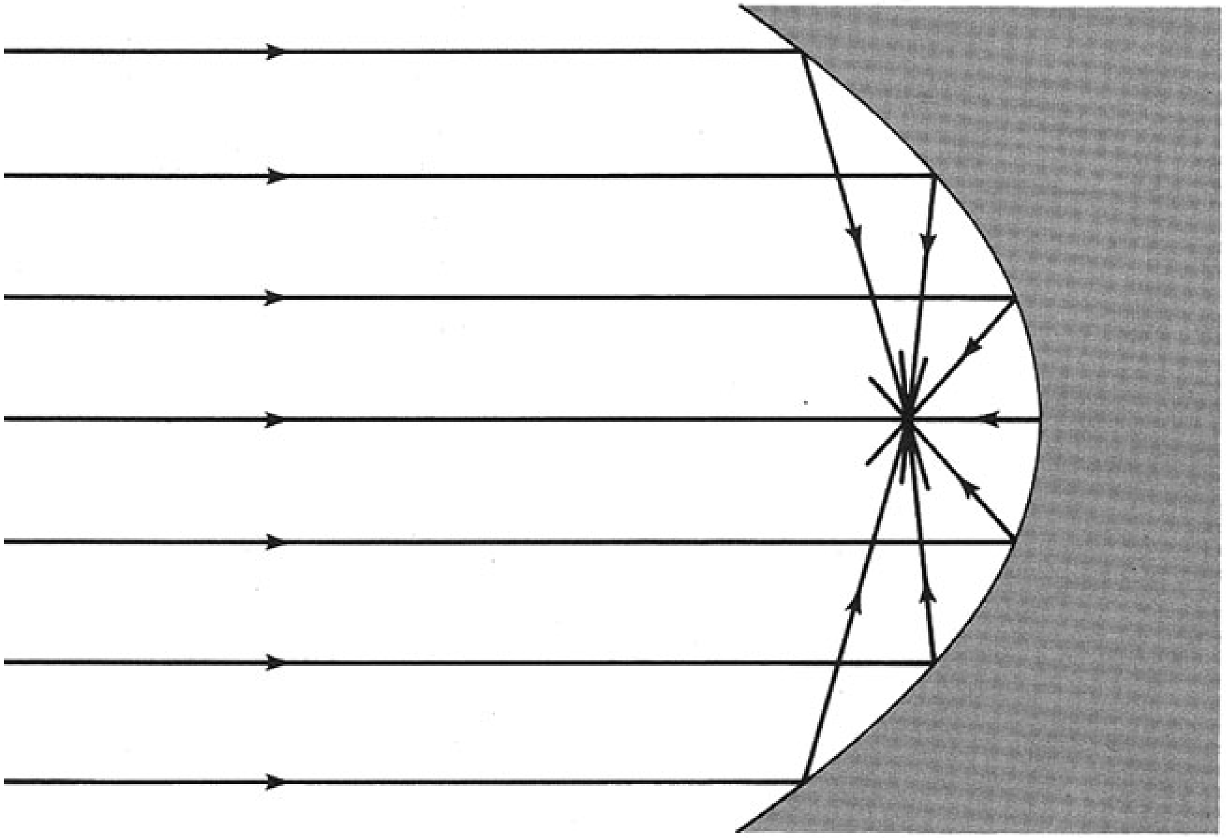


FIGURE 3.36

A parabolic reflector has no aberrations if the object is on the axis and very far away.

