

# AY1 Announcements

- Quiz 4: Thursday June 8 (last day of class)
- Final: June 14, 4pm -7pm

# June 1: Mandel Lecture

Mandel Lecture June 2017

## Einstein, Gravitational Waves and Black Holes

Dr. Gabriela Gonzales

DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS AND THE UC OBSERVATORIES PRESENT THE FOURTH MANDEL LECTURE

UC SANTA CRUZ

**EINSTEIN  
GRAVITATIONAL  
WAVES AND  
BLACK HOLES**

More than 100 years ago, Einstein predicted that there were ripples in the fabric of space-time traveling at the speed of light: gravitational waves. In 2015, gravitational waves created by the merger of two black holes were detected for the first time by the LIGO detectors in Hanford, Washington, and Livingston, Louisiana. These observations marked the beginning of gravitational wave astronomy. We will describe the history and details of the observations and the bright future of the field.

Astrophysicist **Gabriela González**

astro.ucsc.edu

**THURSDAY, JUNE 1, 2017, 7:30 PM RIO THEATRE**  
THE RIO THEATRE IS LOCATED AT 1205 SOQUEL AVENUE, SANTA CRUZ. **FREE AND OPEN TO THE PUBLIC.**

No tickets or reservations will be required for entry. Seating availability is on a first come, first served basis. Theater will be open for seating starting at 7:00 pm. The lecture is presented as a part of the Mandel Lecture Series, which promotes public awareness and appreciation for astronomy. We seek your help and support in keeping these lectures going. For information on giving, please visit: [astro.ucsc.edu/support-us](http://astro.ucsc.edu/support-us). For more information or for disability-related needs, please call 831-424-3271. 6/1/17

Thursday, June 1, 2017

7:30 pm

Rio Theatre - 1205 Soquel Avenue, Santa Cruz

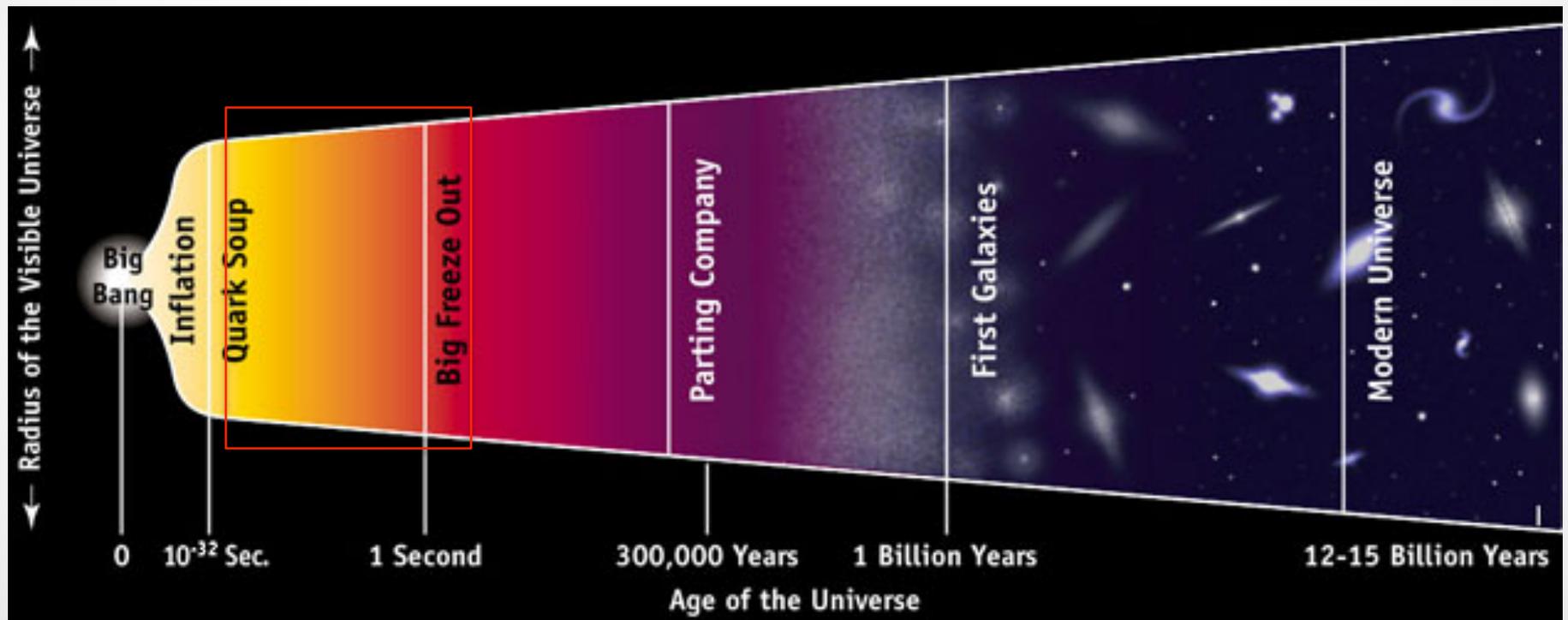
Admission is FREE and open to the general public.

No tickets or reservations will be required for entry. Seating availability is on a first come basis.

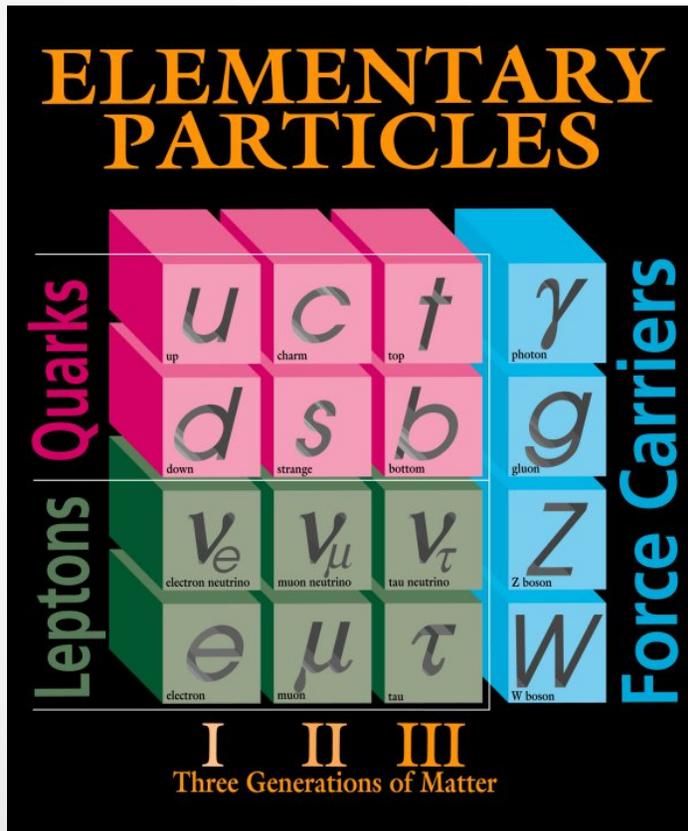
Theater will be open for seating starting at 7:00 pm.

# From Last Time

# Universe Timeline



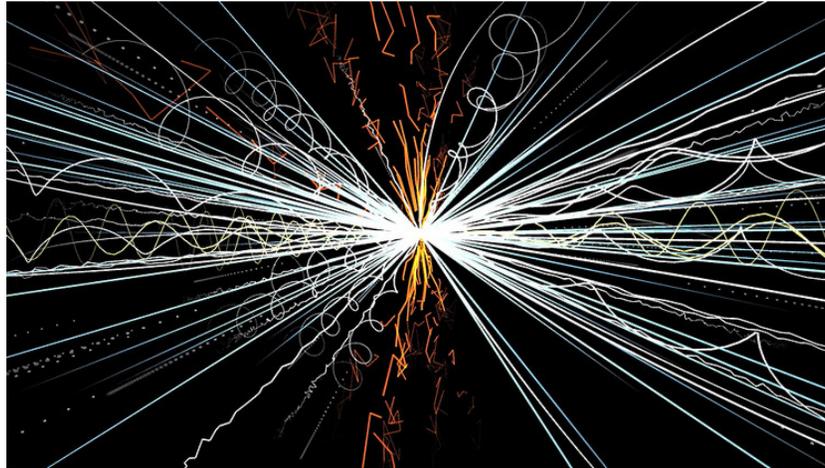
# Table of Fundamental Building Blocks



Fermilab 95-759

- Fermions: fundamental particles.
  - Leptons
  - Quarks
- Hadrons: combinations of Fermions (i.e. proton, neutron)
- Bosons: particles that exchange forces

# Creation of Matter



- The threshold energy for creating a particle is its “rest mass energy” given by  $E=mc^2$
- In the early universe, as the temperature goes up so does the mass of particles created

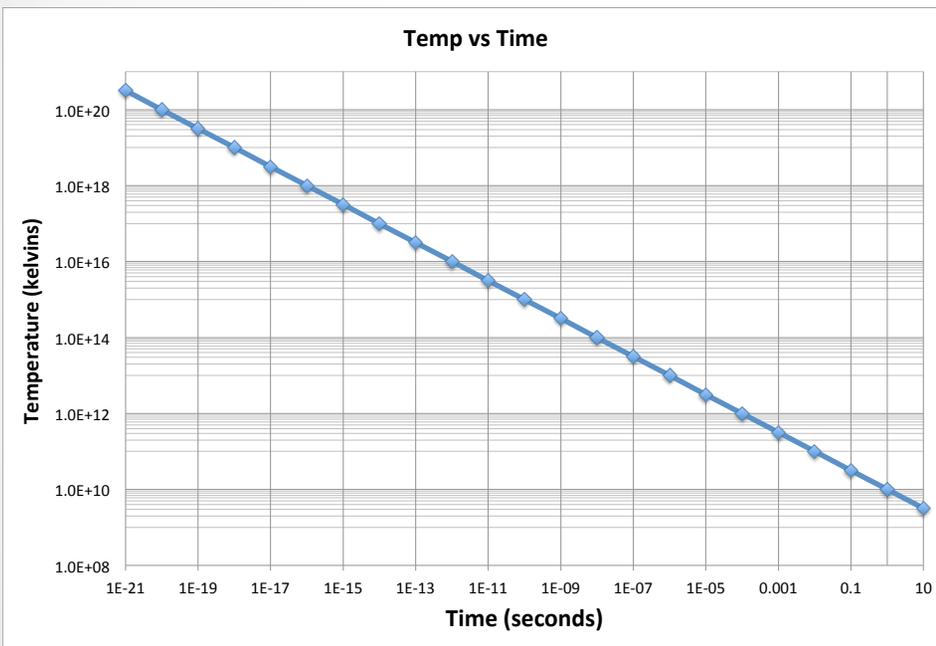
# Putting these ideas to work

- The temperature of the Universe after the Big Bang is given by:

$$T(\text{kelvin}) = \frac{10^{10}}{\sqrt{t \text{ (seconds)}}}$$

- The average energy of the universe is given by:

$$E(\text{eV}) = \frac{8.6 \times 10^5}{\sqrt{t(\text{seconds})}}$$



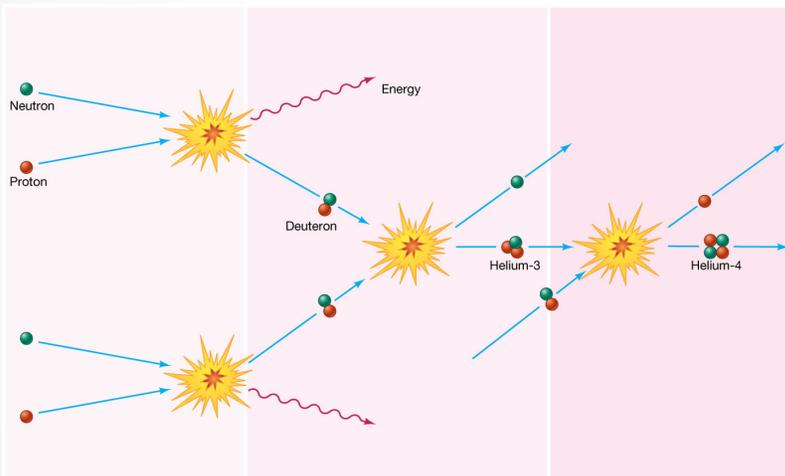
# Energy to matter

- Very early on, the Universe is dominated by energetic radiation and matter-antimatter pairs are formed and broken apart
- As the average energy per volume of the Universe drops, matter starts to dominate
- When the average energy drops below 2x the rest mass energy of a particle, that particle is no longer created

# Primordial Nucleosynthesis

Principle fusion chain:

- $p^+ + n^0 \rightarrow H^2$  (deuterium)
  - No electrical repulsion
- $H^2 + H^2 \rightarrow H^3 + p^+$
- $H^3 + H^2 \rightarrow H^4 + n^0$

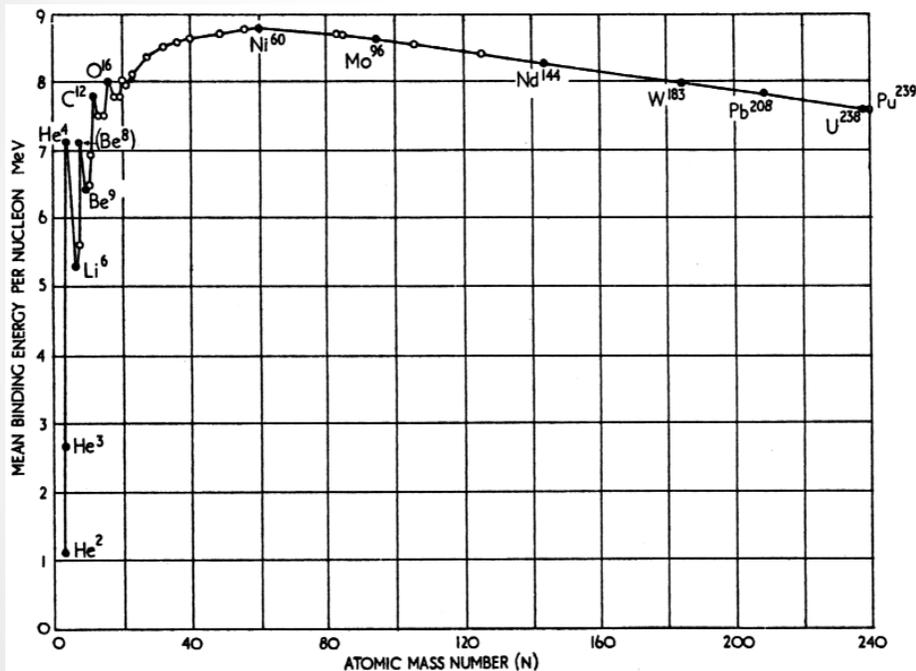


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$H^2$  is relatively easily broken apart by energetic photons.

