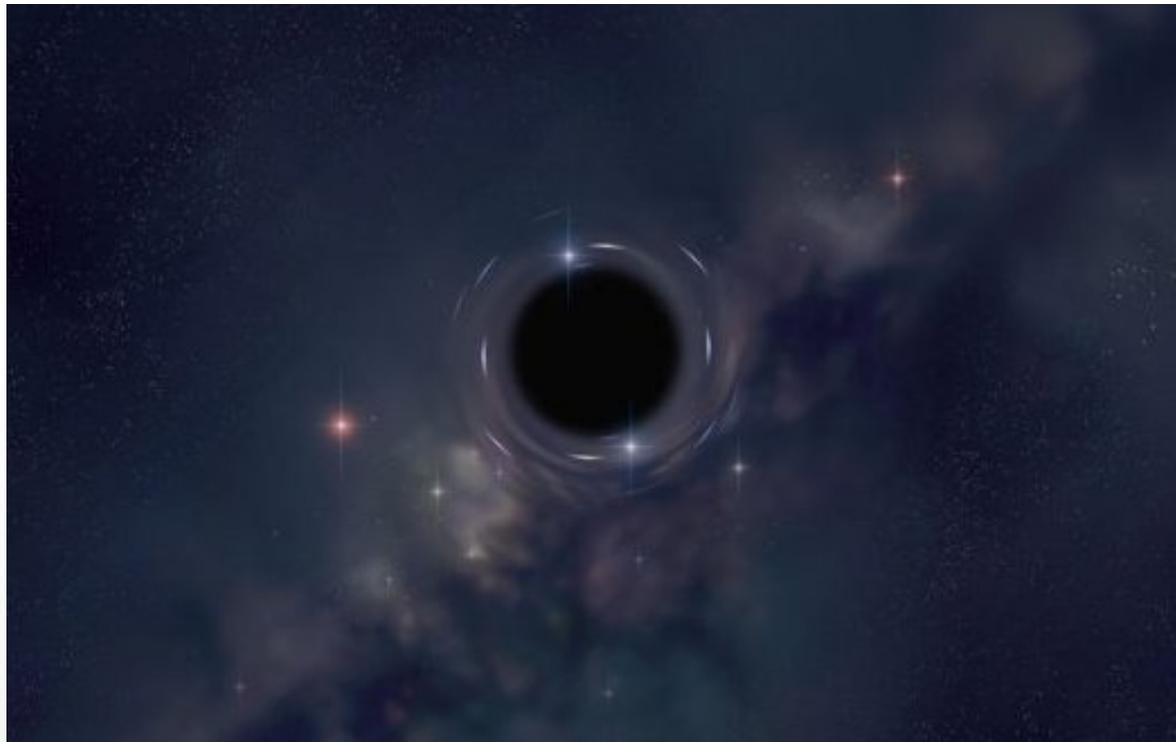


AY1 Announcements

- Next
 - Einstein's theory of relativity
 - Stellar mass black holes

Last option for stellar evolution

- Initial stellar mass $0.1M_{\text{Sun}} - 8M_{\text{Sun}}$: White Dwarf supported by e^- degeneracy (WD mass $\leq 1.4M_{\text{Sun}}$)
- Initial stellar mass $> 8M_{\text{Sun}}$: neutron star supported by neutron degeneracy



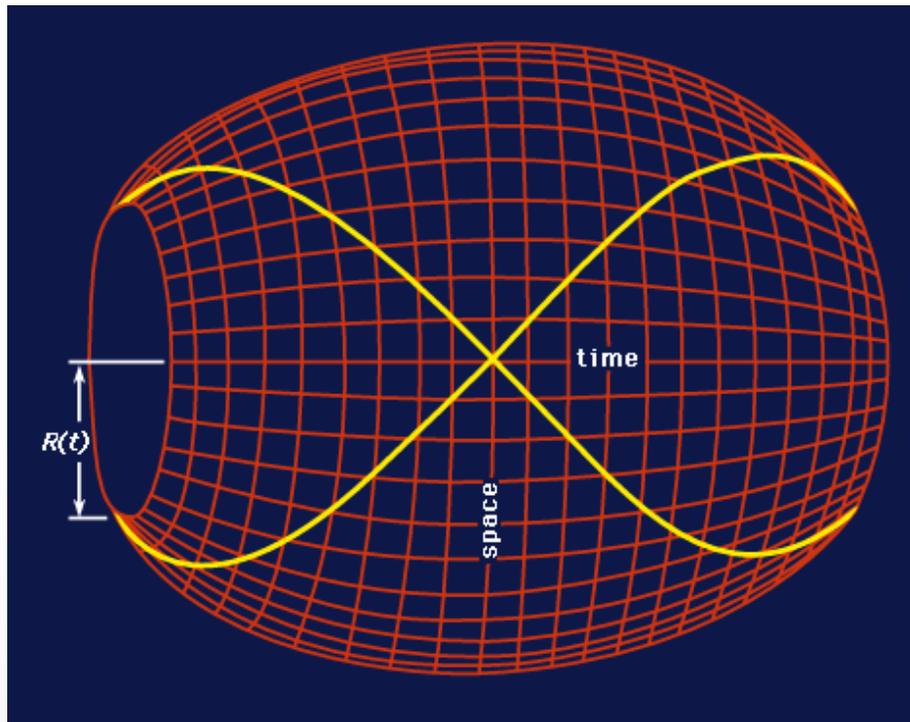
Is there a limit to neutron degeneracy?

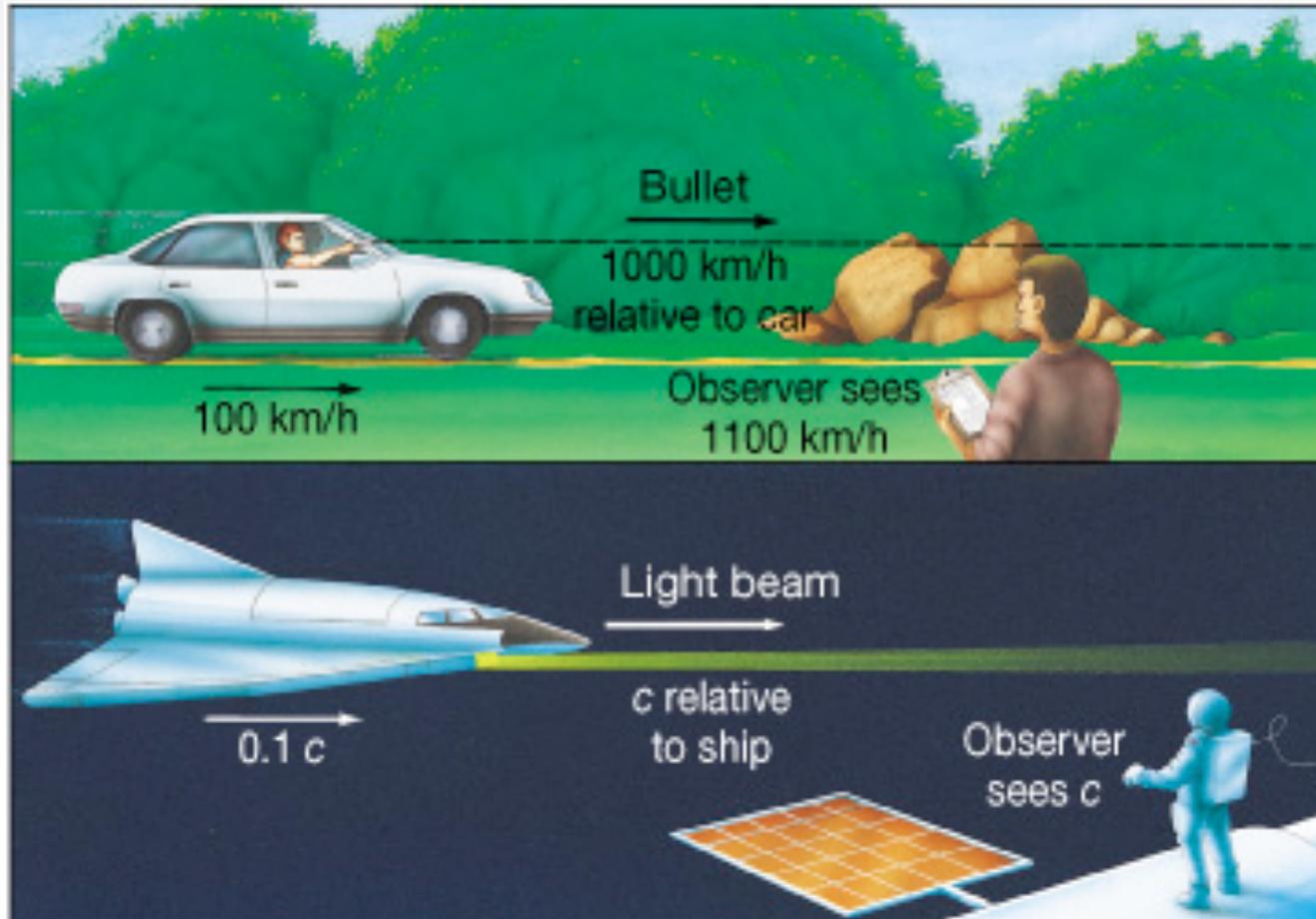
- Yes! Gravity wins the final battle. The current best estimate for the maximum mass of a neutron-degenerate star is $3M_{\text{Sun}}$.
- If a neutron star exceeds this mass it will collapse into an infinitely small volume called a black hole.
- But, this story starts with Einstein's theories of special and general relativity.



Special Relativity

- Various experiments starting in the late 1800s suggested that the speed of light was constant, *independent of the motion of the observer.*
- This is very counter-intuitive.



