# XI. 21cm Radiation

## A. Physics

- Proton has a spin and therefore a magnetic moment

$$\vec{\mu}_p = \frac{g_p e}{2 m_p c} \vec{I} \tag{1}$$

  - $\vec{I}$ is the nuclear spin
  - $g_p \approx 5.5$

- Hyperfine interaction with the electron

$$H = -\vec{\mu}_p \cdot \vec{B} \tag{2}$$

  - Spin aligned has highest energy
  - To estimate $\vec{B}$, model the electron orbit as a magnetized sphere

$$\vec{B} = \frac{8\pi}{3} \vec{M} = \frac{8\pi}{3} \vec{\mu}_e \Psi_0^2 \tag{3}$$

  - Substituting

$$H = \frac{16\pi}{3\hbar^2} \left( \frac{e\hbar}{2 m_e c} \right)^2 \frac{m_e}{m_p} g_p \Psi_0^2 \vec{I} \cdot \vec{s} \tag{4}$$

- Introduce $\vec{F} = \vec{I} + \vec{s}$
  - ala spin-orbit coupling

$$\vec{I} \cdot \vec{s} = \frac{1}{2} \left( F^2 - I^2 - S^2 \right) \tag{5}$$

    - $I = \frac{1}{2} = s$
    - $F = 0, 1$ for spins aligned, anti-aligned
  - For a 1s state

$$\Psi_0^2 = \frac{1}{\pi a_0^3} \tag{6}$$

- Transition Energy
  - Energy

$$\begin{aligned} h\nu &= U_+ - U_- & (7) \\ &= \frac{8\pi}{3} \left( \frac{e\hbar}{2 m_e c} \right)^2 \frac{m_e}{m_p} g_p \frac{2I+1}{\pi a_0^3} & (8) \\ &= \frac{8}{3} \alpha^2 g_p \frac{m_e}{m_p} \mathrm{Ryd} & (9) \end{aligned}$$

- ◇ Frequency: $\nu_{21cm} = 1420.403$ MHz
- ◇ Wavelength: $\lambda = 21.1$ cm
- ◇ Approx energy: $\Delta E = 10^{-5}$eV

- Transition probability
  - ◇ Spontaneous emission coefficient

$$A_{10} = 2.85 \times 10^{-15}\ \mathrm{s}^{-1} \tag{10}$$

  - ◇ Lifetime

$$\tau_{21cm} \approx 10^{7}\ \mathrm{yr} \tag{11}$$

  - ◇ Excited state is primarily de-excited by collisions

$$< \sigma v > \sim 10^{-10}\ \mathrm{cm}^3\,\mathrm{s}^{-1} \text{ for T} \sim 100\mathrm{K} \tag{12}$$

$$\tau_{coll} \sim 3 \times 10^{2}\ \mathrm{yr} \tag{13}$$

- Level population
  - ◇ Boltzmann

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} \exp\left(-\frac{h\nu}{kT}\right) \tag{14}$$

$$= 3\ \exp\left(\frac{-T_*}{T}\right) \tag{15}$$

$$\approx 3\left[1 - \frac{T_*}{T} + \mathcal{O}\left(\frac{T_*}{T}\right)^2\right] \tag{16}$$

  - ◇ $T_* = 0.068$K
  - ◇ Majority of H gas will have 3/4 of the atoms in the excited state and only 1/4 in the ground state
  - ◇ Note: Cannot neglect stimulated emission altogether

## B. Radiative Transfer

- Players
  - ◇ Intensity: $I_\nu d\nu d\Omega dA dt$ is the energy of those photons which during a time interval $dt$ passes through the area $dA$ whose frequency lies within $d\nu$ of $\nu$ whose direction is within the solid angle $d\Omega$
  - ◇ Absorption coefficient (opacity): $\kappa_\nu I_\nu ds d\nu d\Omega dA dt$ is the energy absorbed from a beam of photons along the distance $ds$ which during a time interval $dt$ passes through the area $dA$ whose frequency lies within $d\nu$ of $\nu$ whose direction is within the solid angle $d\Omega$

- ◇ Emission coefficient: $j_\nu ds d\nu d\Omega dA dt$ is the energy emitted by the volume element $dsdA$ during a time interval $dt$ whose frequency lies within $d\nu$ of $\nu$ and whose direction is within the solid angle $d\Omega$

- Transfer equation

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu \tag{17}$$

- ◇ Optical depth

$$d\tau_\nu = -\kappa_\nu ds \tag{18}$$

- ◇ Consider radiation from a single cloud with optical depth $\tau_{\nu r}$ and incident radiation $I_\nu(0)$

$$I_\nu = I_\nu(0)\mathrm{e}^{-\tau_{\nu r}} + \int_0^{\tau_{\nu r}} \frac{j_\nu}{\kappa_\nu}\mathrm{e}^{-\tau_\nu} d\tau_\nu \tag{19}$$