*This is what starts the nucleosynthesis chain going: Universe cools to the point where very energetic photons are rare and  $H^2$  is available*

# The Surprise



Natural next steps:



Be<sup>8</sup> is unstable with a half life of  $7 \times 10^{-17}$  seconds!

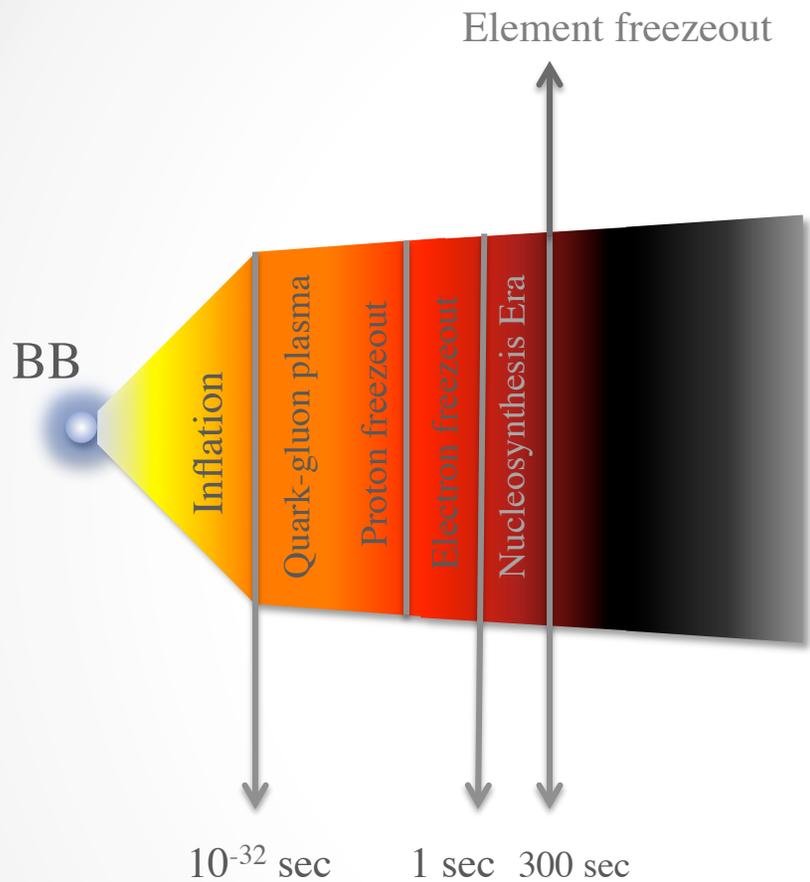


Unstable again

- Other isotopes of Be and Li are stable, but energetically not favored. Makes these in small amounts

# The Surprise

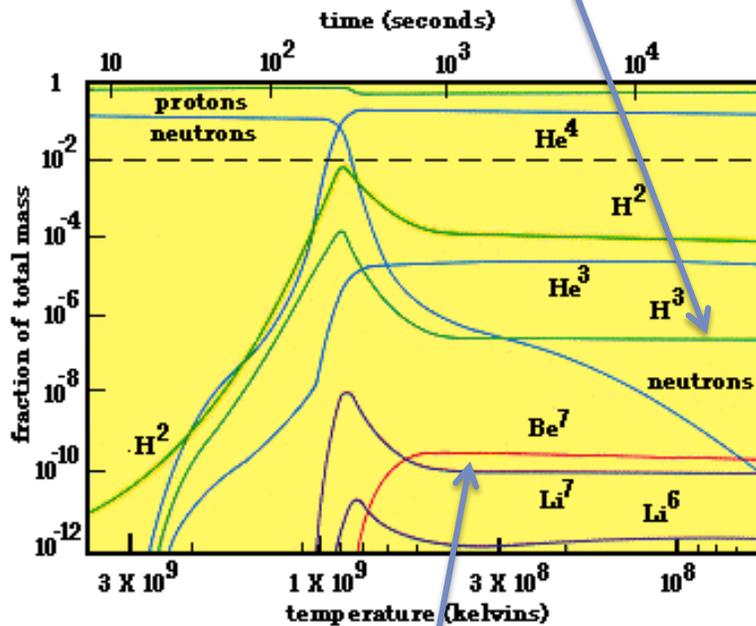
- By  $10^3$  seconds after the BB, the temperature of the Universe has dropped to values too low to support fusion reactions
- Produced lots of hydrogen, 25% by mass of  $\text{He}^4$ , some  $\text{H}^2$ ,  $\text{H}^3$ ,  $\text{He}^3$ ,  $\text{Li}^7$ ,  $\text{Li}^6$ ,  $\text{Be}^7$
- This was used as an argument against the Hot Big Bang model: where did all the other elements come from?
- Required a slight asymmetry in matter vs anti-matter



- Pair-production produces matter-anti-matter pairs from photon field (conservation laws)
- Hadron Era with production of  $p^+$  and  $n^0$  ends when energy of the photon field is  $< 2 \times 938.3 \text{ MeV}/c^2$
- Lepton Era lasts longer as  $e^-+e^+$  total mass is  $2 \times 0.511 \text{ MeV}/c^2$  and it takes the Universe longer to cool to this equivalent energy
- Nucleosynthesis starts when deuterons ( $p^++n^0$ ) are no longer being blasted apart by energetic photons
- Nucleosynthesis ends when temperature is too low for fusion

# Big Bang Nucleosynthesis

$H^3$  decays into  $He^3$  (12 years)



$Be^7$  decays into  $Li^7$  (53 days)

- BB+1 second: electrons, photons, neutrons, protons
- BB+2 minutes: some  $H^2$  (p+n) produced
- BB+4 minutes: He production+tiny amount of Be, B and Li
- That's all! Universe has expanded to  $10^9$ K and a density of only  $10 \text{ g/cm}^2$

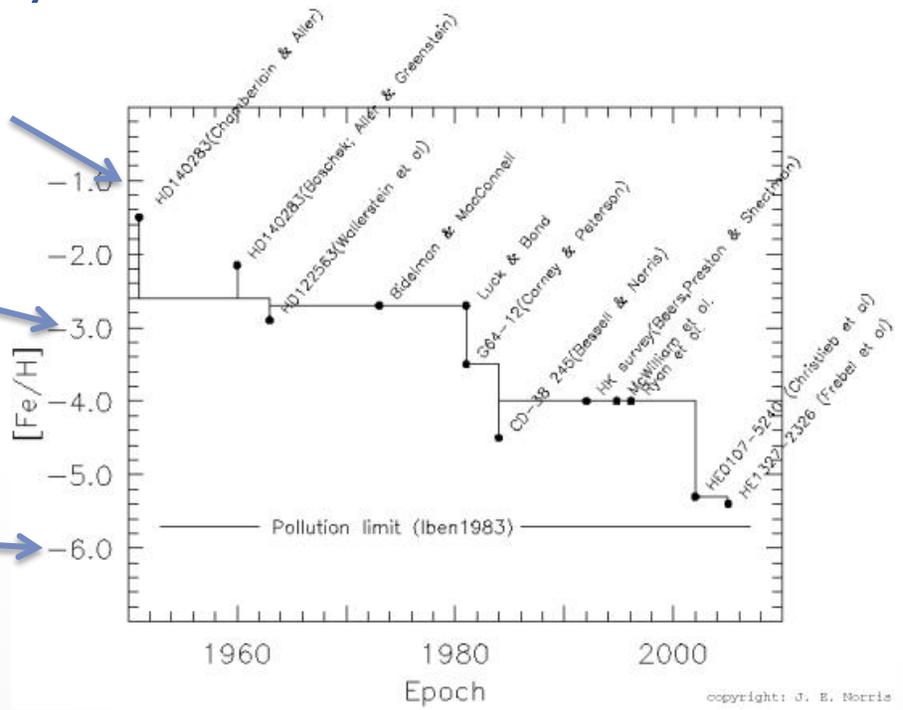
# Stellar Abundance Record

- 1950, first “chemically deficient” stars were discovered (another astro-lady: Nancy Roman).

Factor 10 lower abundance of Fe compared to the Sun

Factor  $10^3$  lower abundance of Fe compared to the Sun

Factor  $10^6$  lower abundance of Fe compared to the Sun



# New Material

# Pristine Gas in the Early Universe

physicsworld.com

Fill

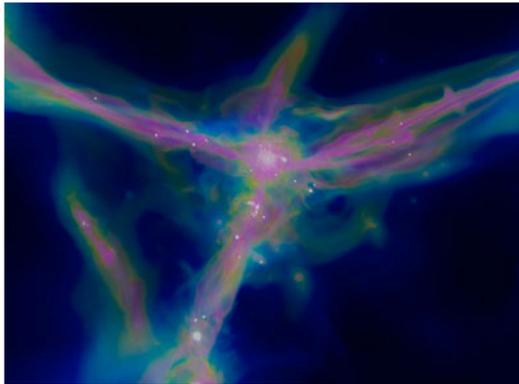
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## News archive

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## Pristine relics of the Big Bang spotted

Nov 10, 2011 [27 comments](#)

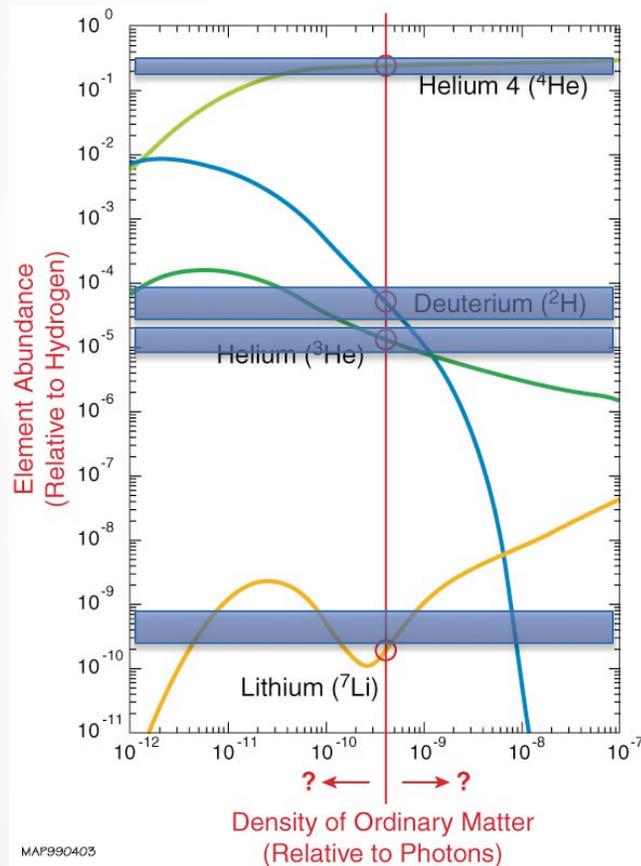


An artist's impression of an ancient cloud forming into a star

For the first time, astronomers have discovered two distant clouds of gas that seem to be pure relics from the Big Bang. Neither cloud contains any detectable elements forged by stars; instead, each consists only of the light elements that arose in the Big Bang some 14 billion years ago. Furthermore, the relatively high abundance of deuterium seen in one of the clouds agrees with predictions of Big Bang theory.

- In addition to looking for old stars made of primordial material, can look directly back in time at large look-back times
- 2011, UCSC astronomers made the first discovery of pristine clouds of primordial material

# HBB Nucleosynthesis



- Calculations make specific predictions for the “primordial” abundance of H,  $\text{H}^2$ ,  $\text{He}^3$ ,  $\text{He}^4$  and  $\text{Li}^7$
- Depends on the ratio of “baryons” to photons at the time of nucleosynthesis (plus a few other interesting things like the number of neutrino families)

# HBB nucleosynthesis looks pretty good!



- Predictions of the HBB nucleosynthesis are verified by observations in detail



# New: Next HBB test

- Cosmic Microwave Background



# Cosmic Microwave Background

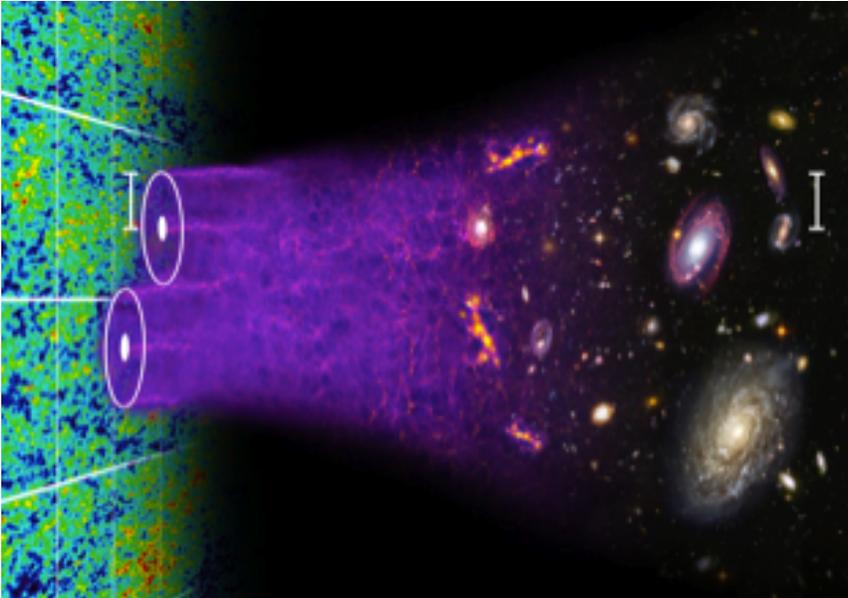


- Gamov and his collaborators continued their investigations and noted that after the era of nucleosynthesis, the Universe remained hot enough for almost 380,000 years for hydrogen atoms to be ionized
- Photons travel a very small distance before being “scattered” off electrons and protons in this plasma
- The Universe was foggy

# CMB

- At 370,000 years after the Big Bang, the Universe has cooled to 3000K and electrons combined with nuclei
- Now, the vast majority of photons no longer are absorbed by atoms or scattered by free electrons: the Universe became transparent
  - As we look back in time we should see the “surface of last scattering” in every direction redshifted by a large factor
  - And/or radiation of the 3000K plasma fills the Universe and has been diluted by the subsequent expansion to look like that from a 3K object

# CMB



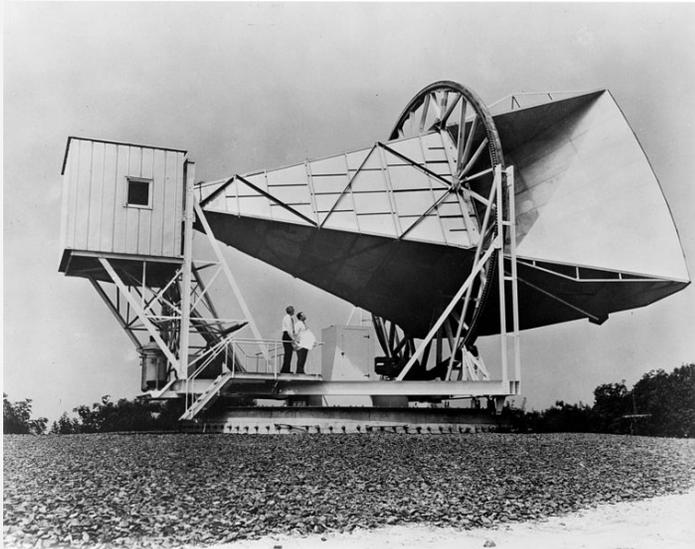
Wien's Law:  $\lambda_{\text{max}}(\text{m}) = 3 \times 10^{-3} / T(\text{k})$

$\lambda_{\text{m}} = 1\mu$  for 3000k plasma at 370,000 years

$\lambda_{\text{m}} = 1\text{mm}$  for 3k equivalent today

- Gamov, Alpher, Hermann work was rediscovered by Russian astronomers and Robert Dicke at Princeton in the early 1960s
- A group from Princeton started to build a radio telescope to measure the “3°” background radiation or “cosmic microwave background”

# CMB



As the Princeton group started to build a radio telescope to test this hypothesis, two physicists at Bell Labs in 1964 had already detected the radiation without quite realizing what they were looking at

# Penzias and Wilson



“A measurement of excess antenna temperature at 4080 Megacycles per second”

1978 Nobel Prize

- Arno Penzias and Robert Wilson at Bell Labs developing a microwave horn antenna designed to measure reflected radio signals from Echo balloon satellites
- Background noise in the receiver was a factor of 100 higher than expected
- Saw it everywhere in the sky, day and night
- Cooled detectors to liquid-helium temperatures, removed “dielectric material” from the horn. No change.