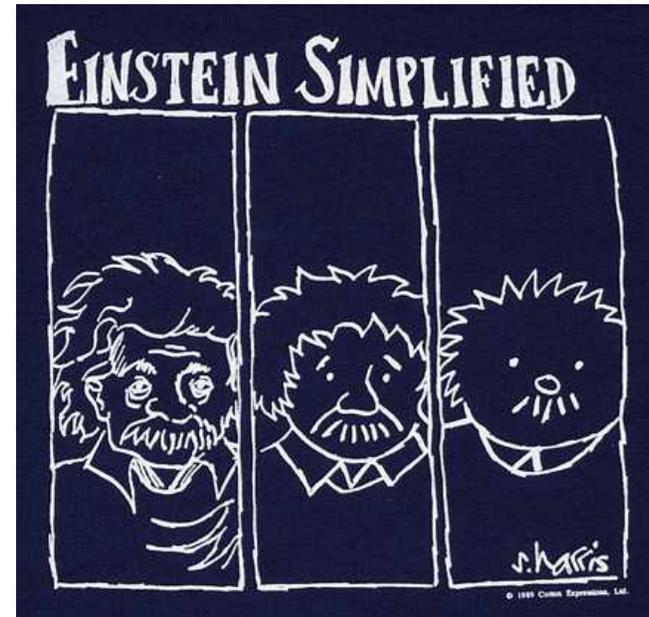


The spaceship traveling in the same direction of a photon measures the photon zooming away at the speed of light NO MATTER how fast the spaceship is traveling!

Special Relativity

- Einstein (and others before him) decided to take the speed of light as an invariant and not make any assumptions about the two properties that go into determining speed:

Space and Time



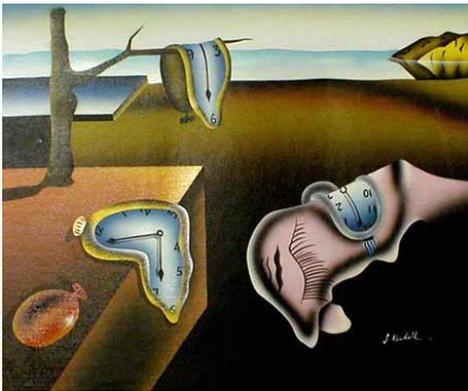
Time Dilation and Length Contraction

The invariance of the measured speed of light independent of the motion of the observer can be understood if:

- (1) Clocks run more slowly as speed increases
- (2) Metersticks shrink as speed increases

Time Dilation

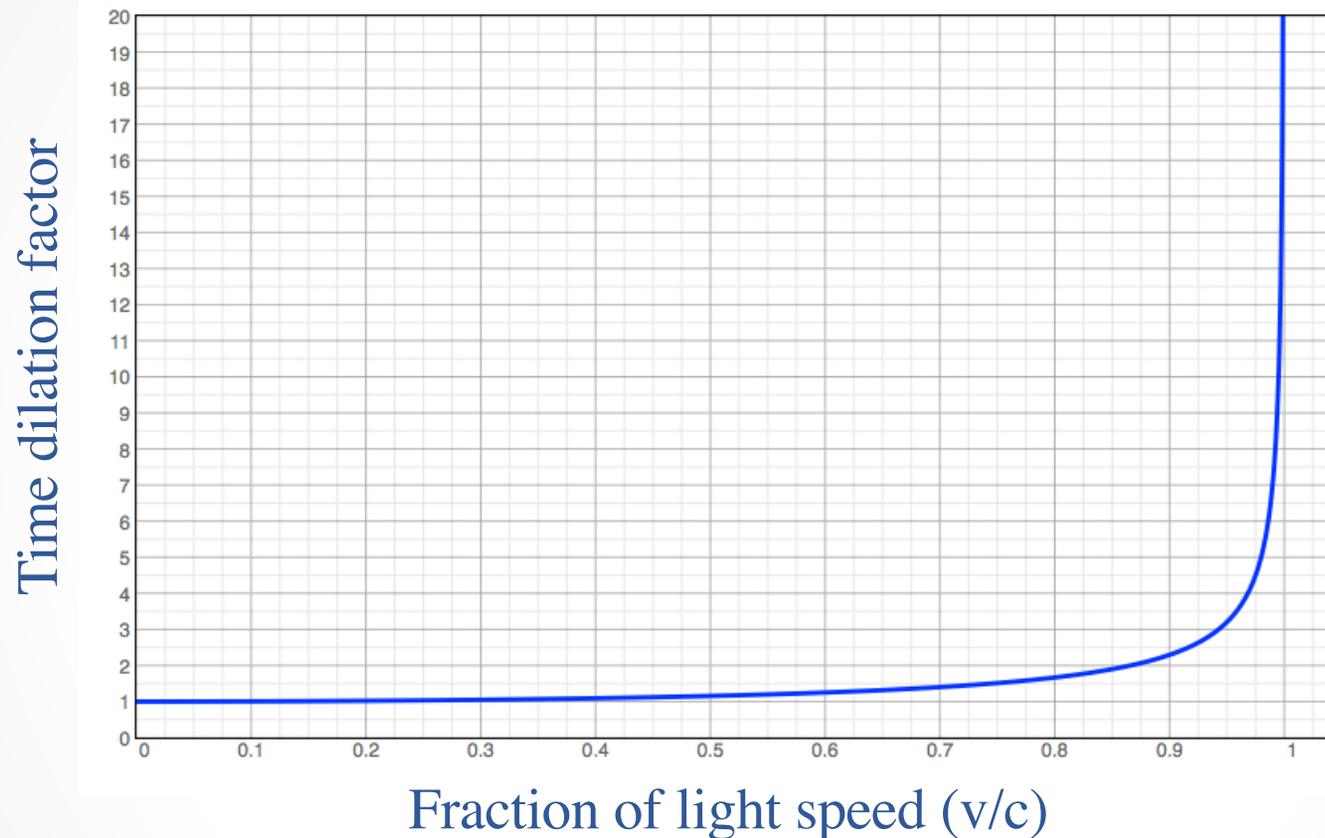
As your speed with respect to another observer increases, your watch runs more slowly than the observer's watch. This is called 'time dilation'



$$T = \frac{T_0}{\sqrt{1 - (v/c)^2}}$$

Note, when $v \ll c$, $T = T_0$

Time Dilation



As $v \rightarrow c$, $v/c \rightarrow 1$ and the denominator goes to zero. Dividing by zero gives infinity.

As v approaches c , time grinds to a halt!

Q. Suppose you measure an event that lasts for 1 second by your watch. What will your friend in a spaceship moving at $0.98c$ measure as the duration of the event?

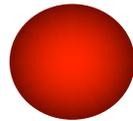
$$T = \frac{T_0}{\sqrt{1 - (0.98)^2}} = 5.02T_0$$

- Time has been stretched by a factor of 5 for your friend.

Length Contraction

- In the same way, metersticks (space) contracts in the direction of motion.

$$L = L_0 \sqrt{1 - (v/c)^2}$$



- But wait, there's more!

Mass

Mass grows with speed.

$$M = \frac{M_0}{\sqrt{1 - (v/c)^2}}$$

Constant Speed of Light

- The shrinking rulers and slowing clocks conspire to let observers in any moving frame measure the same speed of light.

Travel beyond the Galaxy will be Difficult

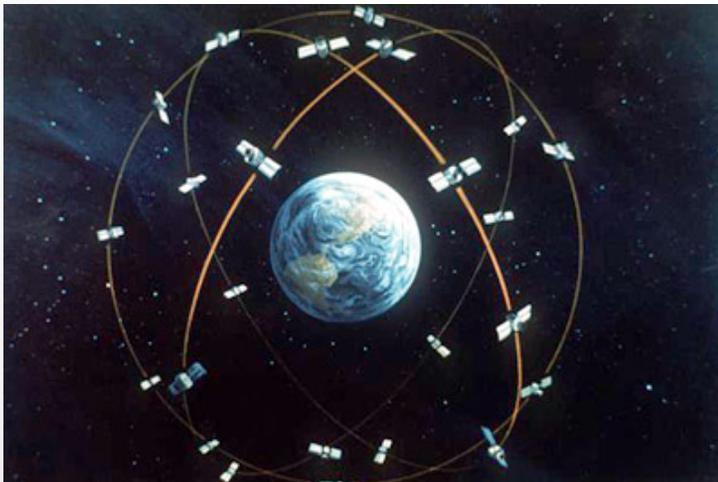
- The slowing clocks and increasing mass conspire to make it impossible for objects with mass to ever reach the speed of light.
- The increasing mass requires an ever-larger force to accelerate to larger speed and the force needed would become infinite.
($F=ma$)
- Even if you could find the force, your clock would slow and slow and the last step would take an infinitely long time

Is this right?

- Yes! There are many tests of Special Relativity.
- In particle accelerators, mass increase and time dilation effects are routinely measured
- There have been tests flying very accurate clocks in high-speed jets that show time dilation directly.
- We might not be here if not for time dilation in the frame of cosmic rays called muons that get launched at $0.98c$ and have decay lifetimes stretched by a factor of 5 to allow them to reach the ground

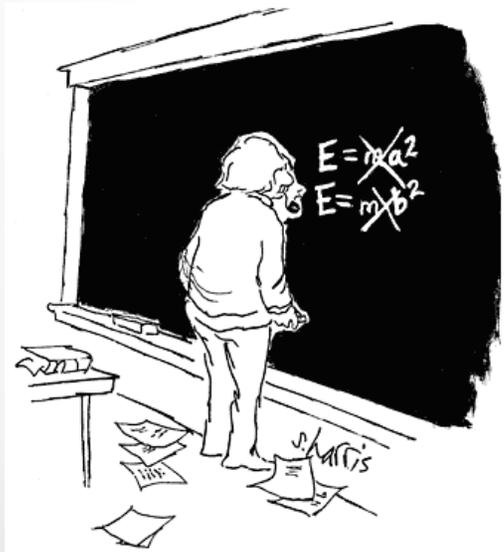
GPS and relativity

- Commercial GPS requires timing good to ~ 20 nanosec (20×10^{-9} seconds). The relative speed of GPS satellites to stationary receiver is $\sim 14,000$ mph and the time dilation is $\sim 7 \times 10^{-6}$ seconds per day.
- Combined with a General Relativity effect, GPS would be off by ~ 10 km per day if these effects were not accounted for!



