Homework Set #2

1. A box containing a particle is divided into a right and left compartment by a thin partition. If the particle is known to be on the right (left) side with certainty, the state is represented by the position eigenket $|R>(|L>)$, where we have neglected spatial variations within each half of the box. The most general state vector can then be written as:

$$|\alpha> = |R><R|\alpha> + |L><L|\alpha> \quad ,$$

where $<R|\alpha>$ and $<L|\alpha>$ can be regarded as "wave functions". The particle can tunnel through the partition; this tunneling effect is characterized by the Hamiltonian

$$H = \Delta\,(\,|L><R| + |R><L|\,)$$

where Δ is a real number with dimension of energy.

   (a) Find the normalized energy eigenkets. What the corresponding energy eigenvalues?

   (b) In the Schrodinger picture the base kets $|R>$ and $|L>$ are fixed, and the state vector moves with time. Suppose the systems is represented by $|\alpha>$ as given above at $t = 0$. Find the state vector $|\alpha, t_0 = 0; t>$ for $t > 0$ by applying the appropriate time-evolution operator to $|\alpha>$.

   (c) Suppose at $t = 0$ the particle is on the right side with certainty. What is the probability for observing the particle on the left side as a function of time?

   (d) Write down the coupled Schrodinger equations for the wave functions $<R|\alpha, t_0 = 0; t>$ and $<L|\alpha, t_0 = 0; t>$. Show that the solutions to the coupled Schrodinger equations are just what you expect from (b).

2. Consider an isotropic harmonic oscillator in two dimensions. The Hamiltonian is given by:

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}\left(x^2 + y^2\right)$$

   (a) What are the energies of the three lowest-lying states? Is there any degeneracy?

   (b) We now apply a perturbation

$$V = \delta m\omega^2 xy$$

   where δ is a dimensionless real number much smaller than unity. Find the zeroth-order energy eigenkets and the corresponding energy to first order for each of the three lowest-lying states.

   (c) Solve the $H_0 + V$ problem exactly. Compare with the perturbation results obtained in (b).

Note

$$<n'|x|n> = \sqrt{\frac{\hbar}{2m\omega}} \left(\sqrt{n+1}\delta_{n',n+1} + \sqrt{n}\delta_{n',n-1} \right)$$

3. The ions C^{+3}, O^{+5}, and Mg^{+} all exhibit strong doublet transitions in the ultraviolet at 1550Å, 1030Å, and 2800Å respectively.

   (a) Express the ground-state configuration of each ion. Guess at the excited level for each of these doublets (ignore the splitting). Check Verner et al. (1994) to confirm (or refute) your guess.

   (b) Calculate the expected splitting of the excited state due to spin-orbit coupling. How do these energies compare to the experimental values? Explain (take your best shot) and significant differences between your theoretical expectation and the experimental values.