# Penzias and Wilson



- In a conversation with MIT physicist Bernie Burke, they realized it was the radiation the Princeton group was looking for
- Burke put them in touch with Jim Peebles and Robert Dicke and they realized the observations matched the noise data almost perfectly.

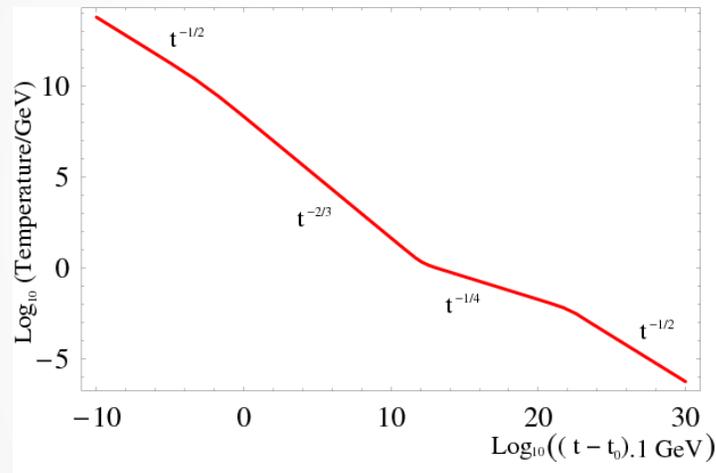


# 3-degree Microwave Background

If due to the expansion of the Universe, the temperature of the Universe is given by:

$$\text{Temp} = 2.725(1+z)$$

At 380,000 after the Big Bang, the redshift of the Universe is about 1100 hence the spectrum of the 3000K hot plasma is redshifted and looks like the spectrum from a 2.75K plasma

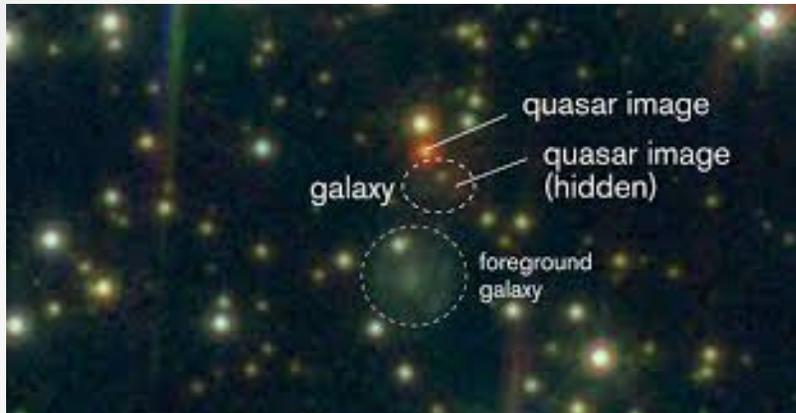


# A check on the evolution of temperature

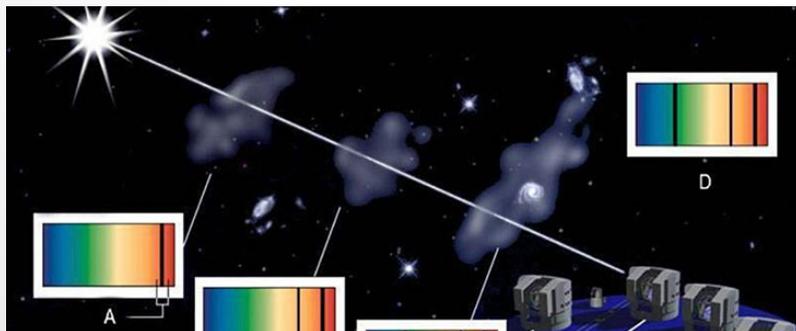


- Andrew McKellar in 1941 measured the state of the molecules CN and CH in the interstellar medium and concluded there was an ambient “temperature” of empty space of 2.35K
- He measured the excitation of the molecules by the cosmic microwave background!

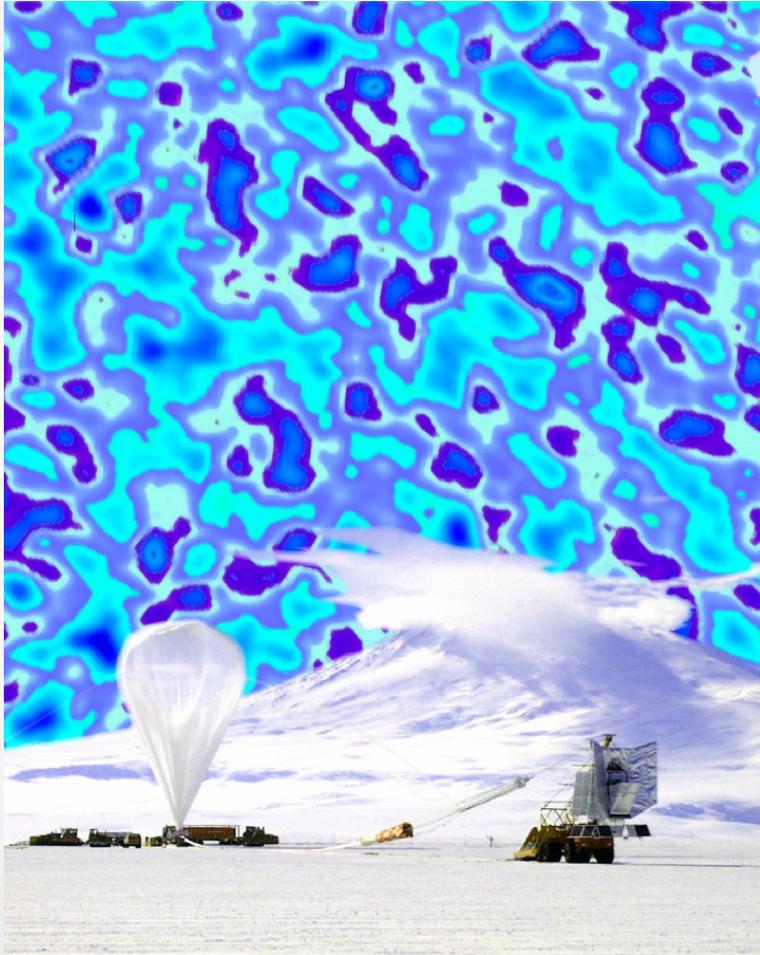
# Double Check



- The baseline background temperature of the Universe has been now measured via this technique for gas at seen at 7 billion years ago and 11 billion years ago
- 7Gyr:  $T=5.10\text{K}$  (5.15K predicted)
- 11Gyr:  $T=9.15\text{K}$  (9.3K predicted)



# CMB Fluctuations



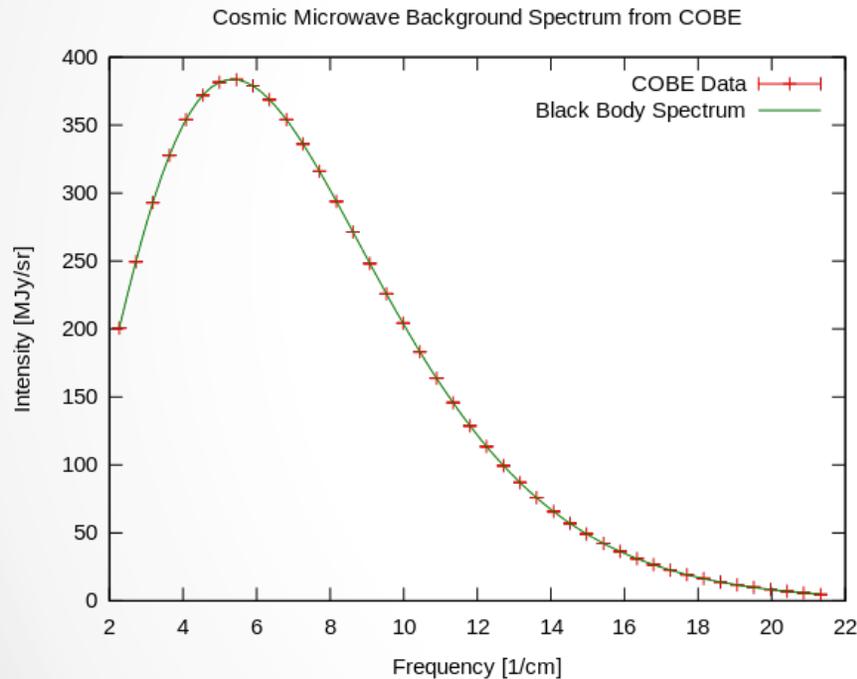
- It was recognized that the CMB could not be strictly uniform and still have the cosmic structures that we see in the Universe
- The CMB was predicted to have “hot” and “cool” spots at the level of *one part in 100,000*
- If the fluctuations existed, all competing HBB theories were finished
- This was a very hard measurement from the ground and for a decade, researchers pushed closer and closer to the 1/100,000 limit without seeing anything

# CMB Fluctuations



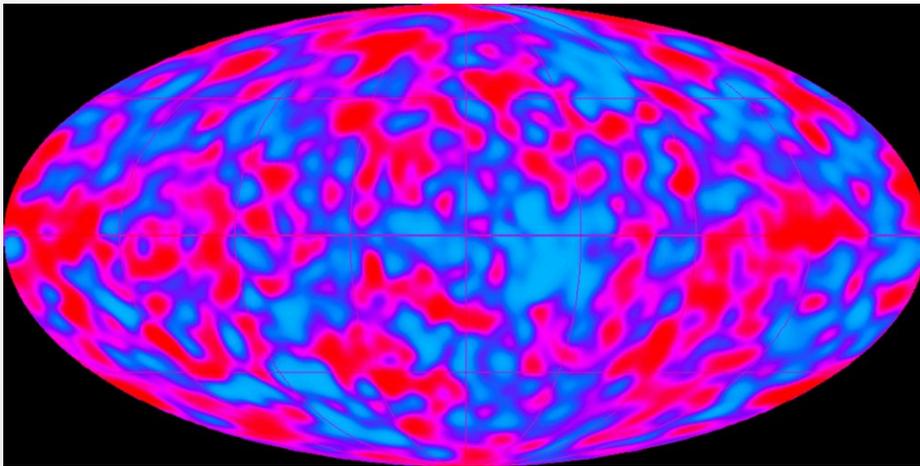
- COBE was a satellite designed specifically to measure the CMB.
- Launched on a Delta rocket in 1989
- Soviet's had launched a satellite to make the same measurement in 1983, but did not initially see fluctuations in the data

# CMB Fluctuations



First COBE result was that the cosmic background radiation spectrum was spectacularly fit by a “blackbody” spectrum for a 2.725K source just as predicted by the HBB and an expansion of the Universe by a factor of  $\sim 1100$  since decoupling

# CMB Fluctuations

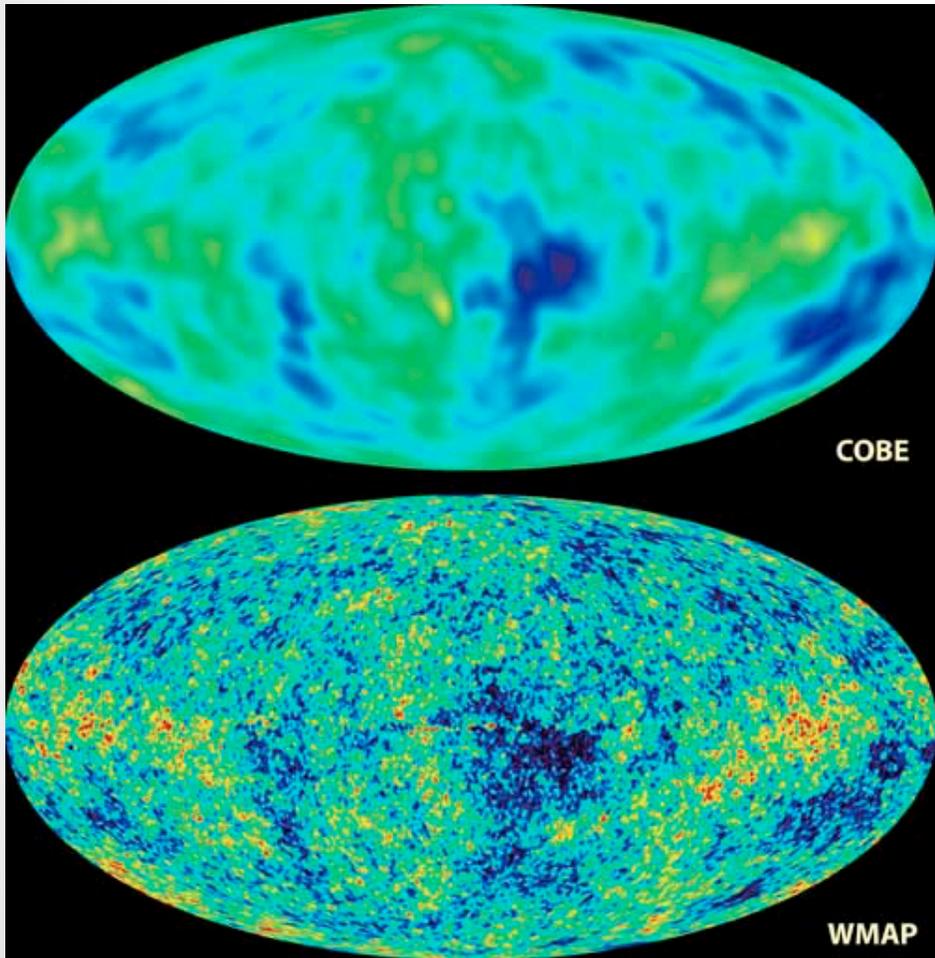


- Second result of COBE was the measurement of CMB fluctuations at the predicted level of 1 part in  $10^5$
- More Nobel Prizes! George Smoot and John Mather
- The growth of these tiny fluctuations into galaxies and clusters of galaxies today is the subject of the rest of the class
- There are  $\sim 400$  microwave photons per cubic centimeter everywhere in the Universe