Einstein II: General Relativity

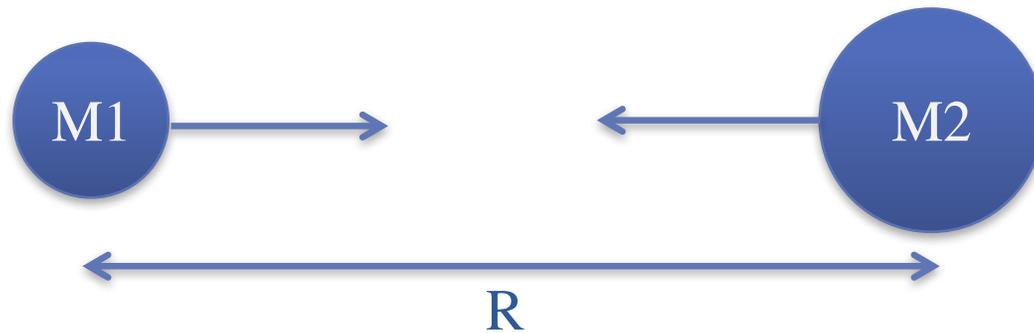
- Einstein's theory of General Relativity is a theory of gravity
- The basic idea is to drop Newton's idea of a mysterious force between masses and replace it with the 4-dimensional **SpaceTime** continuum that is warped by the presence of mass



Think about what needs to happen to have an event in the Universe and you can understand the need to specify 4-dimensions

Newtonian Gravity

- Newton proposed a theory of gravity to explain why things fell to Earth and the orbits of planets

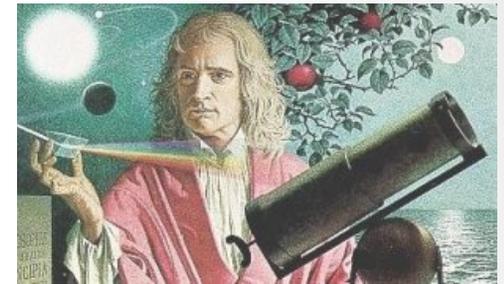
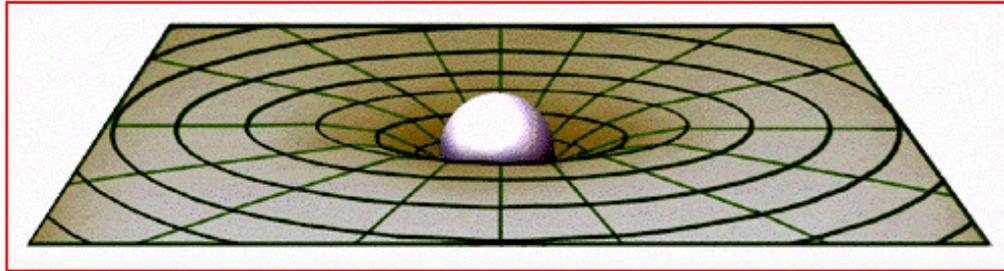


$$Force_{gravity} = \frac{G \times M_1 \times M_2}{R^2}$$



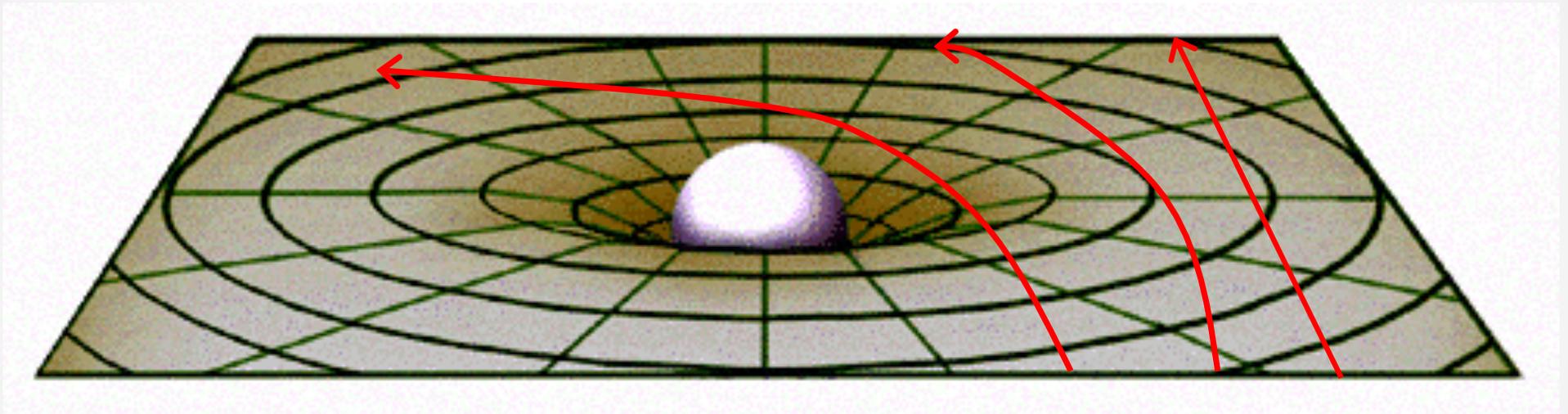
General Relativity

- In GR, mass (or energy) warps the spacetime fabric of space.

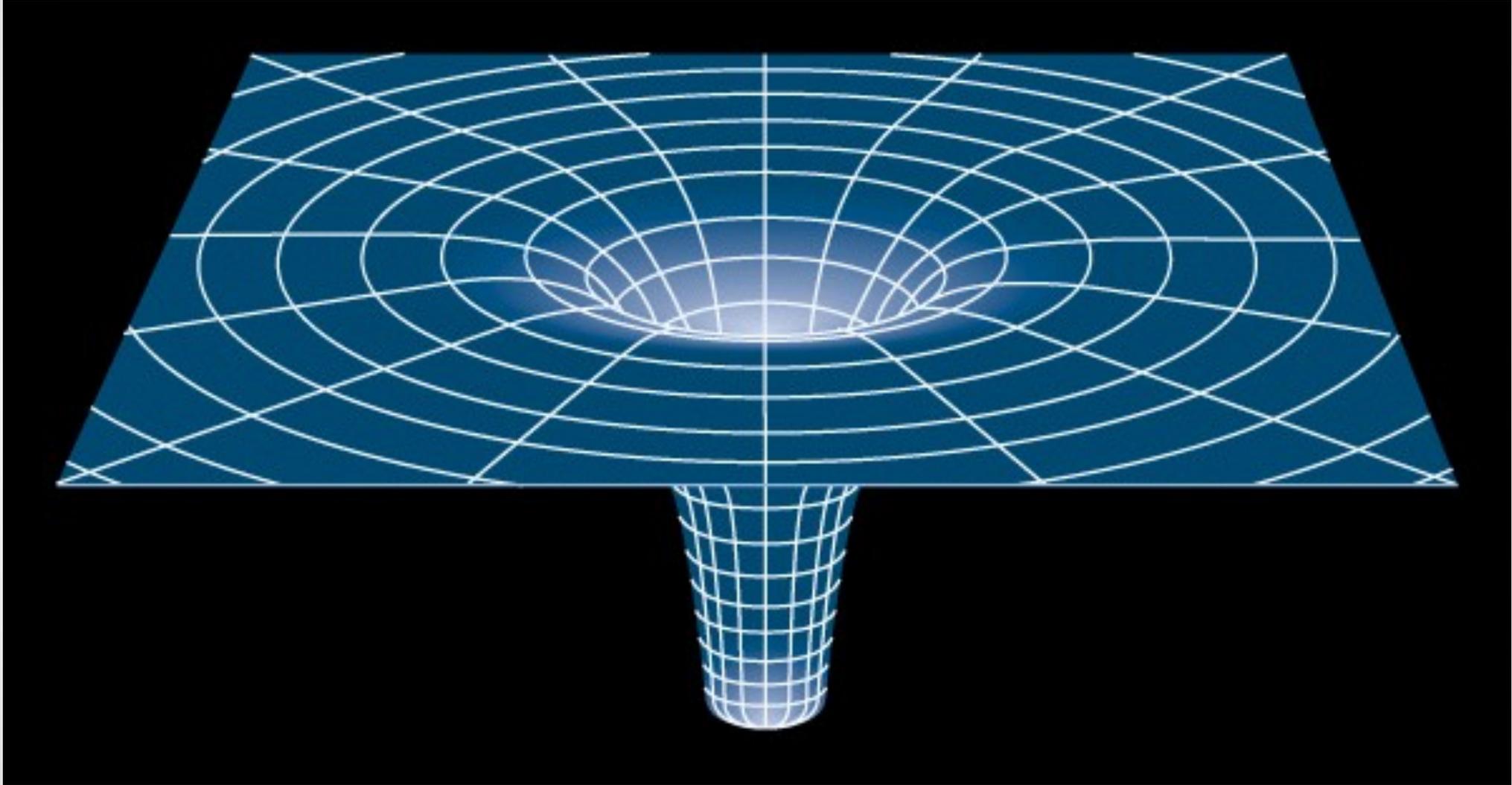


- Orbits of planets around stars are not due to a central force, but rather the planets are traveling in straight lines through curved space