- ◇ If one has strict thermodynamic equilibrium, Kirchoff's law applies

$$\frac{j_\nu}{\kappa_\nu} = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\mathrm{e}^{h\nu/kT}-1} \tag{20}$$

- ◇ In the low frequency regime, $h\nu \ll kT$

$$B_\nu(T) = \frac{2\nu^2 kT}{c^2} \tag{21}$$

  - ▲ Define "brightness temperature", $T_b$ is the temperature where $I_\nu = B_\nu(T)$
  - ▲ Radiation transfer equation becomes

$$T_b = T_{b0}\mathrm{e}^{-\tau_{\nu r}} + T\left(1 - \mathrm{e}^{-\tau_{\nu r}}\right) \tag{22}$$

## C. Astrophysics: Spin Temperature

- Opacity of H I gas to 21cm radiation

$$\kappa_\nu = \tilde{\kappa}_\nu\left(1 - \mathrm{e}^{-T_*/T}\right) \tag{23}$$

- ◇ 2nd term is due to stimulated emission
- ◇ Essential to probing the galaxy
- ◇ Again, $T_* \ll T$

$$\begin{aligned}
\kappa_\nu &\approx \tilde{\kappa}_\nu \frac{T_*}{T} && (24)\\
&= \frac{h\nu_{10}B_{01}n_0\phi_\nu}{4\pi}\frac{T_*}{T} && (25)\\
&= \frac{h\nu_{10}}{4\pi}\frac{g_1}{g_0}\left(\frac{A_{10}c^2}{2h\nu_{10}^3}\right)\frac{1}{4}n_H\frac{T_*}{T}\phi_\nu && (26)
\end{aligned}$$

- ▲ Hot regions are less opaque

- Optical depth
  - ◇ Frequency

$$\tau_\nu = \int_0^L \kappa_\nu ds \tag{27}$$

$$= \frac{(\text{const})}{T} \int_0^L n_H ds \tag{28}$$

  - ◇ Velocity

$$\tau_v = \frac{N(\text{HI})}{1.825 \times 10^{18}} \frac{\phi(v)}{T} \tag{29}$$

    - ▲ $\phi(v)$ is in km/s
    - ▲ $N(\text{HI})$ is in cm$^{-2}$
  - ◇ Measure $N(\text{HI})$ via Ly$\alpha$ and $\tau_v$ from 21cm
    - ▲ Compare to get $T_s$, the spin temperature
    - ▲ WNM: $T_s \approx 8000$K
    - ▲ CNM: $T_s \approx 100$K

### D. Astrophysics: Galactic rotation curve

- Diagram

  - ◇ $\Theta_0 = \omega_0 R_0 =$ rotational velocity at our position
  - ◇ $\Theta = \omega R =$ rotational velocity at radius $R$
  - ◇ $r$ is the distance to a given 'cloud'

- ◇ $\ell$ is angle off the center of the galaxy

- Observationally, we measure the velocity projected along the sightline

$$
\begin{aligned}
v(r) &= \Theta \cos\left[\frac{\pi}{2} - (\,\ell + \theta\,)\right] - \Theta_0 \sin\ell & (30)\\
&= \omega R\,[\,\sin\theta\cos\ell + \sin\ell\cos\theta\,] - \Theta_0\cos\ell & (31)
\end{aligned}
$$

  - ◇ Constraints

$$
\begin{aligned}
R\sin\theta &= r\sin\ell & (32)\\
R\cos\theta + r\cos\ell &= R_0 & (33)
\end{aligned}
$$

  - ◇ Rewrite

$$
\begin{aligned}
v(r) &= \omega R_0\left[\frac{R}{R_0}\sin\theta\cos\ell + \frac{R}{R_0}\sin\ell\cos\theta\right] - \Theta_0\sin\ell & (34)\\
&= \omega R_0\sin\ell - \Theta_0\sin\ell & (35)\\
&= R_0\sin\ell\,[\,\omega(R) - \omega_0\,] & (36)
\end{aligned}
$$

    - ▲ If the Galaxy were rotating as a solid body (i.e. $\omega(R) = \omega_0$), then every parcel of gas would have zero velocity on the los
    - ▲ The optical depth would be extremely large
    - ▲ We would be unable to see through the Galaxy!

- Observations of $v(r)$
  - ◇ Fig
  - ◇ Velocity peaks when $\vec{R} \perp \vec{r}$
  - ◇ In this case, $R = R_c$

$$
\begin{aligned}
R_c &= R_0\sin\ell & (37)\\
\Theta_c &= \omega(R_c)R_c = v_T + \Theta_0\sin\ell & (38)
\end{aligned}
$$

- Knowing $R_0$ and $\Theta_0$, we can observe at a range of $\ell$ to map out the rotation curve
  - Fig
  - Solid body in the interior
  - Flat at $R > 2$ kpc