(*) $\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2}$ → note $\vec{h} = \vec{r} \times \dot{\vec{r}}$

$|h| = r^2\dot{\theta}$

trick: $u = \frac{1}{r}$, use h conservation

+ differentiation of r using the chain rule

$$\dot{r} = -\frac{1}{u^2}\frac{du}{d\theta}\dot{\theta} = -h\frac{du}{d\theta}$$

$$\ddot{r} = -h\frac{d^2u}{d\theta^2}\dot{\theta} = -h^2u^2\frac{d^2u}{d\theta^2}$$

Eqn (*) above can be re-written

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2}$$

- given two parameters $\mu = GM$ and $h = r^2\dot{\theta}$ this 2nd order dE tells how r varies with θ gives the orbital figure

This Eqn. is clearly related to simple harmonic motion – an orbit is an oscillation!

AMPLITUDE
constant of integration

solution $u = \frac{\mu}{h^2}[1 + e\cos(\theta - \varpi)]$

constant of integration
PHASE

↓ recover r from $\frac{1}{u}$.

The constant P = "semilatus rectum" $= \frac{h^2}{\mu}$

$$\boxed{r = \frac{P}{1 + e\cos(\theta - \varpi)}}$$

The general solution to the relative motion in the two body problem is the equation of a conic.