Ellipsoidal reflector

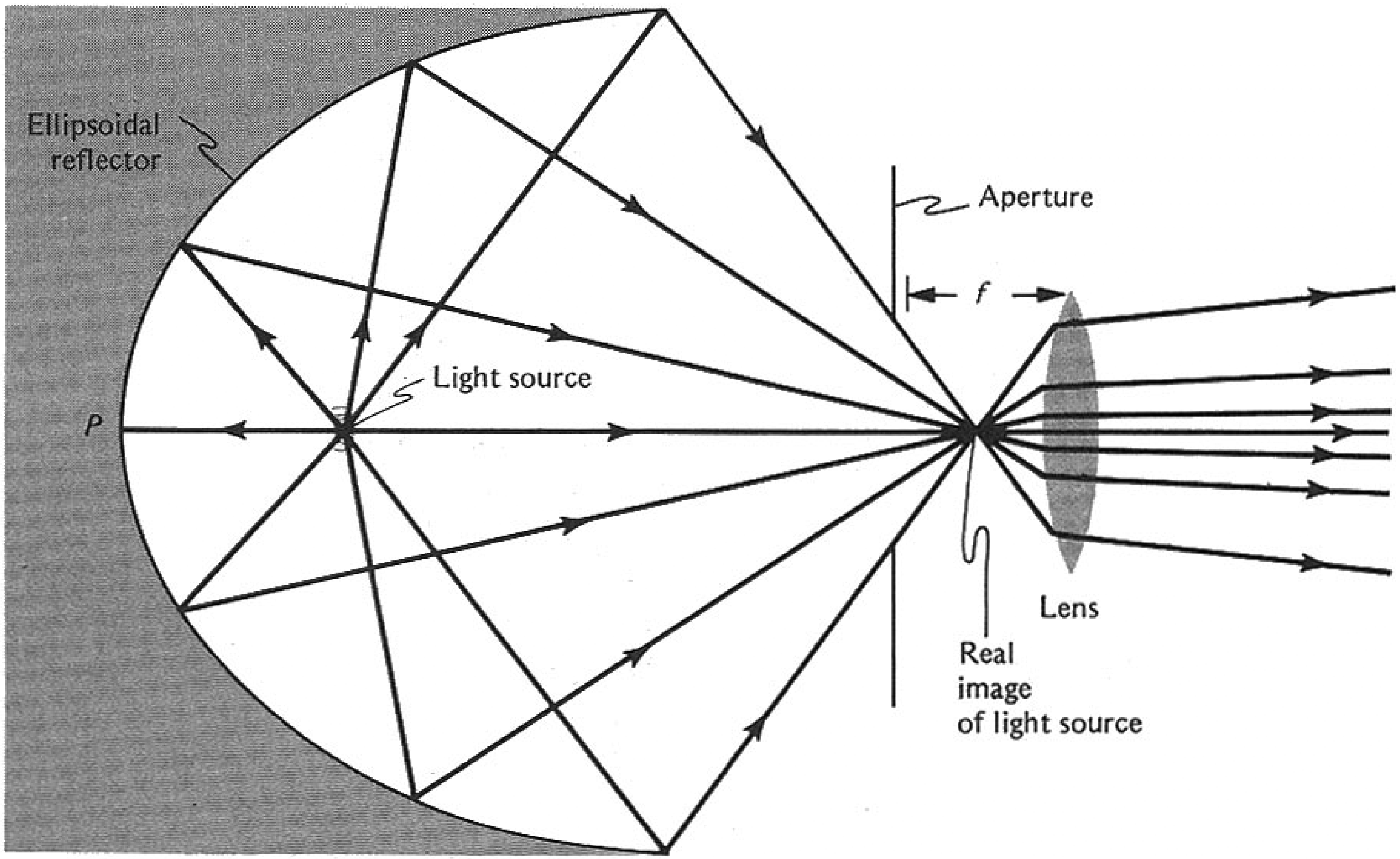
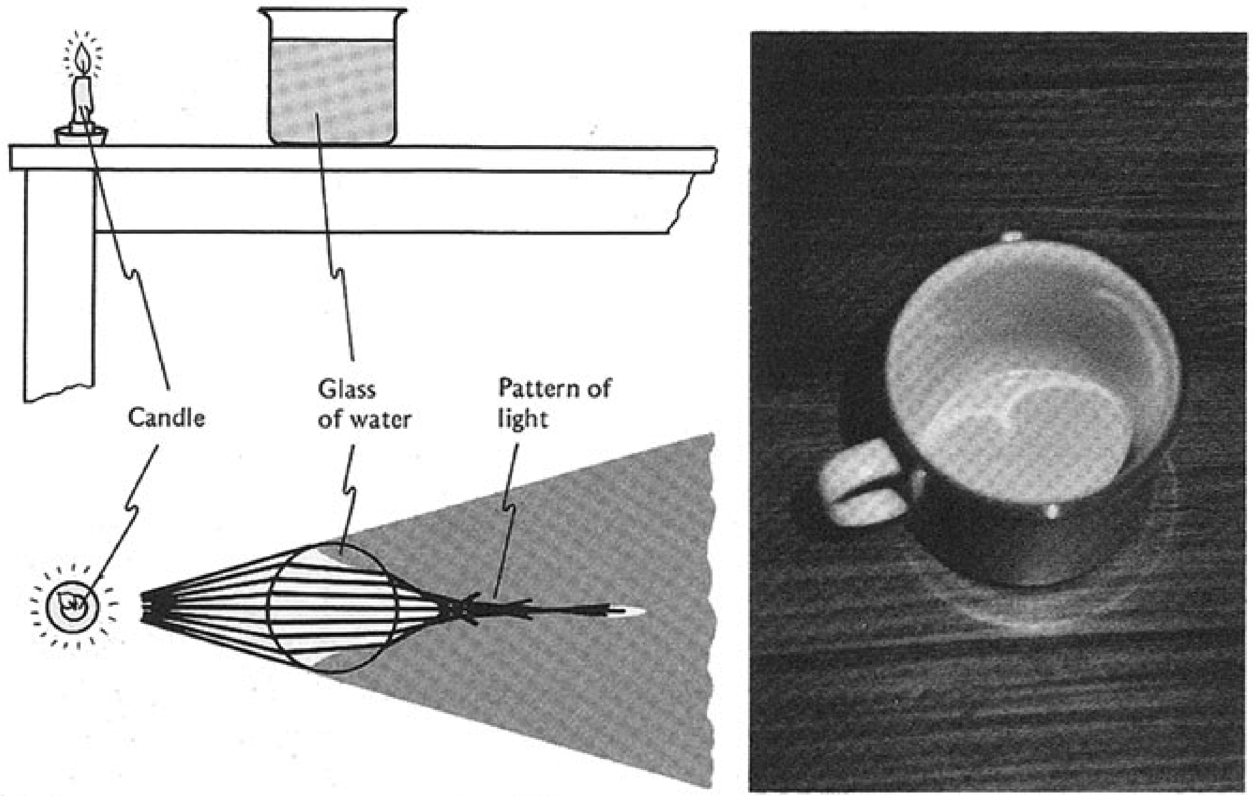