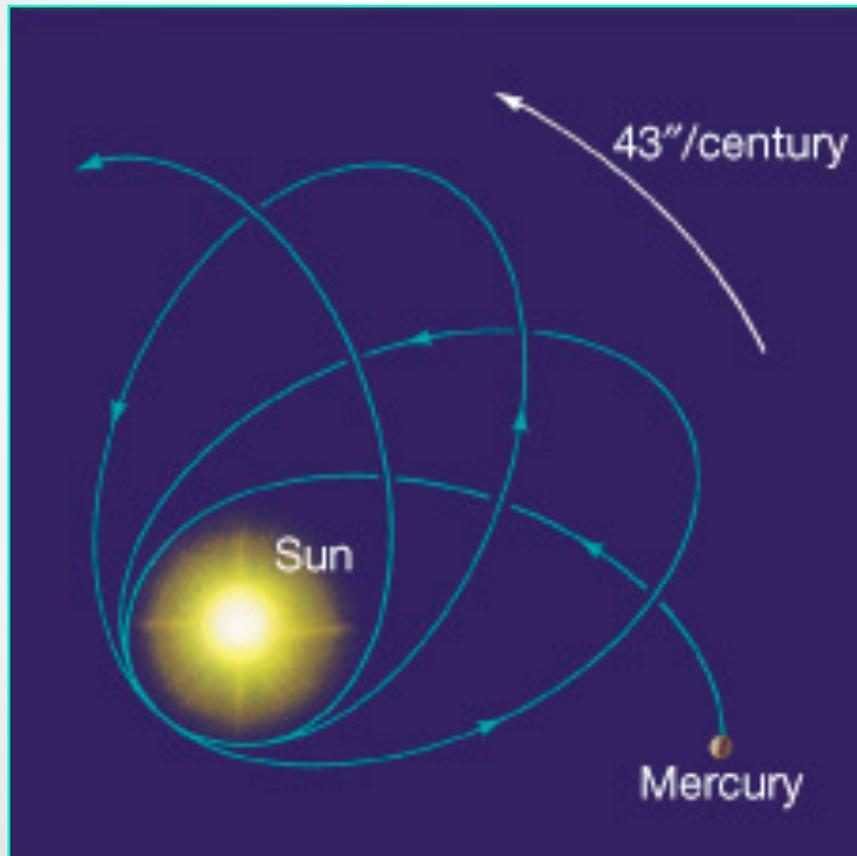
Imagine tossing a shotput onto your bed and rolling marbles at different speeds and distances from the shotput. (also imagine that you have a frictionless blanket on the bed).



The marbles that are moving slowly or close will fall down toward the shotput. If you look from above, it will appear as if the marbles were attracted to the shotput.



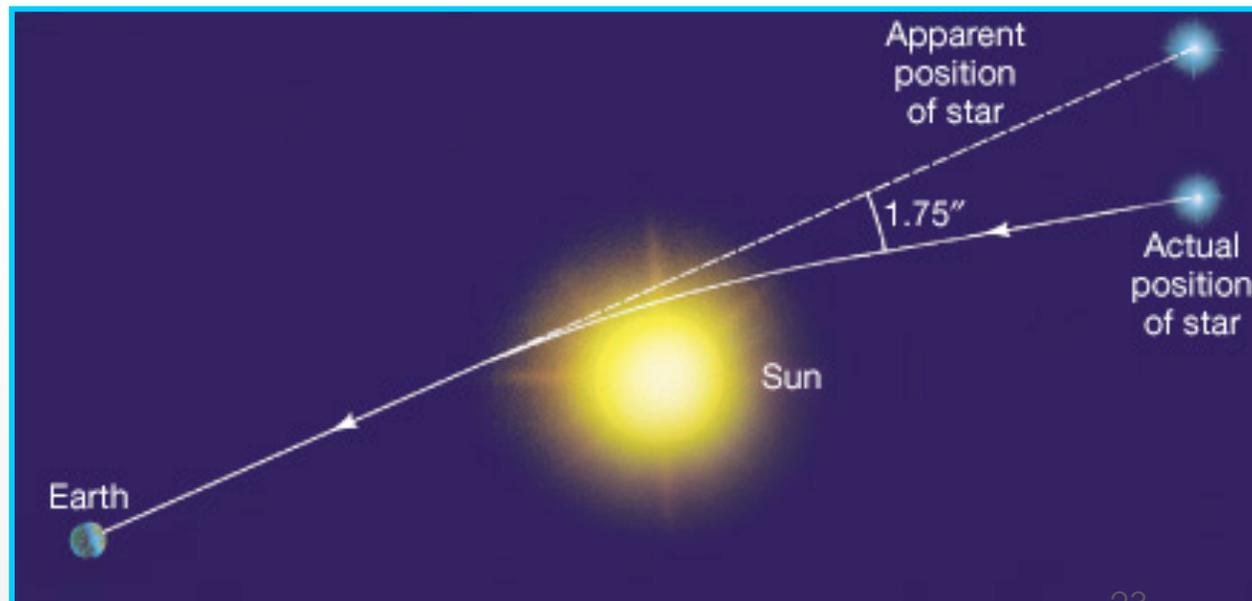
Fabric of Space



- This is a RADICALLY different view of the Universe and gravity
- In regions where space is not strongly curved, GR reduces to Newton's law of gravity
- Einstein pointed out his new theory would explain the Precession of the Perihelion of Mercury

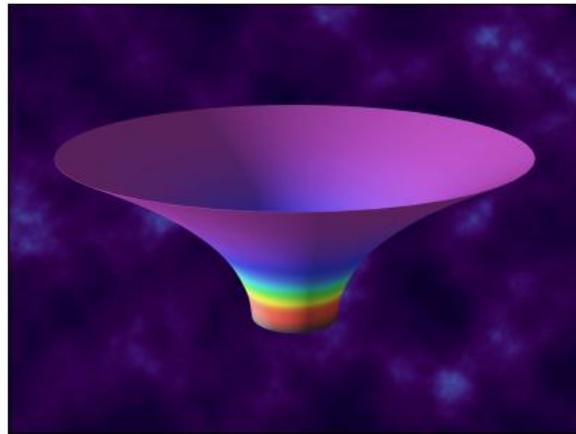
The Deflection of Starlight

- There were several other predictions of GR, one important one was that light rays would also follow straight lines through curved space.



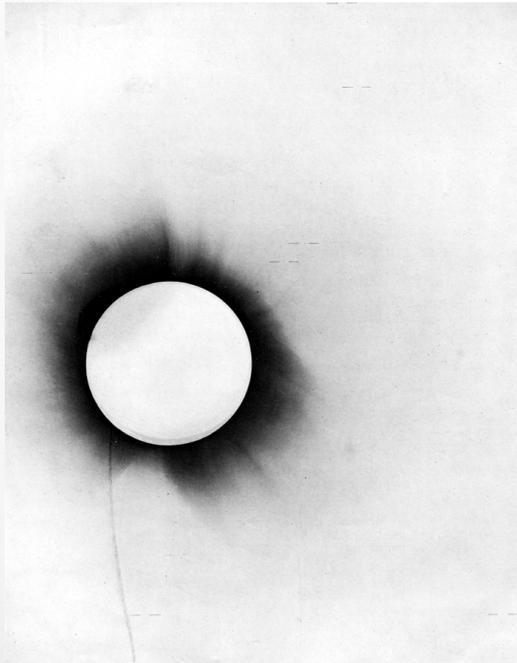
Tests of GR

- In 1919, during a total eclipse of the Sun, the predicted deflection of starlight for stars near to the limb of the Sun was measured and Einstein became a household name.
- GR also predicted that time would slow in strongly curved space. This was verified experimentally in 1958.



The Great Test GR

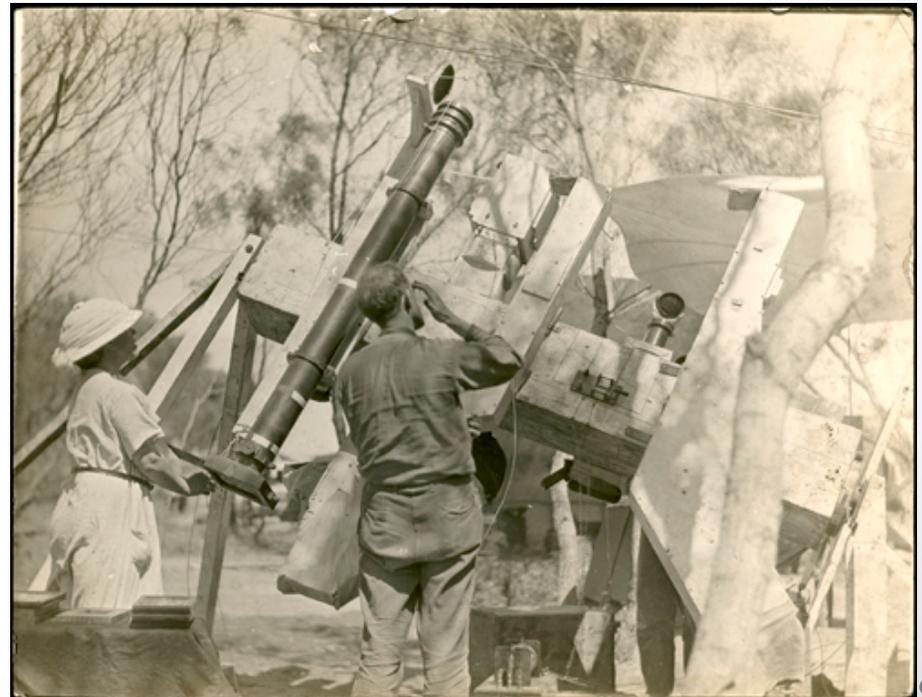
- In 1919, during a total eclipse of the Sun, the deflection of starlight for stars near to the limb of the Sun was measured by Eddington with observations from West Africa and Brazil



- It was announced that Einstein's prediction was correct and it was front-page news worldwide
- However, it was fairly quickly established that the observational errors were too large and the test was inconclusive