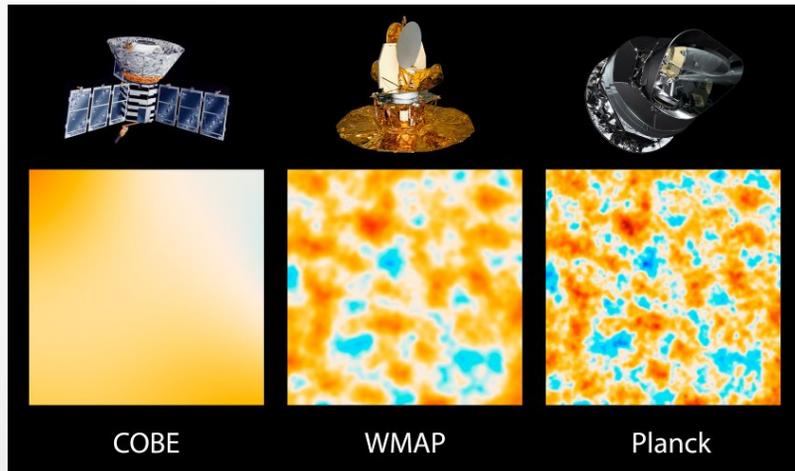
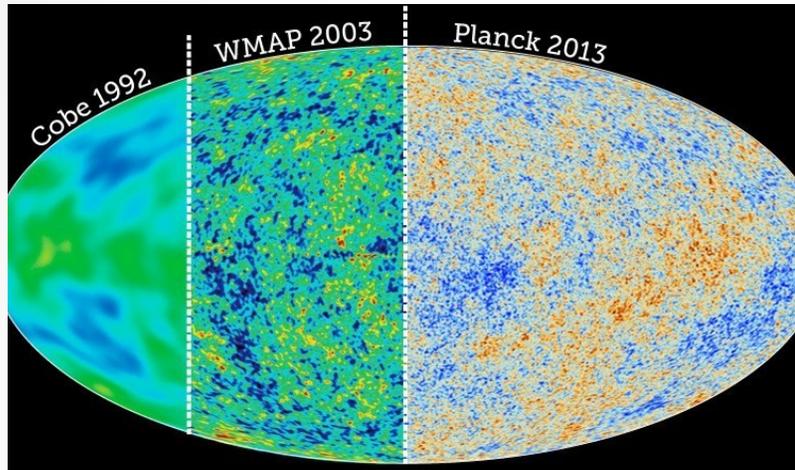
- 2006 Nobel Prize to George Smoot at UC Berkeley
  - Named parking spot at Berkeley
  - Guest appearance on the The Big Bang television show
  - Only Nobel Prize winner to receive The Rolling Wave at a UCB football game
  - \$1M winner on “Are you smarter than a 5th grader?”
- Soundly trounced by MB on Science Jeopardy

# WMAP



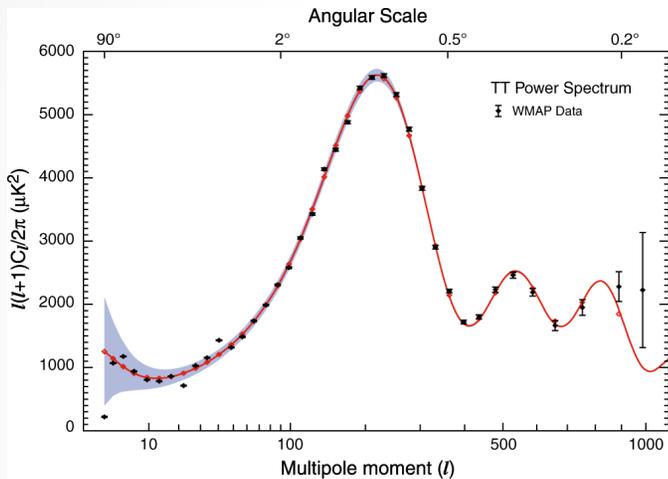
- COBE was followed by two additional space missions with greater sensitivity and higher spatial resolution
- WMAP had 33 times better resolution and 45 times the sensitivity
- Total range in temperature is only 0.0004K

# Planck

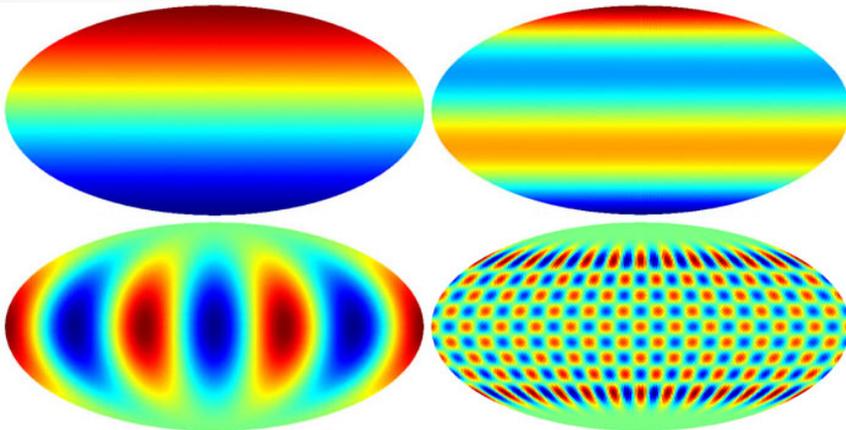


The European Space Agency launched Planck in 2013 and provided an independent measurement with even better precision than WMAP

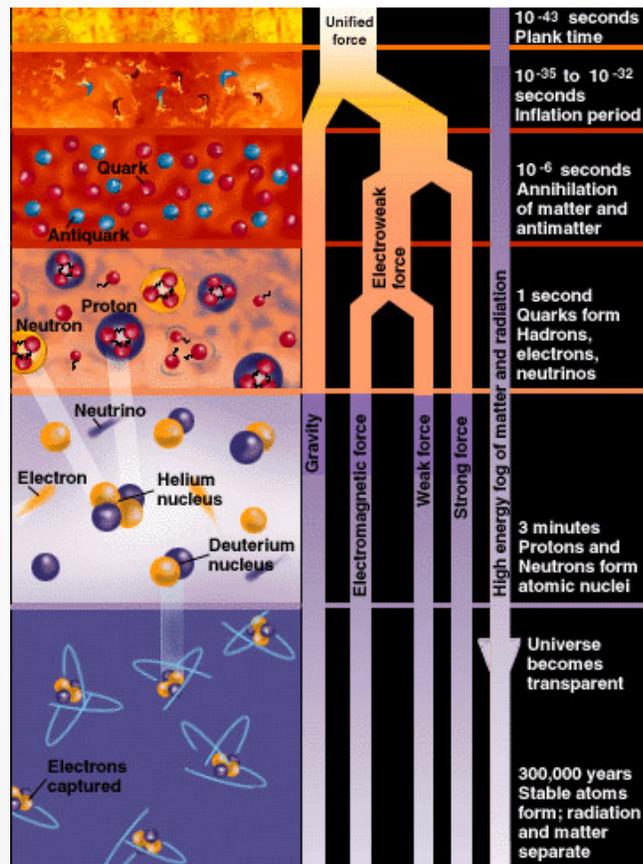
# Quantifying anisotropy



- The “power spectrum” describes the distribution of fluctuations on different spatial scales on the sky.
- The predictions for where the correlations are strong and weak depend on physical properties of the Universe at the time of decoupling