FIGURE 3.37

An ellipsoidal spotlight.

Spherical aberration in glass of water



(a) (b)

FIGURE 3.38

(a) Spherical aberration in a water glass lens (side and top views). **(b)** Photograph of spherical aberration pattern in a teacup reflector.

Field curvature from lens

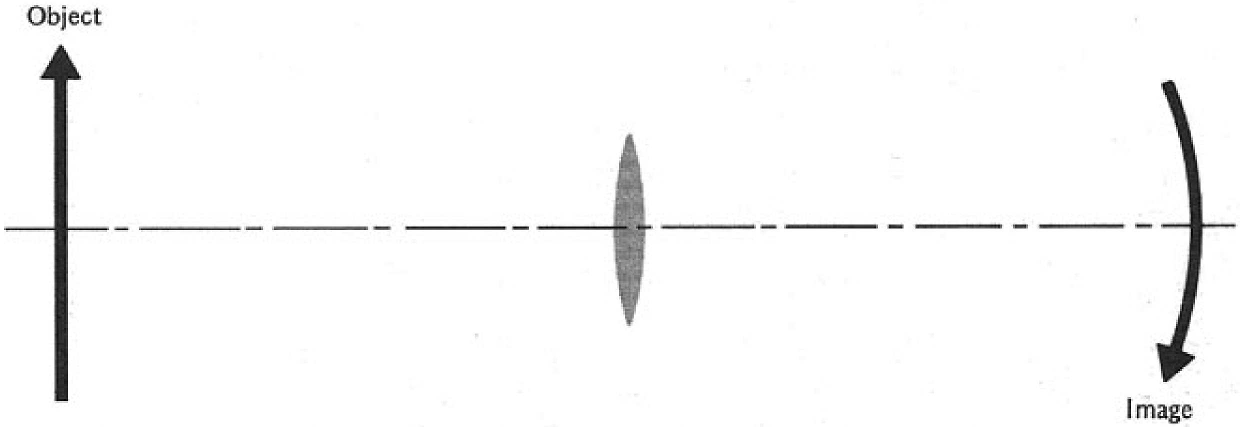


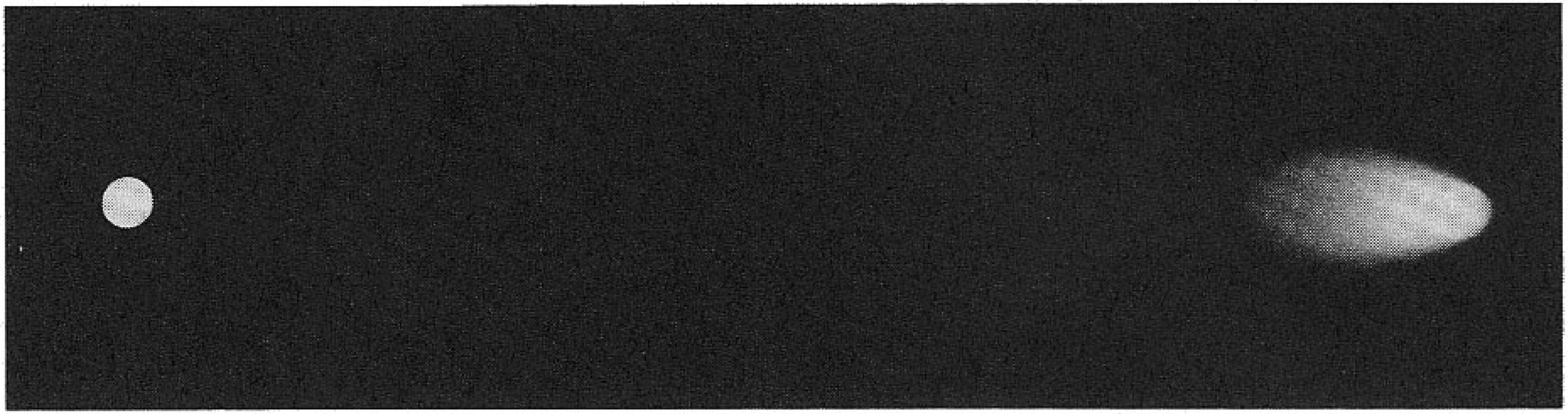