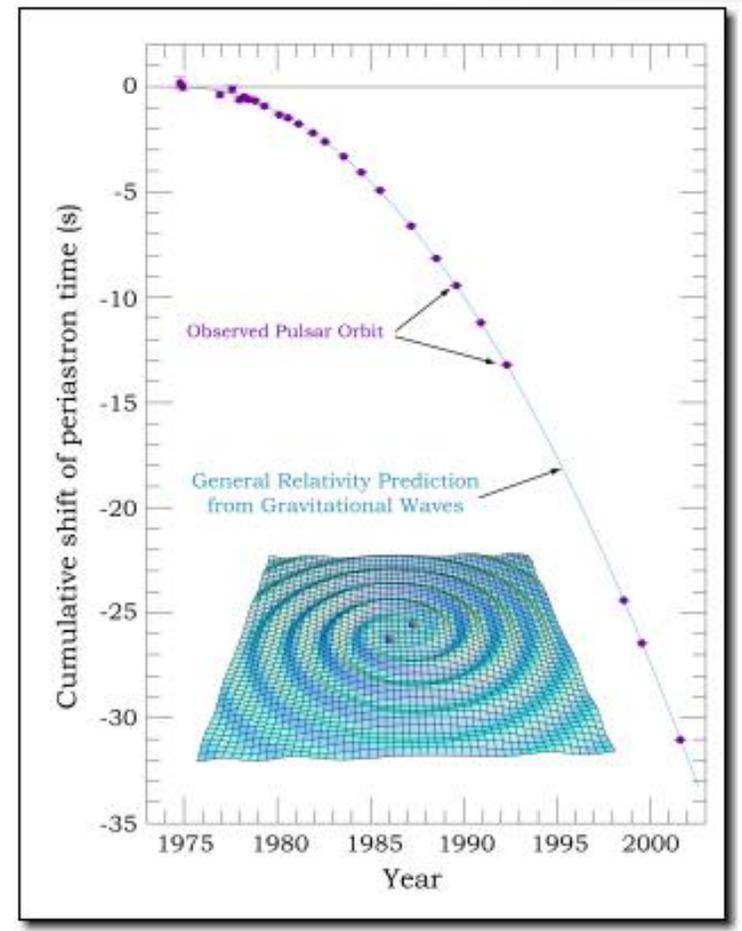
Lick Observatory and GR

The data from the famous eclipse expedition of 1919 weren't good enough to prove or disprove Einstein's theory. Lick Observatory Director WW Campbell made it the goal of the Observatory to carry out the observations that eventually supported GR.



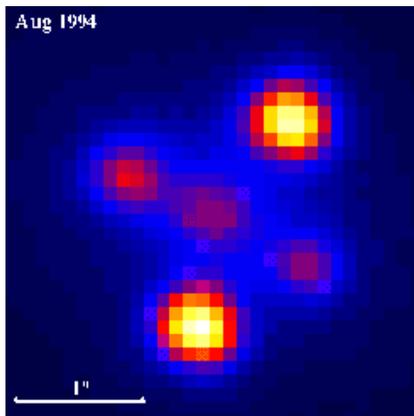
More tests of GR

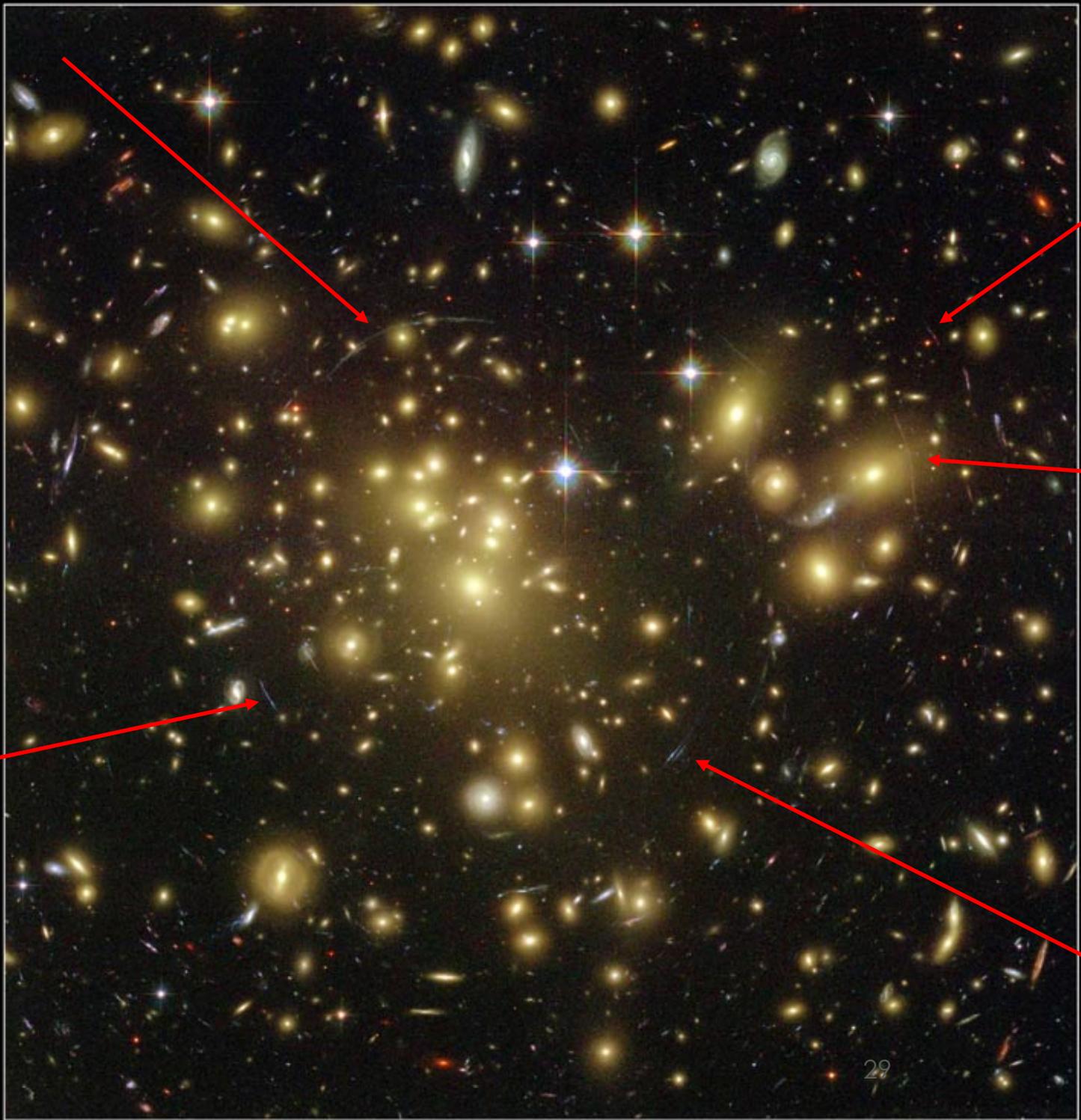
- There is now a long list of predictions made by GR and in every case to date, observations have they have been consistent with the theory.



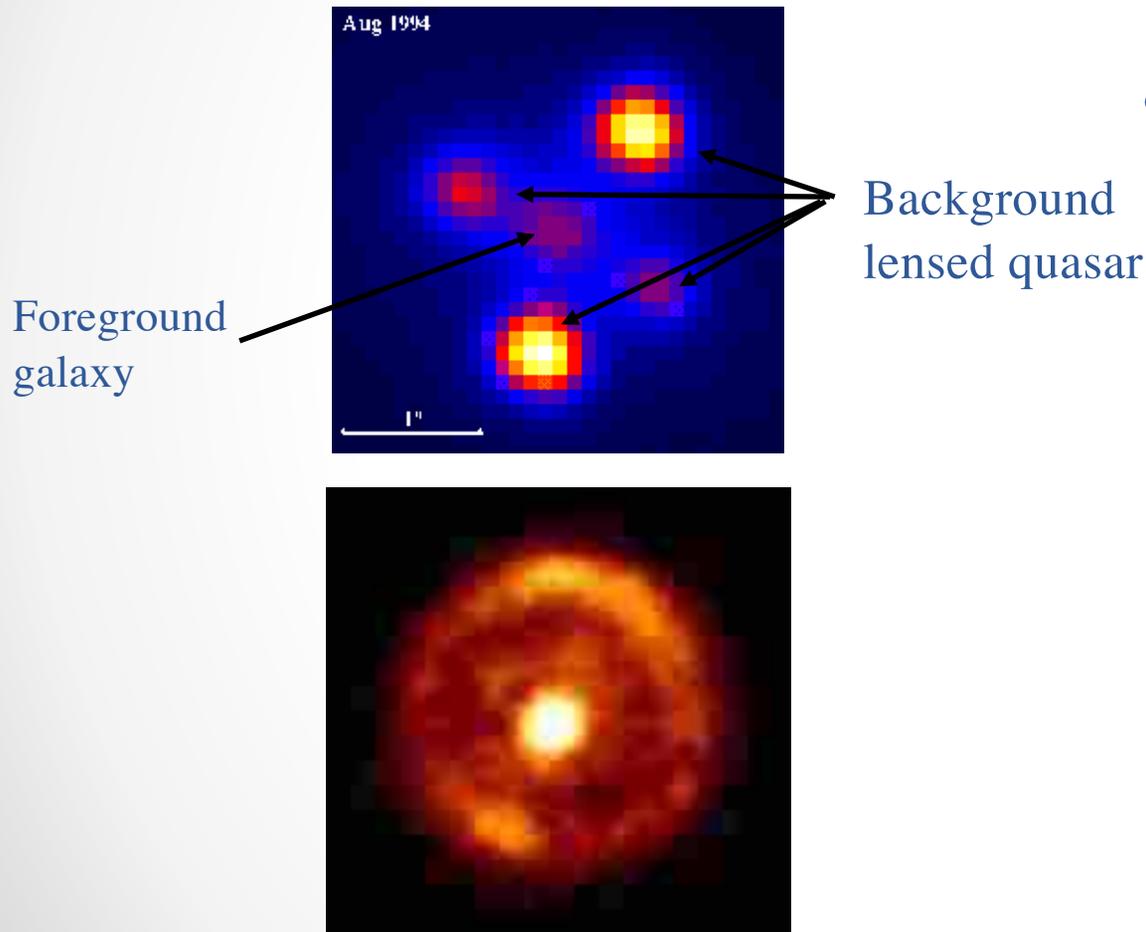
The Bending of Light II

- The bending of light in GR leads to some very useful and interesting phenomenon.
- One is the effect called a gravitational lens.
- The light from a distant galaxy is bent by a large mass along the line of sight to to Earth. If everything is lined up perfectly, you get an “Einstein Ring”. Very useful for identifying and measuring “Dark Matter”









- It is rare to have a close-to-perfect alignment. The more common case is to have a set of discrete images.