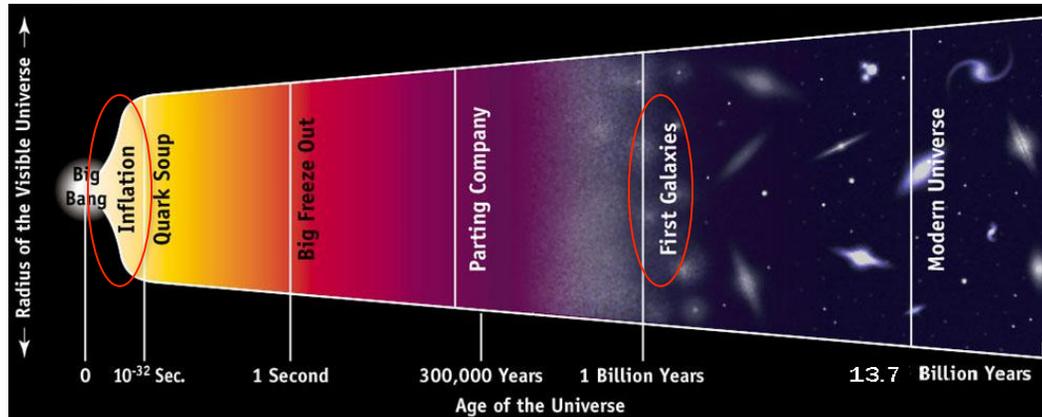


# HBB test #2

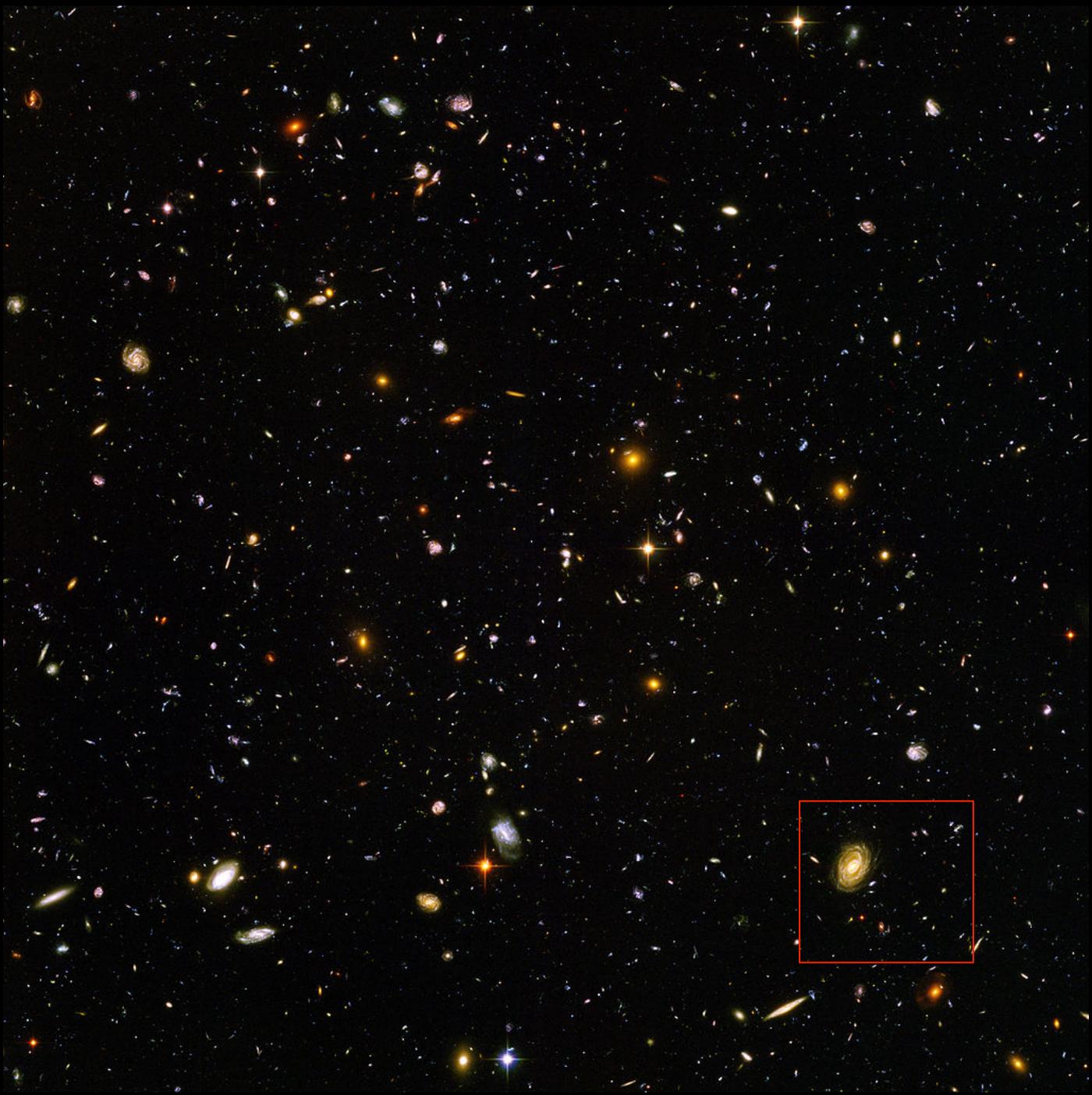


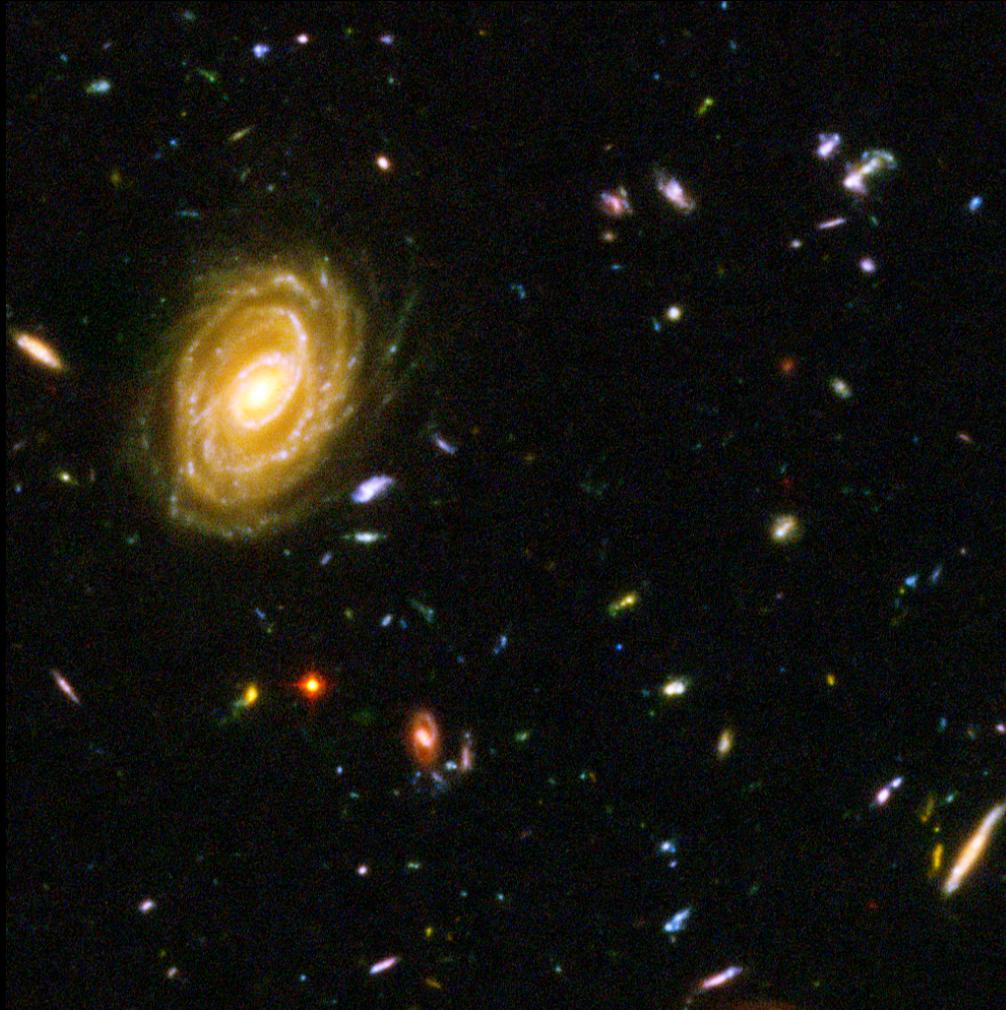
- CMB and fluctuations in the CMB are the second major test of the HBB that it passes with flying colors

# Last topics



- Shape of the Universe
- Formation of structure
- Inflation
- The cold, dark future





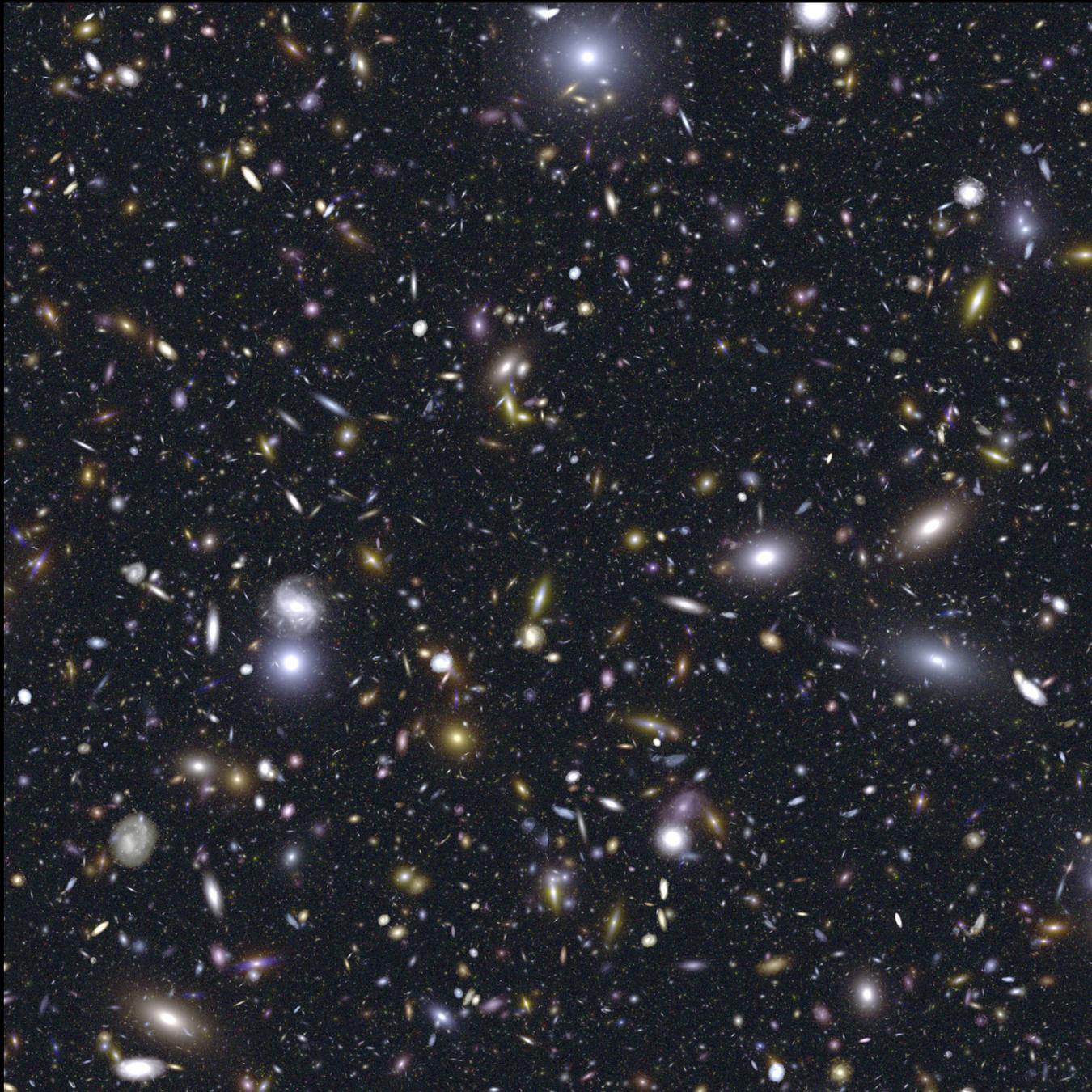
# Hubble Probes the Early Universe

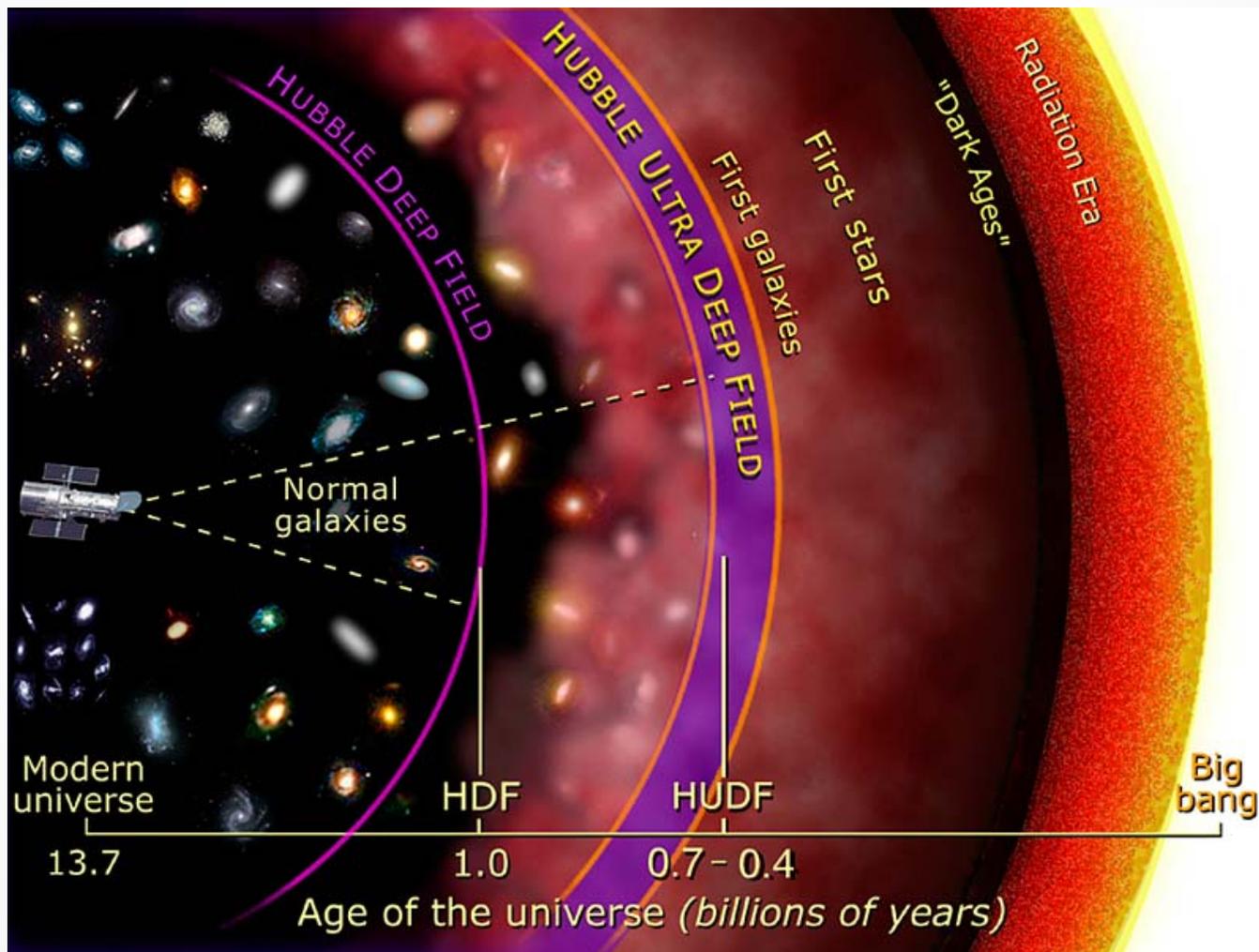


**Redshift (z):**

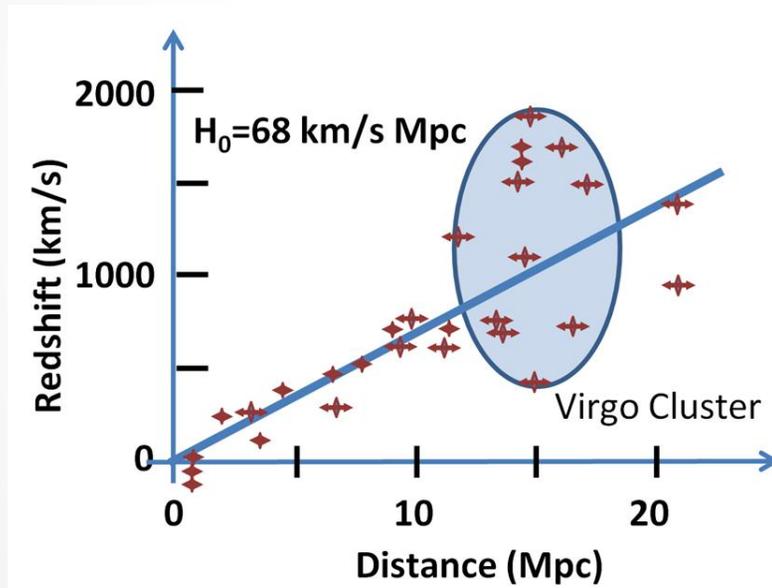
Time after the Big Bang Present

1	4	5	6	7	8	10	>20
6 billion years	1.5 billion years			800 million years		480 million years	200 million years



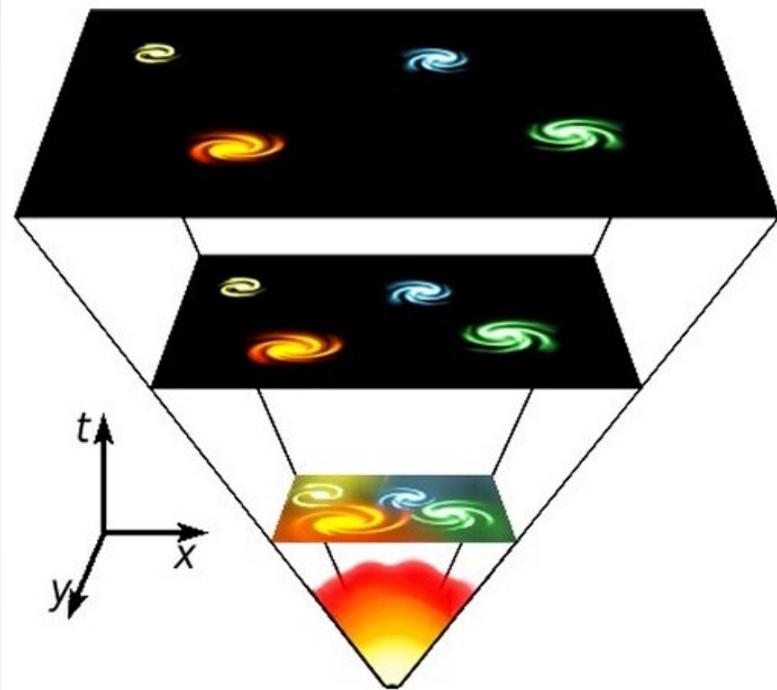


# Back to Hubble's Law



- To map the distribution of galaxies in space and time need to measure the “redshift” of galaxies
- Using Hubble's Law can use velocities to estimate distances
- Get for free information on the star formation history of the Universe, evolution of galaxies, evolution of QSOs and the distribution of matter on large scales

# Redshift Measurements



- The measurement is of the change in wavelength of absorption or emission lines from a galaxy:

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0}$$

$$v = c \times \frac{\Delta\lambda}{\lambda_0}$$

If interpreted as Doppler shift

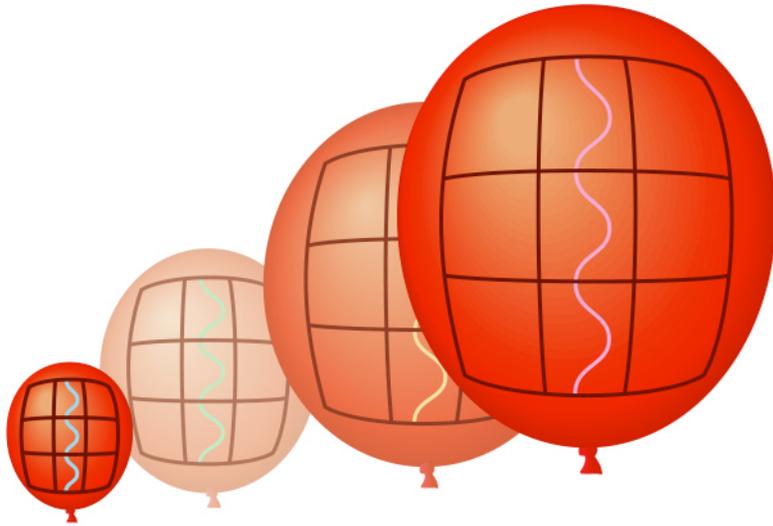
$$v = cz$$

Only holds for  $v \ll c$ . Use relativistic form of Doppler shift for larger velocities

# Scale Factor of the Universe

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - \frac{\lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1$$

$$\frac{\lambda}{\lambda_0} = 1 + z$$



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$(1 + z)$  is the ratio by which the Universe has expanded since the photons left the object

# iclicker

- Suppose you take a spectrum of a distant QSO and determine that it has a redshift  $z = 3$ . How much smaller (compared to today) was the Universe when the light left the QSO? (recall  $\lambda / \lambda_0 = 1 + z$ )
  - A.  $1/3$
  - B. 3
  - C.  $1/4$
  - D. 4
  - E. huh?

$$\lambda/\lambda_0 = \text{the expansion factor} = 1 + 3 = 4$$

The Universe is 4 times larger today so it was  $1/4$  its current size when the light left the QSO

# Hubble's Law: Universe in 3-D



Aside from “peculiar motion”, the Hubble constant can be used to estimate distances to galaxies:

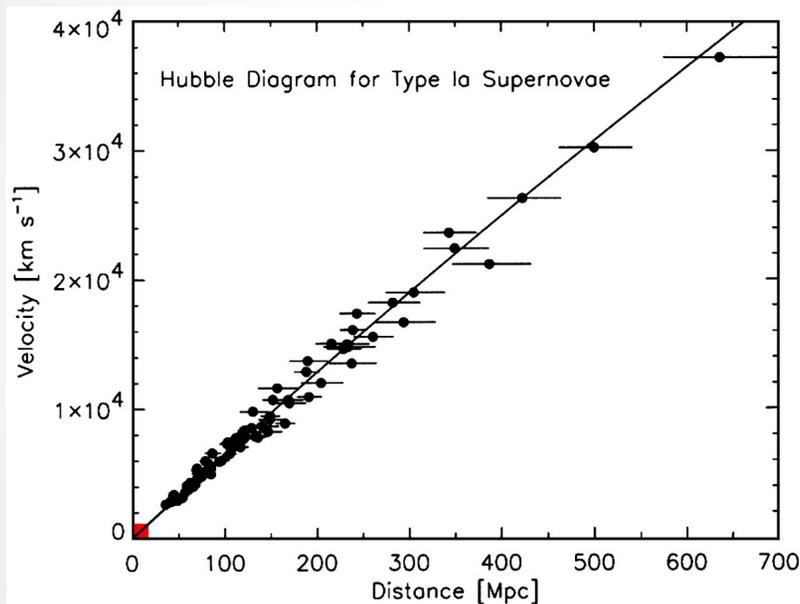
$$v = cz = H_0 d$$

$v$  = velocity measured from spectral line shifts

$d$  = distance to the galaxy

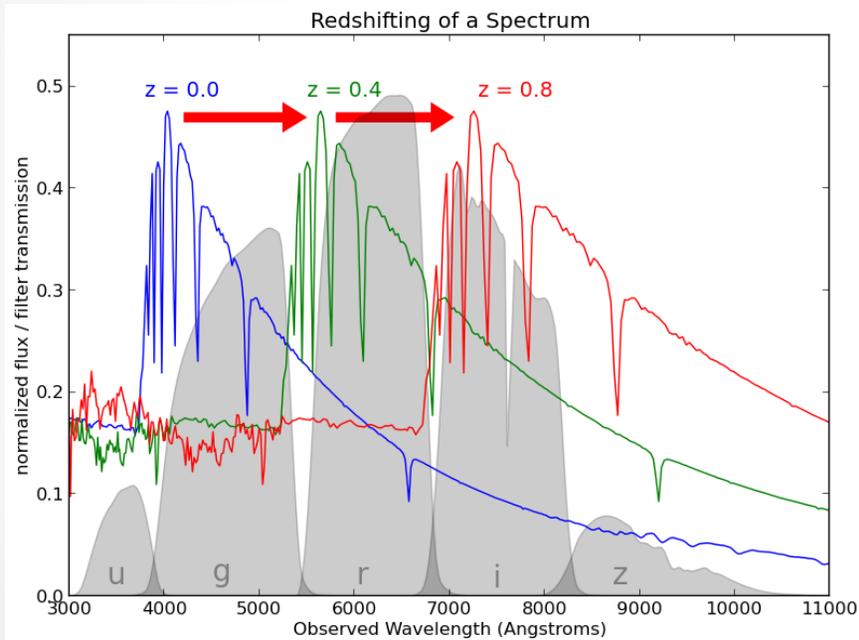
$H_0$  = Hubble constant at current time

$v$  in km/sec,  $H_0$  in (km/sec)/Mpc,  $d$  in Mpc



- $H_0$  is the slope of the plot of velocity vs distance and is often in units of  $(\text{km/sec})/\text{Mpc}$  (1/time)
- Local measurement
- Can track expansion rate at earlier times
- Deviations from a straight line tell us if the expansion is slowing down or speeding up

# Galaxy Distances from $z$



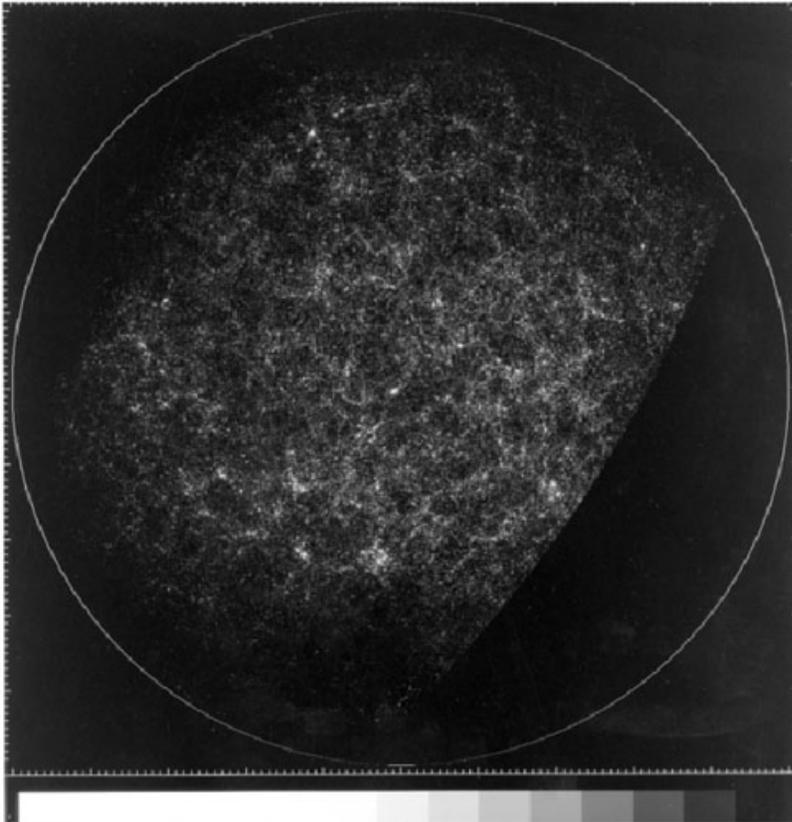
- Take a spectrum, measure speed,  $s$ , from the shift in spectral line position.
- Suppose you measure  $s = 18,000$  km/sec:  
$$d = s / H_0$$
$$= 18,000 / 72 \text{ (Mpc)}$$
$$= 250 \text{ Mpc}$$

# Large-scale Structure



- First hint of complex structure for the Universe was the Shane-Wirtanen survey at Lick Observatory
- Used the double astrograph to photograph the entire accessible sky (1400 photographic plates) from 1949-1957

# Large-scale Structure



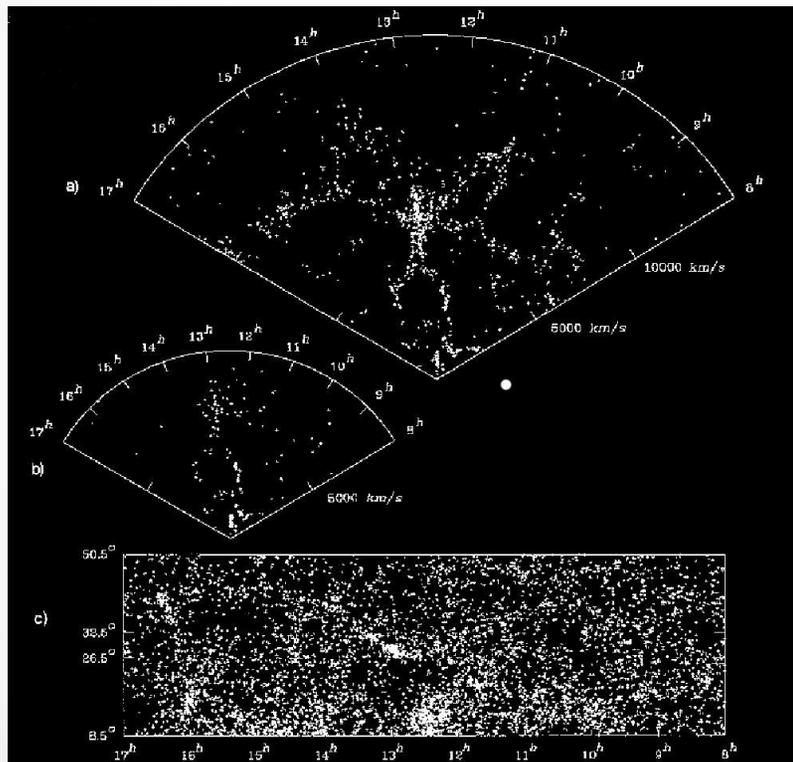
- Even without distances, the Shane-Wirtanen galaxy counts showed that galaxies were distributed in filaments and clumpy structures

# Large Scale Structures



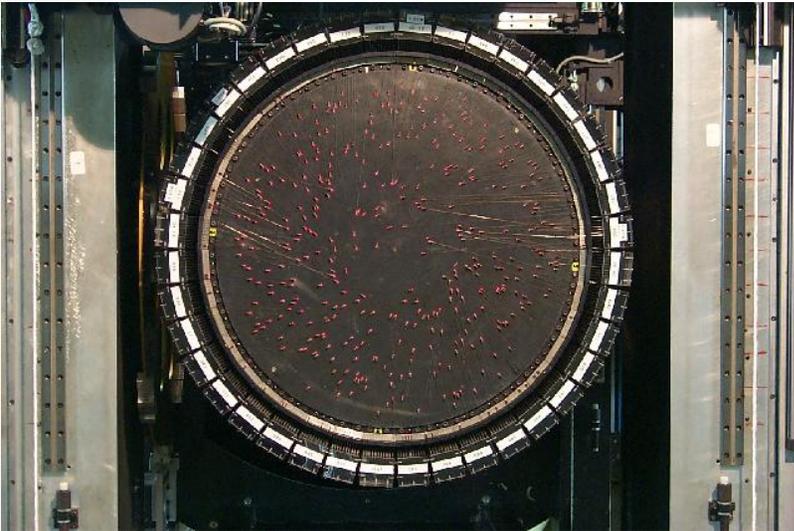
- 1980s John Huchra and Margaret Geller at Harvard started a program to map the 3-d distribution of galaxies
- The initial results took everyone by surprise
- The distribution was not uniform by filled with voids, filaments and clustering on all scales

# Large Scale Structures



- These maps did not look like any of the models of the time for the expected formation of structures formed by gravitational forces
- There was some missing ingredient that was extremely important

# Redshift Surveys



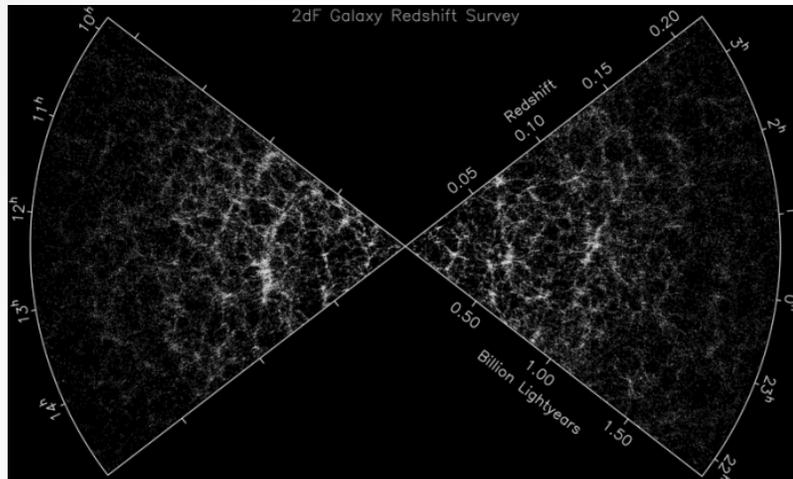
- After the Huchra/Geller survey done the old-fashioned way special instruments and sometimes telescopes were built to obtain spectra of up to thousands of galaxies at the same time

# SDSS DR9 fly through

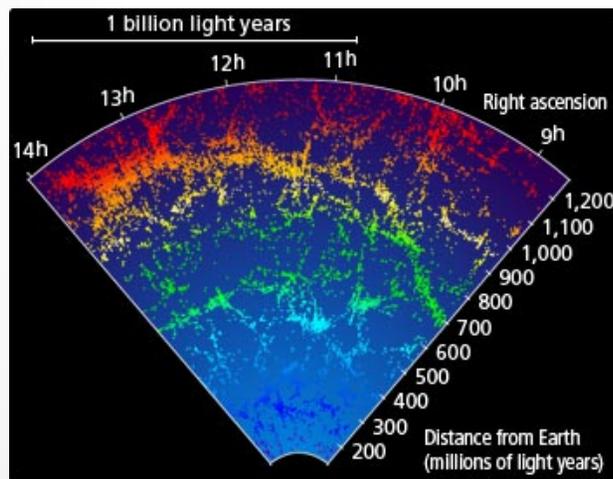
[SDSS flythrough](#)