FIGURE 3.39

Curvature of field of a converging lens.

Comatic aberration

FIGURE 3.40

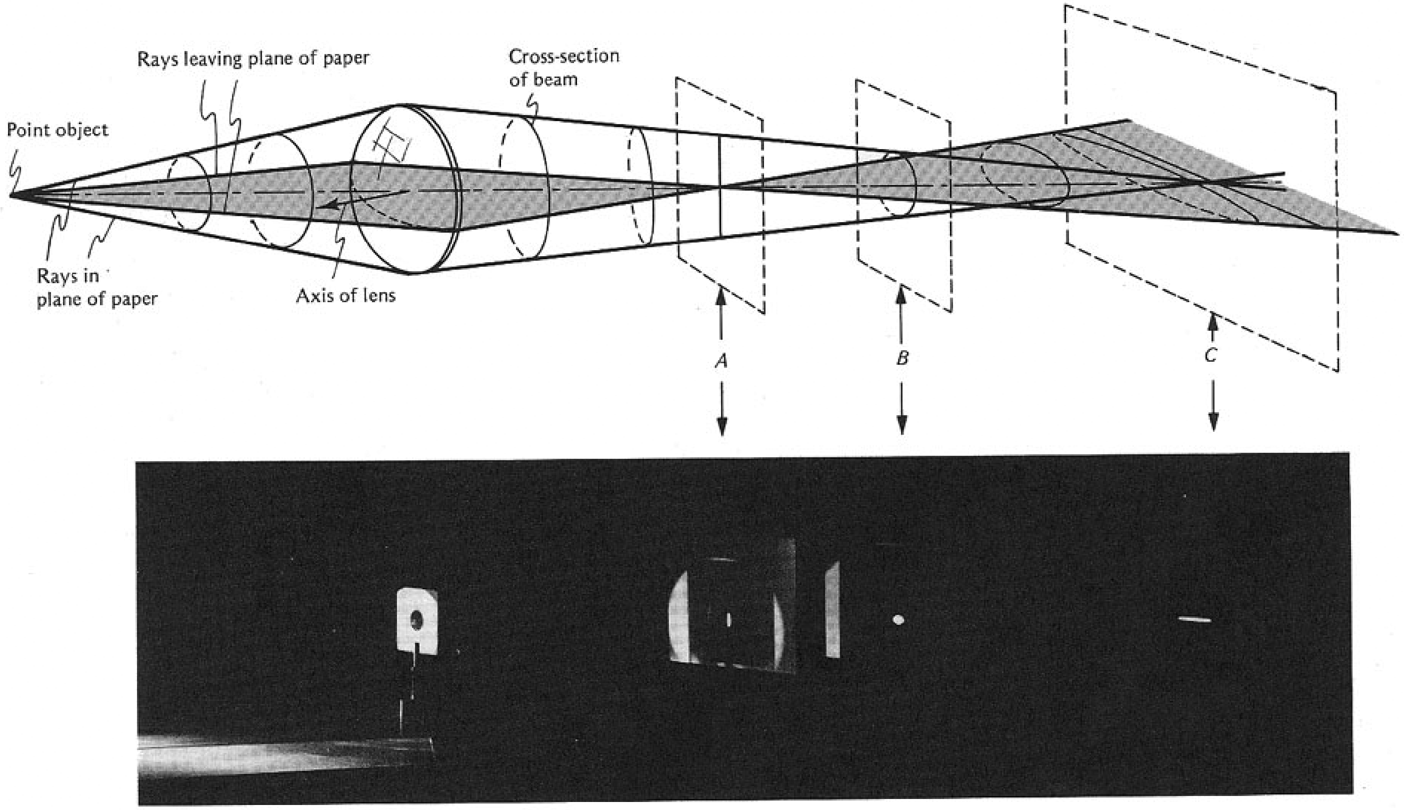
A small circular spot of light *on the axis* is projected by a lens to form the faithful image on the left (no coma). An identical *off-axis* spot of light produces the image with coma, on the right.