On to Black Holes

- The second very interesting aspect of light bending in General Relativity is the idea of Black Holes. It starts with the concept of escape velocity.



Escape Velocity

- Imagine feebly tossing a rocketship up in the air. It falls back to Earth because its kinetic energy was less than its gravitational potential energy.
- However, toss it with a larger and larger velocity and it will go higher and higher before falling back to Earth.
- There is a velocity above which it will not return to Earth -- this is the escape velocity.

Escape Speed

- To determine the escape speed from Earth you set the gravitational potential energy equal to kinetic energy and solve for speed

$$\frac{1}{2}mv^2 = \frac{GmM}{R}$$
$$v_{escape} = \sqrt{\frac{2 \times G \times M}{R}}$$

Mass of the
escaping
object

Mass of the
object from
which you
want to escape

Radius from which you want to escape

Escape Speed II

- Note that the escape speed doesn't depend on the mass of the escaping body.
- For the Earth, put in the mass and radius of the Earth (for escape from the surface of the Earth) and you get:

$$V_{\text{esc}} = 11 \text{ km/sec} = 25,000 \text{ miles/hr}$$

Escape Speed III

Suppose you increase the mass of the Earth by a factor of 4 without changing its radius. What happens to V_{esc} from the surface of the higher-mass Sun? (iClicker quiz)

- A. Escape speed will be higher
- B. Escape speed will be lower

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$
$$V_{esc} \propto \sqrt{M}$$

Escape Speed III

Suppose you shrink the Earth to 1/100 of its current radius (at constant mass). What happens to V_{esc} from the surface of the smaller Sun? (iClicker quiz)

- A. Escape speed is 1/100 of the original
- B. Escape speed is 1/10 of the original
- C. Escape speed is 100 times the original
- D. Escape speed is 10 times the original

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$
$$V_{esc} \propto \sqrt{\frac{1}{R}}$$

Escape Speed

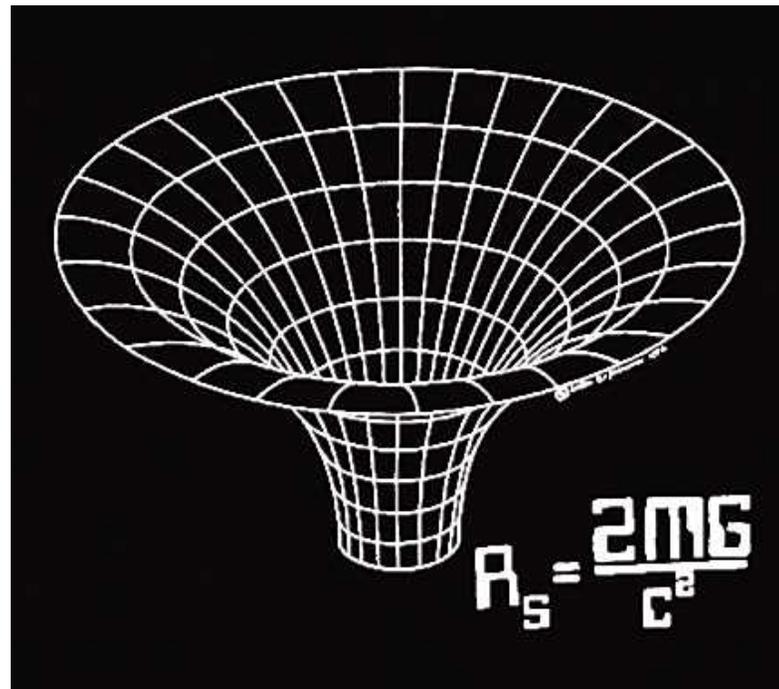
Reduce the radius of the Earth to 1cm and

$$V_{\text{esc}} = c \text{ (speed of light)}$$

- In this new theory of Gravity, where photons are affected by gravity, if the escape velocity equals or exceeds the speed of light, that object can no longer be observed. This is a Black Hole

Black Holes

- The critical radius for which an object of a particular mass has an escape velocity of equal to the speed of light is called the Schwarzschild Radius.
- The surface at this radius is called the “Event Horizon”.



Schwarzschild Radius

- You can easily calculate the Schwarzschild radius for any mass by setting $V_{esc} = c$

$$V_{esc} = c = \sqrt{\frac{2MG}{R_s}}$$
$$c^2 = \frac{2MG}{R_s} \Rightarrow R_s = \frac{2MG}{c^2}$$

- Every object has a radius at which it becomes a Black Hole

Black Holes

- But, it is VERY, VERY difficult to compress an object to its Schwarzschild radius.
- For the Sun, you would have to somehow overcome:
 - thermal pressure
 - e- degeneracy
 - neutron degeneracy

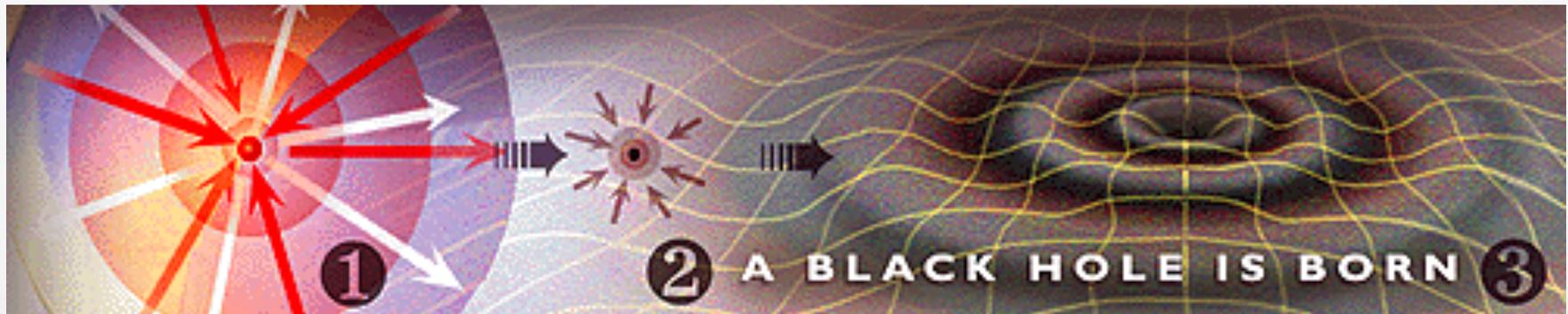
We know of no `cosmic vice' that can do that.

Black Holes

- But, go back to a neutron star and we are building a pretty big vice. Thermal pressure has already been overcome as has e- degeneracy pressure.
- And, there is a limit to the pressure that can be generated by neutron degeneracy. Its hard to calculate, but is probably between $2M_{\odot}$ and $3M_{\odot}$.

Black Holes

- Think about the n-star core of a SNII explosion. If say $1.6M_{\odot}$ of material falls back, the core will exceed the neutron degeneracy limit and undergo collapse to zero volume (what?).