- UCSC Connie Rockosi is one of the leaders of the SDSS

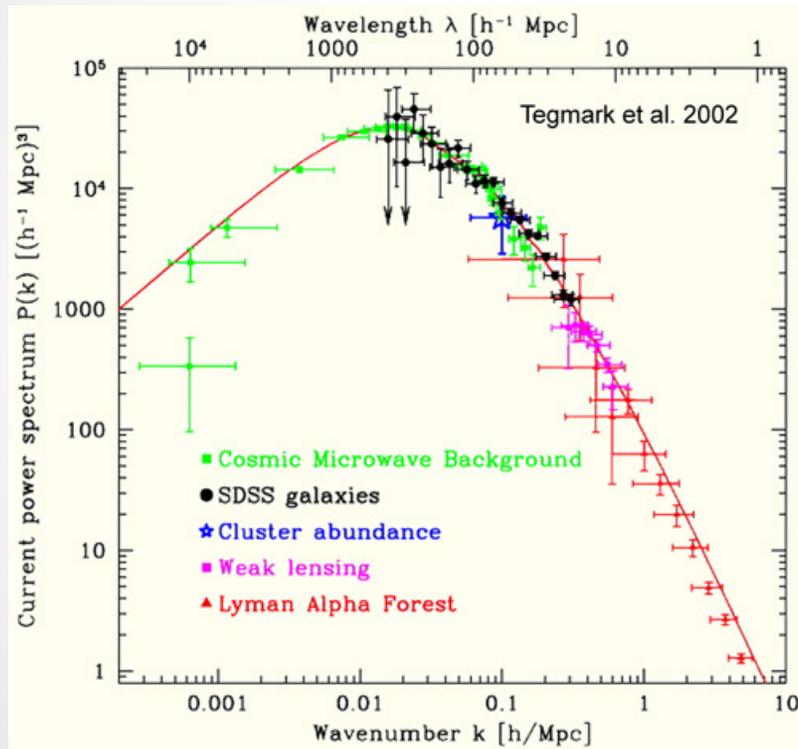
# Large Scale Structure



- The result of all these studies is:
  - quantification of the nature of the distribution of galaxies in space
  - Observations of the evolution of structure with time



# LSS can be quantified



- “Power Spectrum” of galaxies is a statistical representation of the “clumpiness” of the distribution in space
- Build models for the Universe with various ingredients and see if you get the same power spectrum as is measured
- Models in 1970s, 80s were way off!

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## review article

*Nature* 311, 517–525 (11 October 1984); doi:10.1038/311517a0

## Formation of galaxies and large-scale structure with cold dark matter

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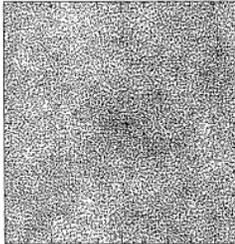
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**The dark matter that appears to be gravitationally dominant on all scales larger than galactic cores may consist of axions, stable photinos, or other collisionless particles whose velocity dispersion in the early Universe is so small that fluctuations of galactic size or larger are not damped by free streaming. An attractive feature of this cold dark matter hypothesis is its considerable predictive power: the post-recombination fluctuation spectrum is calculable, and it in turn governs the formation of galaxies and clusters. Good agreement with the data is obtained for a Zeldovich ( $|\delta_k|^2 \propto k$ ) spectrum of primordial fluctuations.**

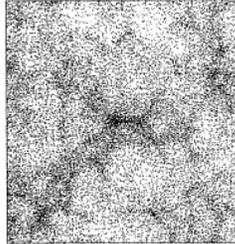


# Dark Matter III

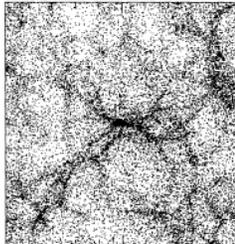
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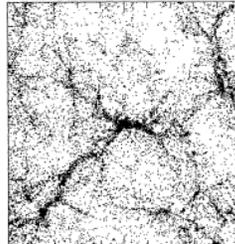
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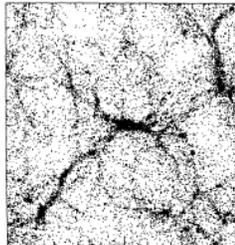
03:  $\Omega_0=0.3$   $\Lambda_0=0.0$



04:  $\Omega_0=0.4$   $\Lambda_0=0.0$



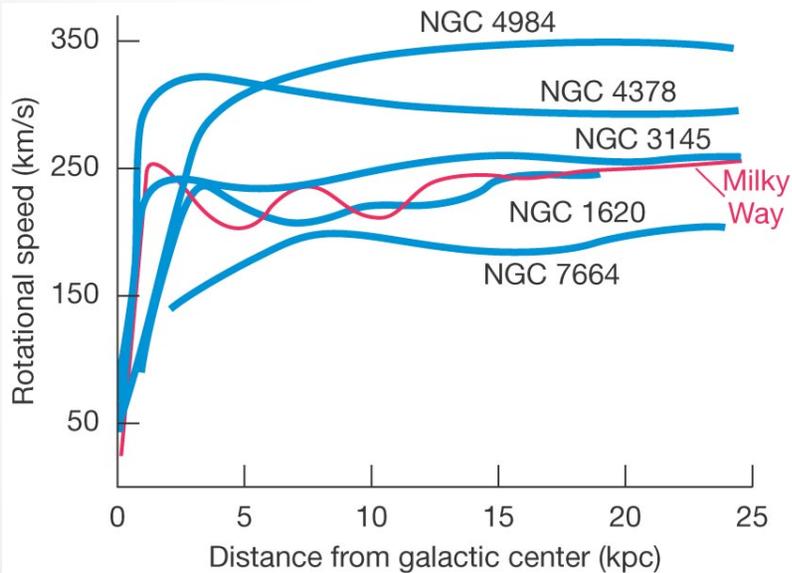
05:  $\Omega_0=0.5$   $\Lambda_0=0.0$



From Cole et al 1997  
MNRAS 289, 37

- Blumenthal et al. did theory with pen and paper
- As computing power expanded, the same physics was put into models of the Universe
- These became great experiments
- Structure formation required lots and lots of dark matter

# Dark Matter I



(b)

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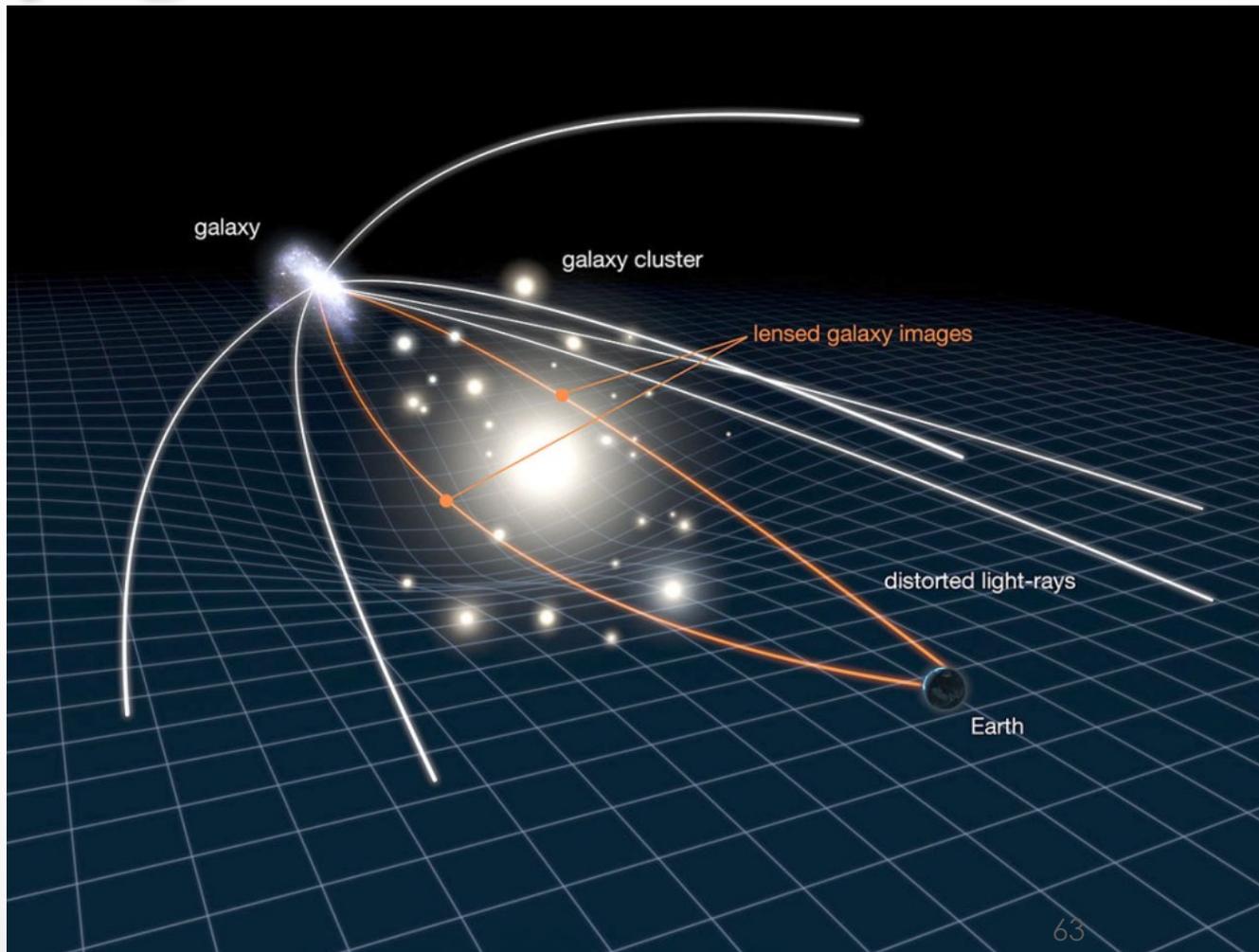
- Rotation curves of galaxies required an extended halo of non-luminous mass to keep the stars at larger radii bound to the galaxies

# Dark Matter II

On the scale of galaxy clusters, Fritz Zwicky measured the “velocity dispersion” and inferred dark matter was required to prevent the galaxies from flying apart

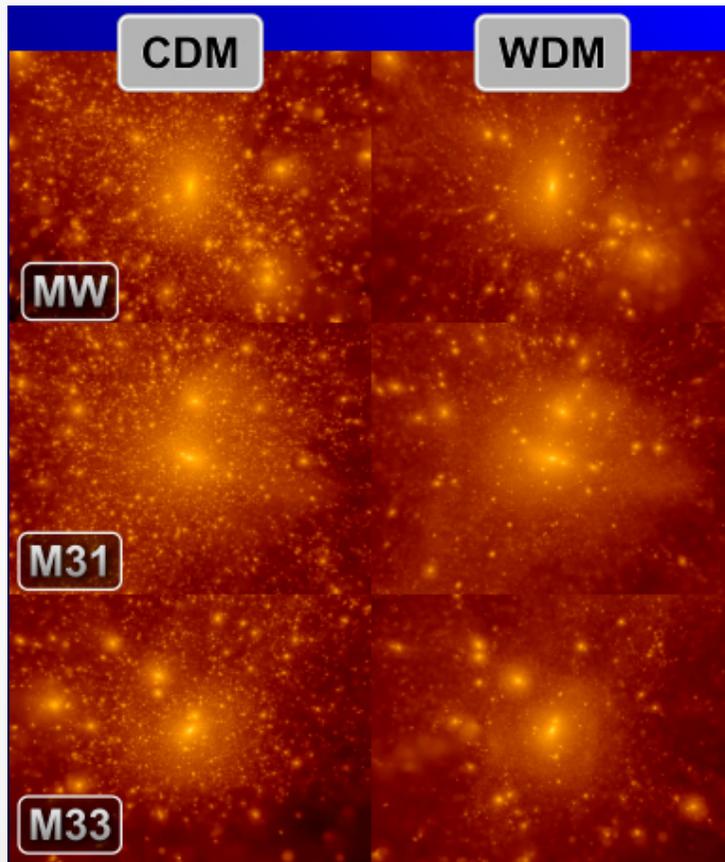


# Dark Matter II verified by gravitational lensing





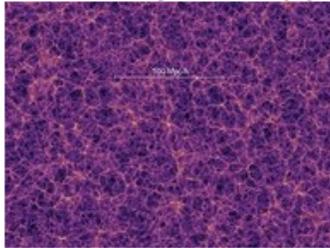
# Dark Matter III return



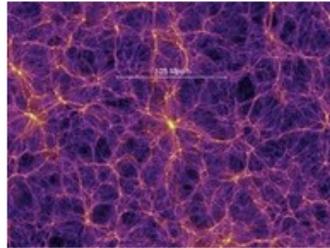
- The presence and nature of dark matter can be inferred by comparing observations to data
  - “hot” vs “cold” dark matter
  - Interaction with photons. Growth of structure depends on reaction to gravity, photon pressure and expansion of the Universe



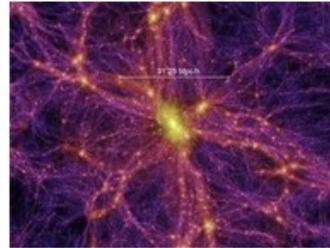
**Redshift  $z=0$  ( $t = 13.6$  Gyr):**



▶ 1024x768 ▶ 2048x1536

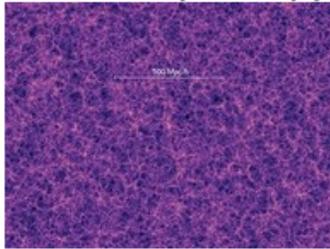


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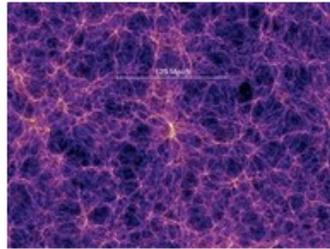


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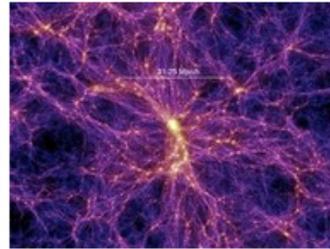
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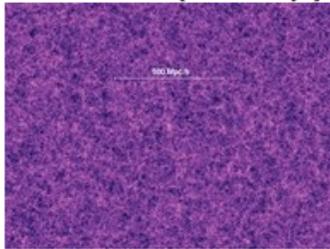


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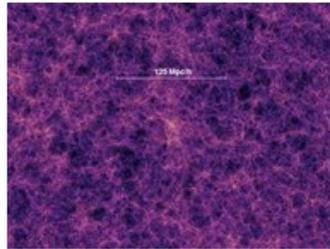