Astigmatism aberration

FIGURE 3.41

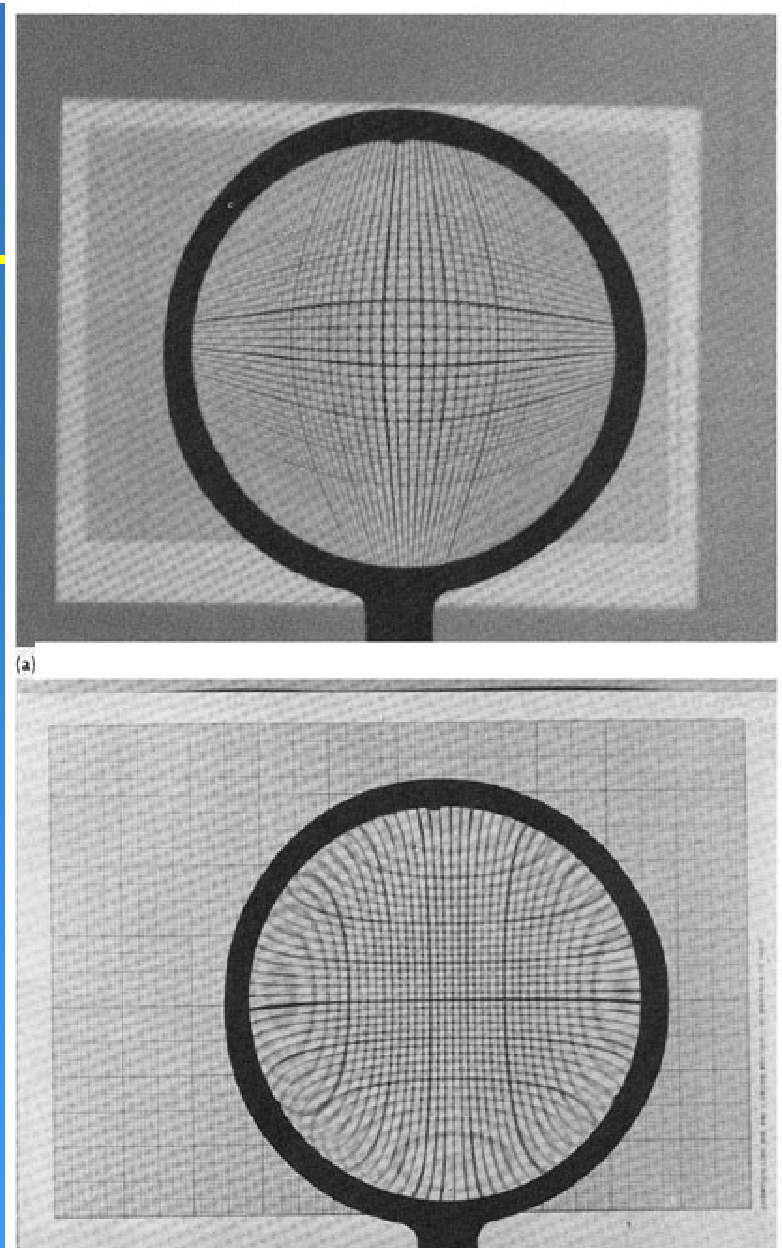
The astigmatic image of a point object in three planes, perpendicular to the axis. When the screen is at *A*, the image is a vertical line, and it is a horizontal line when the screen is at *C*. A screen at *B* gives the smallest circular image. (The photograph was taken by triple exposure, changing the position of a single screen between exposures.)



Distortion

- **Upper image shows barrel distortions**

- **Lower image shows pincushion distortion**



(a)

(b)

FIGURE 3.42

Graph paper with square rulings as seen through a lens that exhibits distortion. (Photographs taken by method described in the TRY IT.) **(a) Barrel distortion. (b) Pincushion distortion.**