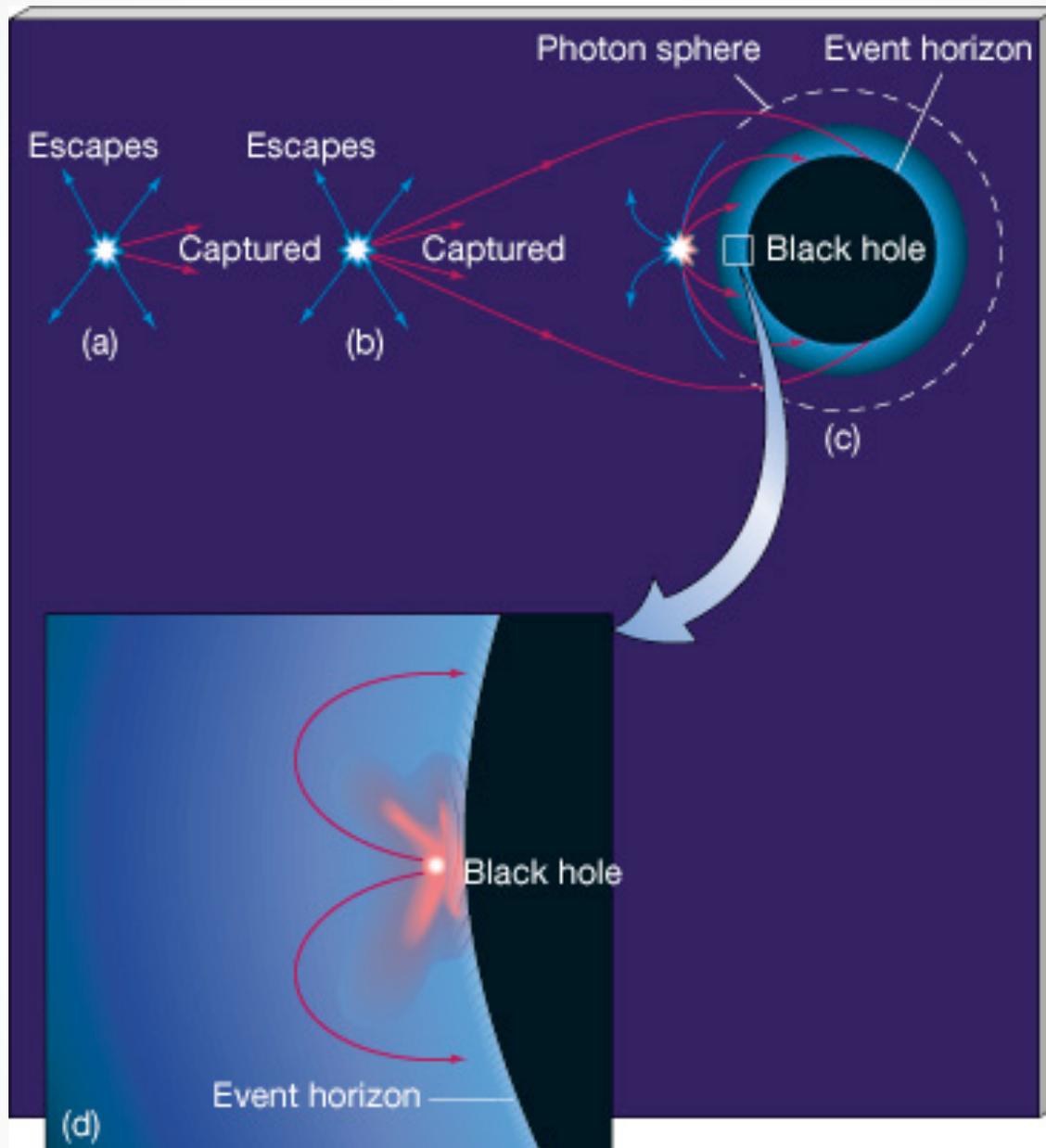
Black Holes

- What is left behind?
 - The gravitational force (i.e. a warp in spacetime) including a “singularity” at the center of the warp
 - An Event Horizon with radius given by

$$R_{\text{Sch}} = 8.9\text{km (for } 3M_{\text{Sun}} \text{ black hole)}$$



Hawking radiation



Black Hole FAQs

- What would happen if the Sun collapsed into a Black Hole, would the Earth be dragged in?

No, the gravitational force at the distance of the Earth would not change

- Is the Event Horizon a physical boundary?

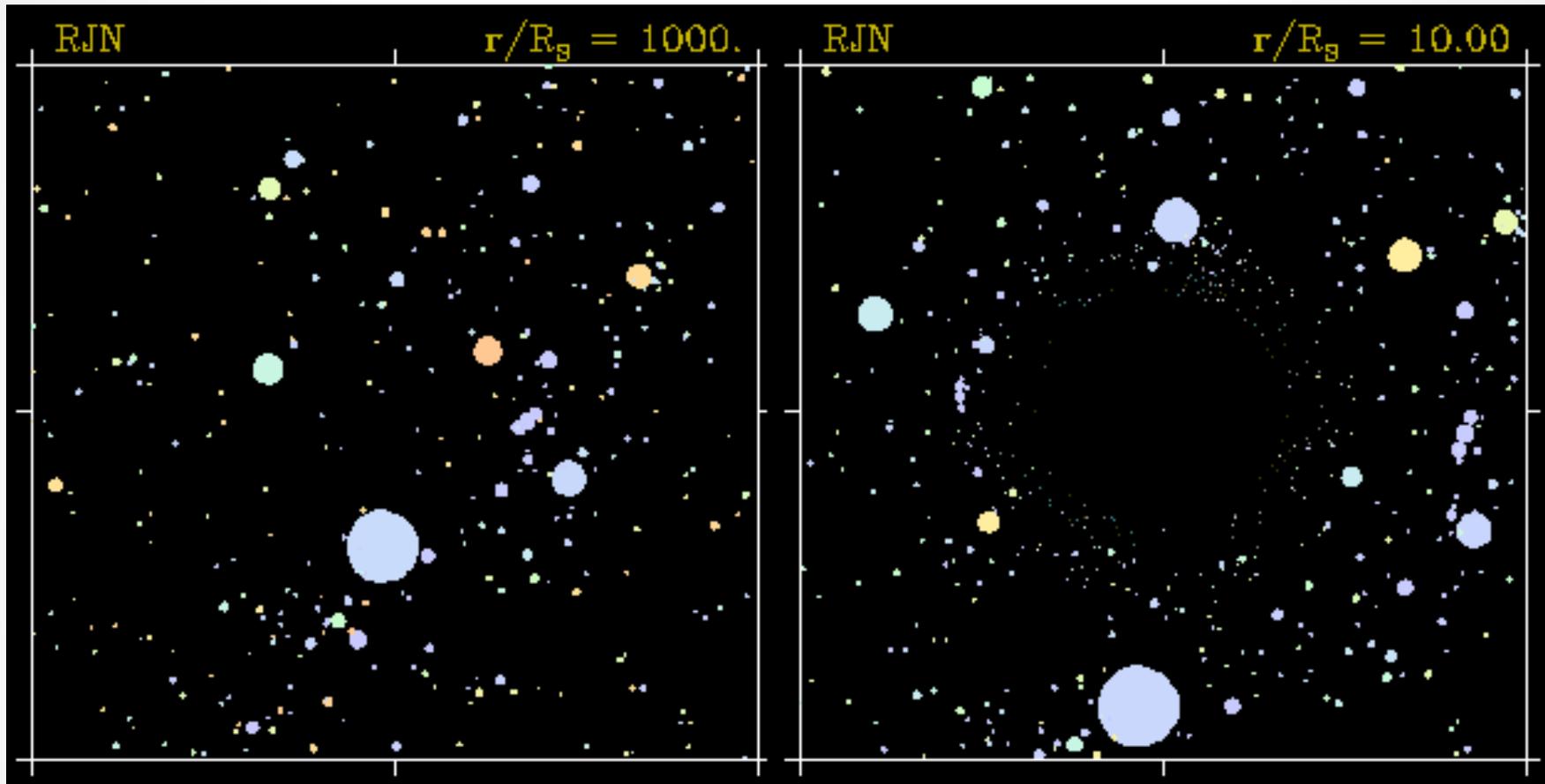
No, it is the distance from the center where the escape velocity of `c`.

- What happens if a Black Hole absorbs some mass?

As M increases, the Schwarzschild radius also increases.

- Is there any reason to believe that Black Holes exist?
Yes!

This would be great. But not too likely...



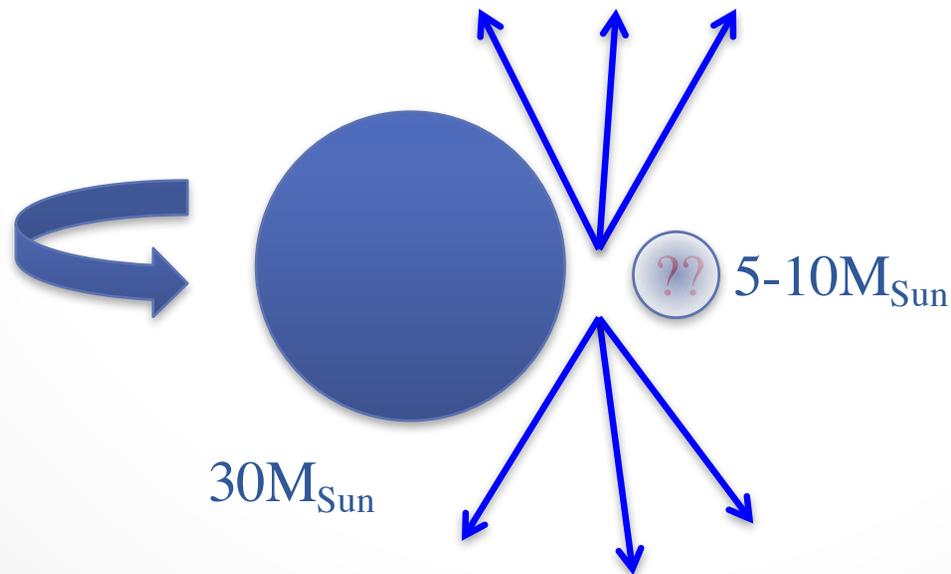
Black Hole Evidence

- The best stellar-mass cases are binary x-ray sources.
Cygnus X-1 is a good example.



Black Hole Evidence

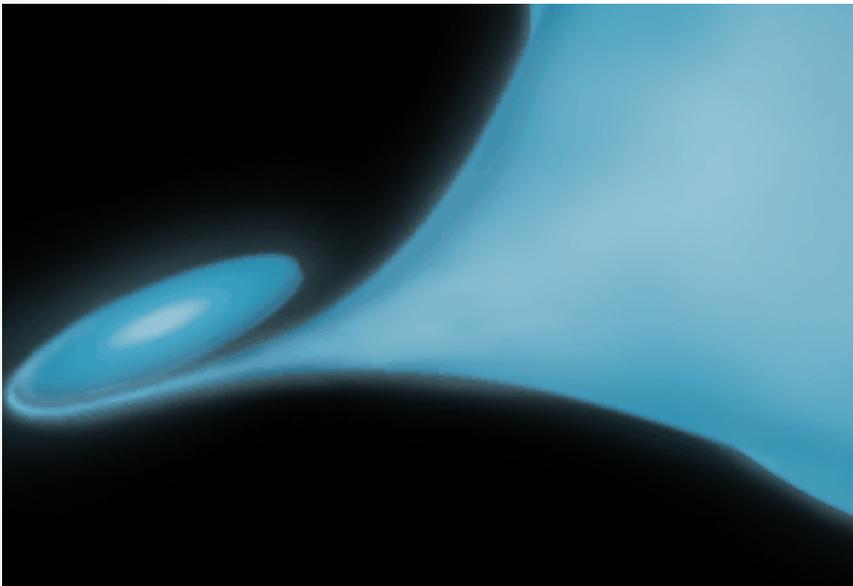
- Cyg X-1 is a bright x-ray source. Look there in the visual part of the spectrum, we see a $30M_{\text{Sun}}$ blue main-sequence star which is a spectroscopic binary with a period of 5.6 days.
- The companion has a mass of between 5 and $10M_{\text{Sun}}$. What is it?