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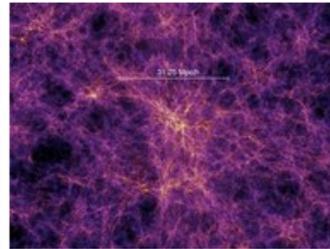
**Redshift  $z=5.7$  ( $t = 1.0$  Gyr):**



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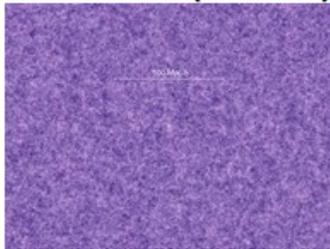


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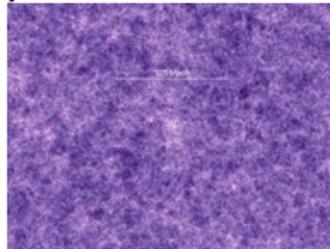


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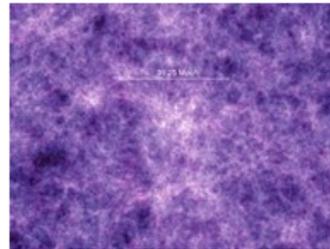
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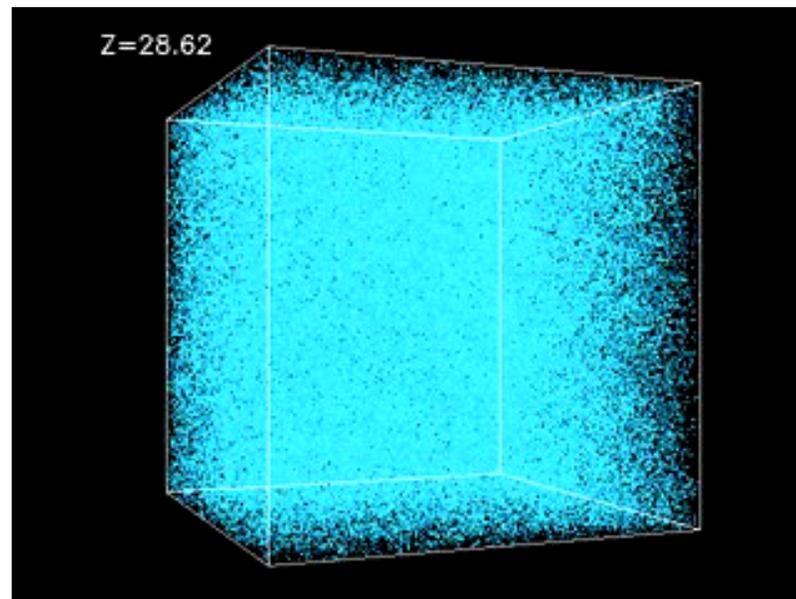
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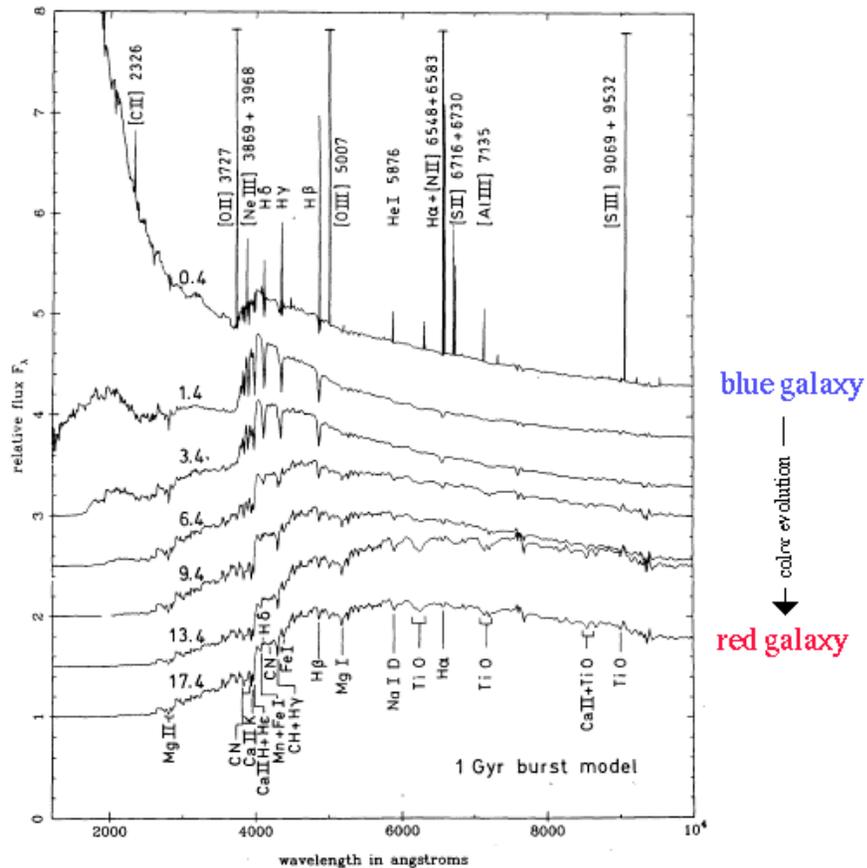
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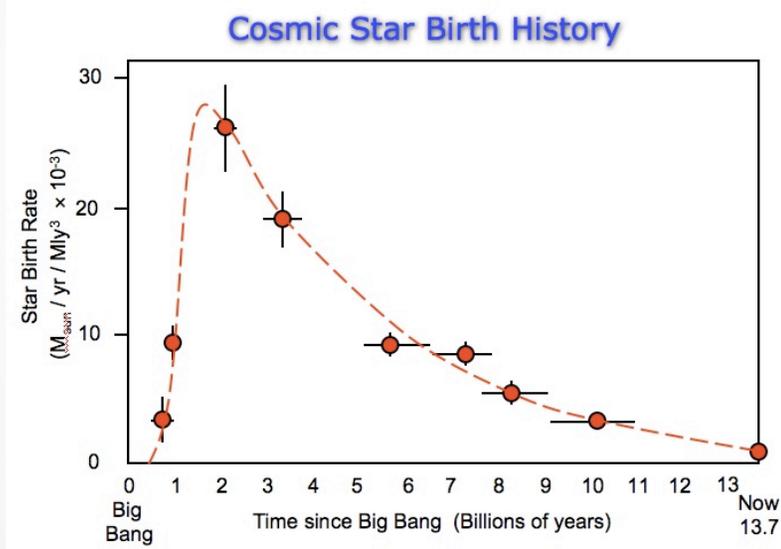


# Aside: History of Star Formation



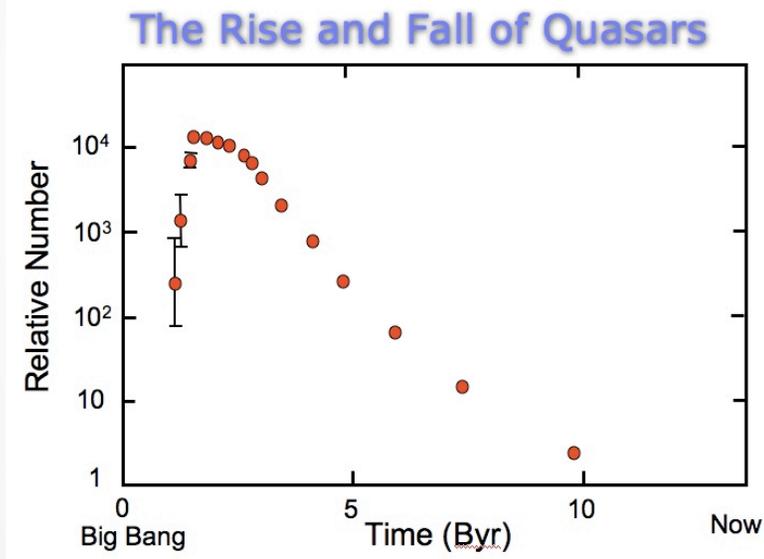
- Bonus aspect of the many spectroscopic surveys of galaxies is the star formation activity has been measured for hundreds of thousands or galaxies
- Star forming galaxies show strong emission lines from hot gas near young stars

# Aside: Madau Plot



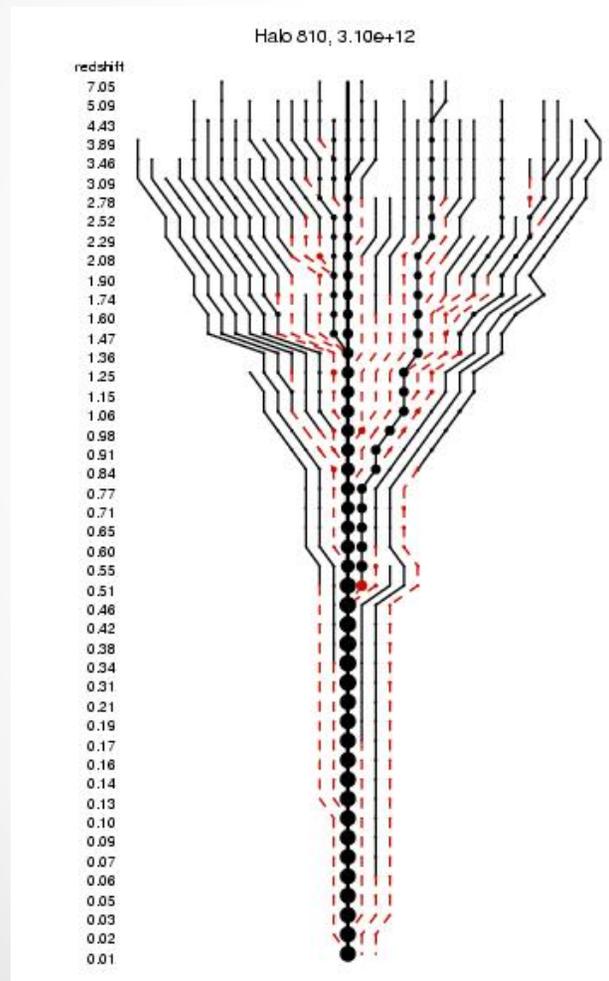
- The answer is that the main star formation epoch of the Universe has already occurred
- Fueled by lots of gas and mergers of galaxies at early times

# Quasar/SMBH History of the Universe

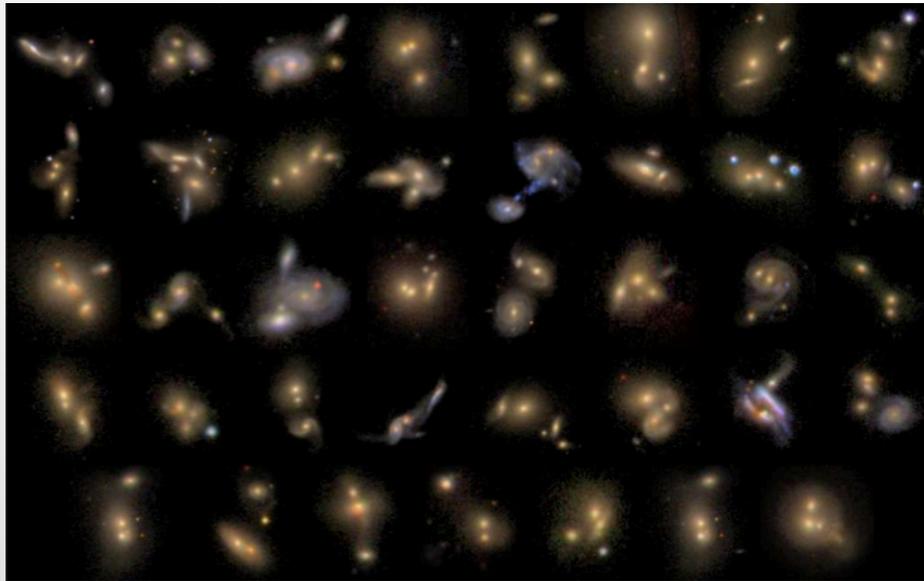
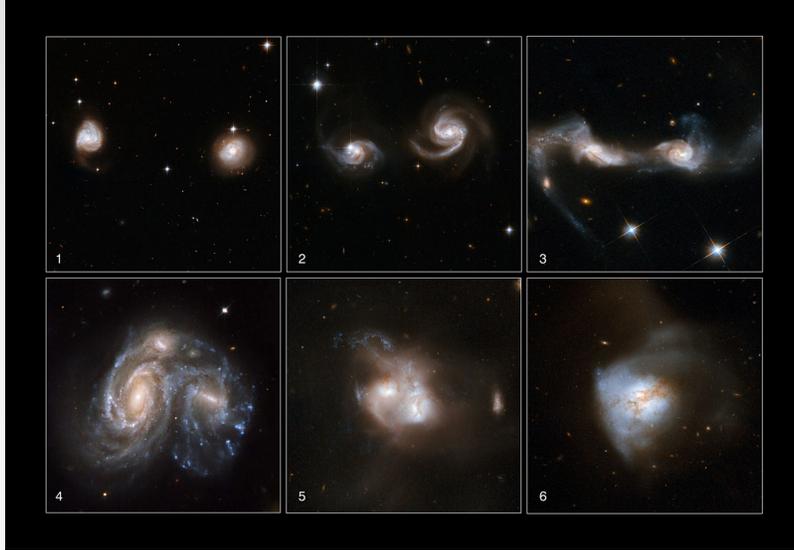


- Get a plot similar to that for star formation history (but not identical)
- Very few QSOs at times  $< 1$  billion years after the BB, strong peak between 1.7 and 3 billion years after the Big Bang

# The Universe is Evolving



- We can explain the star formation and QSO histories by Star-formation history by merger history from Universe simulations
- QSO distribution in time are two examples of a Universe that is evolving in one direction



- Mergers were much more common at earlier time because the Universe was smaller and gas was falling into dark matter halos
- Mergers kick off star formation bursts and sometimes funnel gas to centers to feed super massive black holes

# Age of the Universe

$$H_0 = 72 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$$

$$\frac{1}{H_0} = \frac{1 \text{ s} \cdot \text{Mpc}}{72 \text{ km}} \times \frac{3.1 \times 10^{19} \text{ km}}{1 \text{ Mpc}} = 4.3 \times 10^{17} \text{ s}$$

$$4.3 \times 10^{17} \text{ s} \times \frac{1 \text{ year}}{3.15 \times 10^7 \text{ s}} = 13.6 \times 10^9 \text{ years}$$

- If  $H_0$  was constant for all time the “expansion age” of the Universe is simply  $1/H_0$
- $H_0 = 72 \text{ (km/s)/Mpc}$
- km and Mpc are distance units so  $H_0$  has units of 1/seconds