

Homework Problem #1

1) An $R = 20$ star observed with LRIS (the Keck imaging spectrograph) produces 1890 detected photo-electrons per second. The R-band sky brightness at Mauna Kea is listed at the CFHT WWW site as 20.9 mag/arcsec². The LRIS pixel scale is 0.22 arcseconds/pixel, the readout noise is 8e- and the inverse gain of the system is 2.0 e-/DN.

(a) What is the rate of detected e-/pixel from the sky in the R band?

To determine the e-/arcsecond in the R-band sky:

$$I_{\text{sky}}/I_{m=20} = 10^{-0.4(m_{\text{sky}} - m_{20})}$$

$$I_{\text{sky}} = 1890 \text{ (e-/sec/arcsec}^2) \times 10^{-0.4(20.9 - 20)}$$

$$I_{\text{sky}} = 825 \text{ (e-/sec/arcsec}^2)$$

And then the e-/pixel from the sky in the R Band

$$I_{\text{sky}} = 825 \text{ (e-/sec/arcsec}^2) \times (0.22)^2 \text{ (arcsec}^2/\text{pixel)}$$

$$I_{\text{sky}} = 39.9 \text{ e-/second/pixel}$$

(b) What is the rate of detected e- from a $R = 26$ magnitude star observed at an airmass of 1.2 assuming the extinction coefficient in R is 0.1 mag/(unit airmass)?

$$R_{\text{observed}} = R_{\text{star}} + [(\text{extinction coefficient}) \times (\text{airmass})] = 26.12$$

$$I_{26}/I_{20} = 10^{-0.4(m_{26} - m_{20})}$$

$$I_{26} = 1890 \times 10^{-0.4(26.12 - 20)} = 6.74 \text{ e-/sec}$$

(c) Assume that you are measuring all of the light for the $R = 26$ magnitude star in an aperture with a radius of 7 pixels. At what exposure time does the measurement become sky dominated?

$$\text{Sky noise per pixel } (I_{\text{sky}} \times t)^{1/2}$$

$$\text{Readnoise per pixel: RN}$$

$$\text{Sky dominated for } (I_{\text{sky}} \times t)^{1/2} > 3 \times \text{RN}$$

$$t = (3 \times \text{RN})^2 / I_{\text{sky}} = (3 \times 8)^2 / 39.9$$

$$= 14.4 \text{ seconds}$$

(d) For the sky-dominated case, how does the S/N scale with exposure time?

$$S/N \propto (I_{\text{sky}} t) / (n_{\text{pix}} I_{\text{sky}} t)^{1/2}; S/N \propto (t)^{1/2}$$

(e) How does the S/N scale with seeing (assume you scale the measuring radius linearly with FWHM of point sources).

$$n_{\text{pix}} \propto r^2 \propto \text{"seeing"}^2$$

$$S/N \propto (n_{\text{pix}})^{1/2}$$

(f) Make a table of the source noise, sky noise, readnoise and S/N for exposure times of 1, 60, 600, and 3600 seconds.

$$\text{Source Noise} = (I_{\text{source}} \times t)^{1/2}$$

$$\text{Sky Noise} = (n_{\text{pix}} \times I_{\text{sky}} \times t)^{1/2}$$

$$\text{Read Noise} = (RN \times n_{\text{pix}})^{1/2}$$

$$I_{\text{source}} = 6.74 \text{ e-/sec}; I_{\text{sky}} = 39.9 \text{ e-/sec}; n_{\text{pix}} = 49\pi; RN = 8 \text{ e-/pixel}$$

t (sec)	Source noise	Sky noise	RN	S/N
1	2.6	78	112	0.06
60	20.1	607	112	0.73
600	63.6	1921	112	2.3
3600	115.8	4705	112	5.7

(g) What is the exposure time required to make an observation of this star with a S/N of 20?

Sky noise dominated at this exposure time

$$S/N \approx (I_{\text{source}} \times t) / (n_{\text{pix}} \times I_{\text{sky}} \times t)^{1/2}$$

$$\text{Solve for } t: t = [(20)^2 \times 39.9 \times 154] / [6.7]^2$$

$$t = 54123 \text{ seconds (!)}$$

(f) What is the exposure time required to make an observation of this star with S/N=20 with WFPC2 in the filter that is the closest match to "R"?

Filter that best matches R-band on WFPC2 is F675W. Use the S/N calculator at STScI and you can get an exposure time ranging from ~10,000 seconds to 120,000 seconds! In this sky-limited regime, you are very sensitive to the size of the measurement aperture.