Cygnus X-1

- There is no sign of the companion at any wavelength (but, remember the x-rays) so what is it?
 - 1) A red giant would be easily seen
 - 2) A main-sequence star would be seen with a little effort
 - 3) Can't be a WD because $M > 1.4M_{\odot}$
 - 4) Can't be a n-star because $M > 3M_{\odot}$

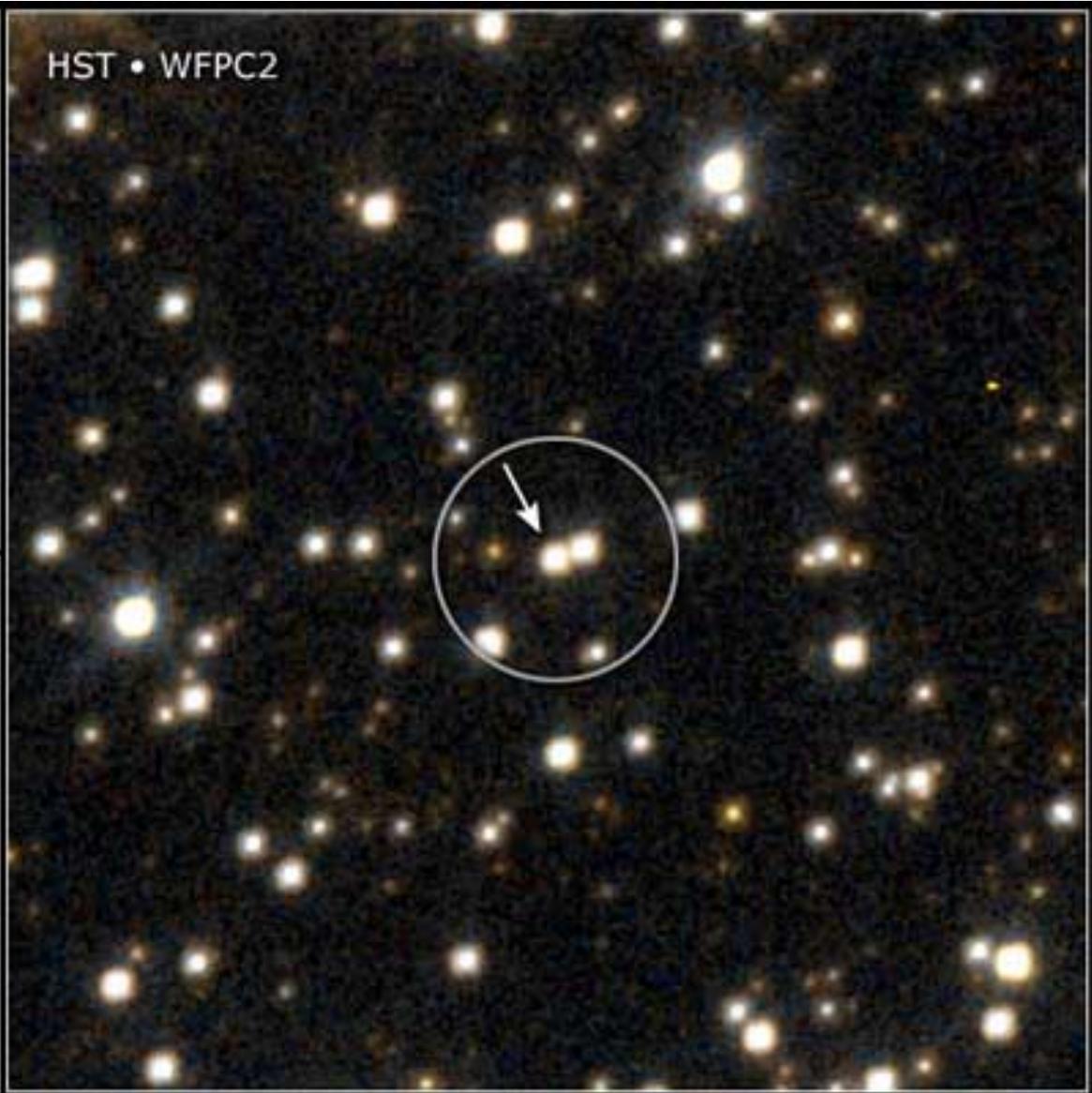
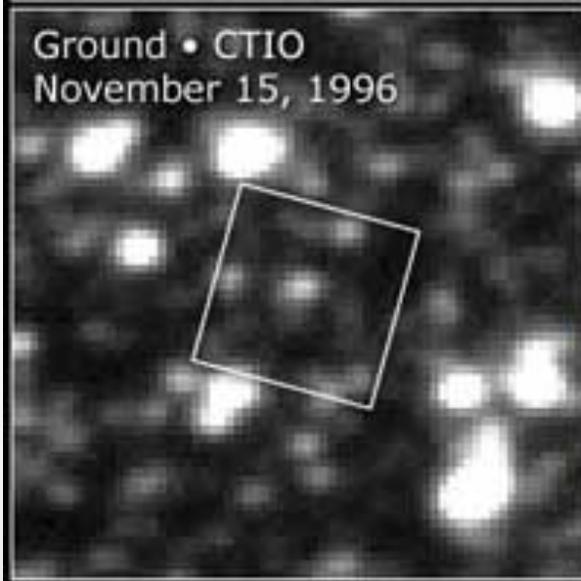
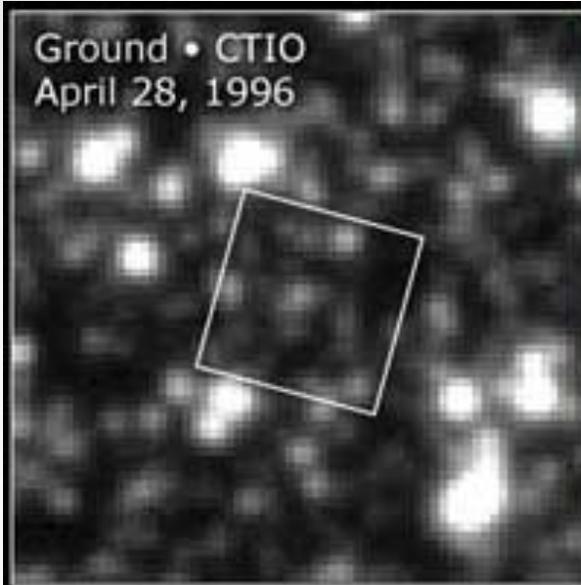
Cygnus X-1



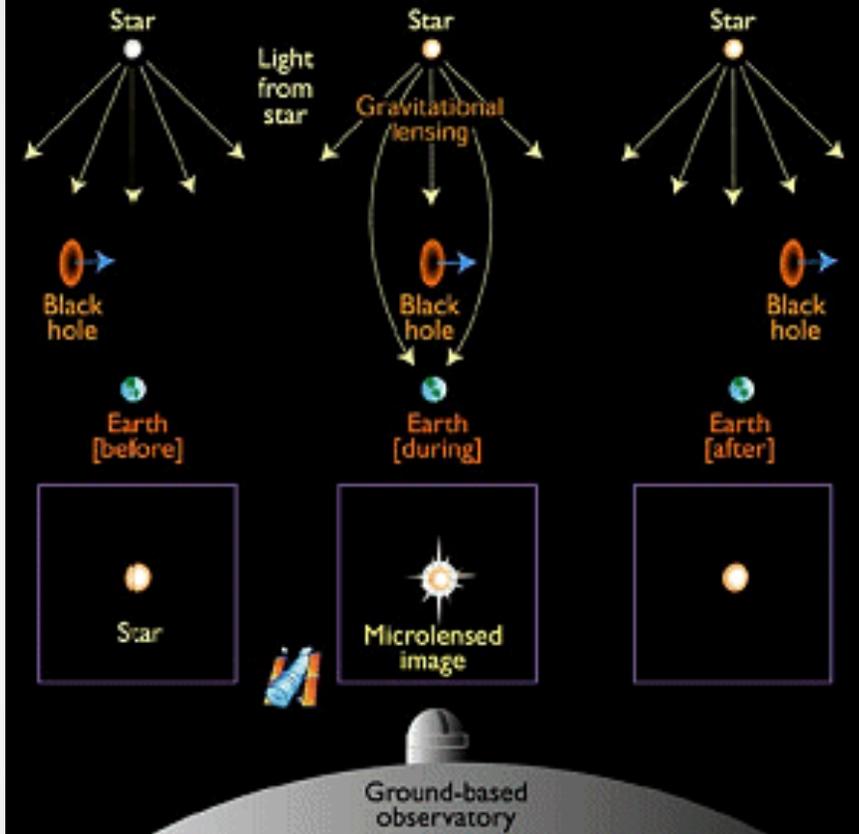
- By elimination, we are left with a black hole
- The x-rays back this up. In an accreting WD we see UV radiation, in a neutron-star we see `soft' x-rays, in Cyg X-1 we see `hard' x-rays because the accreting material falls into a deeper potential well.

Stellar-mass Black Holes

- We now have a few dozen excellent stellar-mass black hole candidates and few people doubt that such objects exist.
- There was a “microlensing” event in 1996 that was ascribed to a blackhole gravitationally lensing a background star.
- There are various claims that x-ray transients are black holes accreting little bits of stuff.



Gravitational Microlensing by Black Hole



- Now have six lensing objects with mass $> 6M_{\text{SUN}}$
- A main-sequence or giant star would be easily visible
- Too massive to be remnant like White Dwarf or Neutron Star