The equation of relative motion is:

$$\frac{d^2\vec{r}}{dt^2} + \mu\frac{\vec{r}}{r^3} = 0$$

Take the cross product:

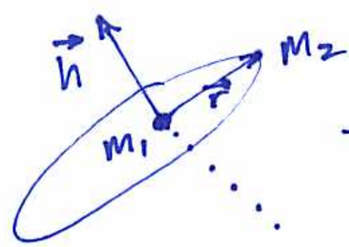
$$\vec{r} \times \ddot{\vec{r}} + \vec{r} + \frac{\mu\vec{r}}{r^3} = 0$$

indicates conservation of angular momentum ($\vec{h}$) + the fact that $\vec{r}$ and $\dot{\vec{r}}$ lie in a plane →

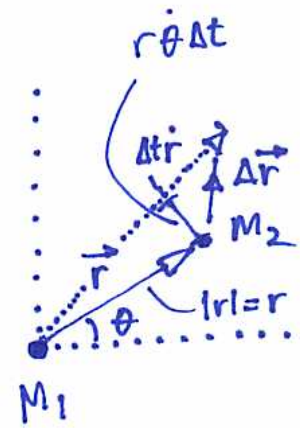
$$\vec{r} \times \ddot{\vec{r}} = 0$$

(integrate)

$$\vec{r} \times \dot{\vec{r}} = \vec{h}$$



→ use polar coords. →



in polar coordinates, the separation vector and its time derivatives are:

$$\vec{r} = r\hat{r}$$

(see diagram) → $\dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

[make sure you can derive this] → $\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta})\hat{r} + \left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right]\hat{\theta}$

↓

substitute this into the equation of relative motion

$$(\ddot{r} - r\dot{\theta})\hat{r} + \left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right]\hat{\theta} + \mu\frac{r\hat{r}}{r^3} = 0$$

look at $\hat{r}$ componet...