Homework Set #3

(1) Find the spectroscopic terms associated with $2p^4 3p$. [Note: a shell more than half filled has the same spectroscopic terms as the complimentary number of electrons needed to fill the shell.]

(2) Estimate (order of magnitude) of the wavelength emitted by a hydrogen atom when the electron's and proton's spins slip from being aligned to be oppositely directed. Recall that the magnetic moment and spin are related by

$$\vec{\mu} = \frac{ge\hbar\vec{s}}{2mc} \tag{1}$$

where $g \approx 2$ for electrons and $g \approx 5.5$ for protons.

(3) Derive the oscillator strength for the Lyα transition:

$$gf = \frac{2^{14}}{3^9} = 0.8324 \tag{2}$$

Note: To calculate gf, one must sum over all degenerate initial and final states.

(4) The optical depth for an absorption line is related to the column density of gas N by:

$$\tau(v) = \frac{\pi e^2}{m_e c} f_{ij}\, \lambda_{ij}\, N\, \phi(v) \tag{3}$$

where f_{ij} is the oscillator strength, λ_{ij} is the wavelength of the transition and $\phi(v)$ is the line profile.

(a) Derive a 'complete' expression for the optical depth of the Lyα transition for a slab of gas with velocity dispersion σ and column density $N(\mathrm{HI})$.

(b) The observed intensity I_0 of a background source I_b penetrating a slab of H I gas will be attenuated according to

$$I_o(v) = I_b(v)\, \mathrm{e}^{-\tau} \tag{4}$$

Overplot $I_o(v)/I_b(v)$ for a series of Lyα transitions with $N(\mathrm{HI}) = 10^{13}, 10^{15}, 10^{18}$ and $10^{21}\ \mathrm{cm}^{-2}$ and $\sigma = 30\ \mathrm{km\,s}^{-1}$. Discuss.

(c) The equivalent width of the absorption feature is defined by

$$W_\lambda = \int (1 - I_o/I_b)\, d\lambda \tag{5}$$

Plot W_λ for $N(\mathrm{HI})$ ranging from 10^{13} to $10^{21}\,\mathrm{cm}^{-2}$ for velocity dispersions of $\sigma = 10$, 30, and 100 $\mathrm{km\,s}^{-1}$.

(d) For one choice of σ, plot $d(W_\lambda/\lambda)/dN$ vs. N.