Math Review!

1. Scientific Notation

Age of universe ~ 14 billion yrs

14,000,000,000 → 1.4E10 yrs

(14E9 yrs)

speed of light 2.998E8 m/s

⇒ 299,800,000. m/s

Gravitational constant G

0.0000000000667 m^3/kgs^2

6.67E-11 m^3/kgs^2

Bohr radius 5.29E-11 m

0.0000000000529 m

___ E ___ = ___ e ___ = ___ $\times 10^{\_\_\_}$

Multiplication & Division

$\times$ = multiply coefficients, add exponents

$\div$ = divide coefficients subtract exponents

2E5 $\times$ 4E6 = 8E11

3E8 $\times$ 2E-8 = 6

6E10 $\div$ 2E4 = 3E6

8E9 $\div$ 4E-2 = 2E11

4E2 $\times$ 2E-6 = 8E-4

2E3 $\div$ 1E5 = 2E-2

2. Metric System

Giga	Mega	kilo	base	centi	milli	micro	nano
G	M	k	base	c	m	μ	n
1E9	1E6	1E3		1E-2	1E-3	1E-6	1E-9
billion	million	thousand		hundredth	thousandth	millionth	billionth

base = meter, second (yr), gram, liter

1 km = 1E3 m = 1000 m
1E-3 km = 0.001 km = 1 m (1/1000 km)

1 cm = 1E-2 m = .01 m (1/100 m)
100 cm = 1E2 cm = 1 m

3. Dimensional Analysis

AKA unit conversions

- if you have a unit on top, put it on bottom when putting in the conversion factor
- cross out units that are on top and bottom
- multiply across top, divide across bottom

5 inches to cm

$$\begin{array}{l|l|} 5\,\cancel{\text{in}} & 2.54\,\text{cm} \\ \hline & 1\,\cancel{\text{in}} \end{array} = 12.7\,\text{cm}$$

6 cm to inches

$$\begin{array}{l|l|} 6\,\cancel{\text{cm}} & 1\,\text{in} \\ \hline & 2.54\,\cancel{\text{cm}} \end{array} = 2.36\,\text{in}$$

35 mph to m/s

$$\begin{array}{l|l|l|l|l} 35\,\cancel{\text{mi}} & \cancel{\text{km}} & 1000\,\text{m} & 1\,\cancel{\text{hr}} & 1\,\cancel{\text{min}} \\ \hline \cancel{\text{hr}} & .62\,\cancel{\text{mi}} & 1\,\cancel{\text{km}} & 60\,\cancel{\text{min}} & 60\,\text{S} \end{array} = 15.65\ \text{m/S}$$

1 g/cm³ to kg/gal

$$\begin{array}{l|l|l|l} 1\,\cancel{\text{g}} & (100\,\cancel{\text{cm}})^3 & 1\,\cancel{\text{m}^3} & 1\,\text{kg} \\ \hline \cancel{\text{cm}^3} & 1\,\cancel{\text{m}^3} & 264.2\,\text{gal} & 1000\,\cancel{\text{g}} \end{array} = 3.785\ \text{kg/gal}$$

how many seconds old is someone on their 21st birthday?

$$\begin{array}{l|l|l|l|l} 21\,\cancel{\text{yrs}} & 365\,\cancel{\text{d}} & 24\,\cancel{\text{hr}} & 60\,\cancel{\text{min}} & 60\,\text{sec} \\ \hline & 1\,\cancel{\text{yR}} & 1\,\cancel{\text{day}} & 1\,\cancel{\text{hr}} & 1\,\cancel{\text{min}} \end{array}$$

$$= 662256000\,\text{s} = 6.62\text{E}8\,\text{s}$$

Math Rules

Ways to manipulate equations:

① You can multiply or divide both sides of an equation by the same value. You can also add or subtract the same value to both sides. (Make sure to put parenthesis around a side that has more than one term when multiplying or dividing, or multiply and divide each term by the same value)

Examples:

$$a = \frac{2}{b} \Rightarrow a \cdot b = \frac{2}{b} \cdot b$$

$$a \cdot b = 2 \Rightarrow \frac{a \cdot b}{b} = \frac{2}{b}$$

$$a \cdot b = 2 \Rightarrow a \cdot b + 4 = 2 + 4 = 6$$

$$a + 3 = b \Rightarrow a + 3 - 3 = b - 3 \Rightarrow a = b - 3$$

$$a = b + 3 \Rightarrow 2 \cdot a = 2 \cdot (b + 3) = 2b + 2 \cdot 3$$

$$a = b + 3 \Rightarrow \frac{a}{2} = \frac{(b+3)}{2} = \frac{b}{2} + \frac{3}{2}$$

(2) You can "cancel out" any terms that are both "on top" (in the numerator) and "on the bottom" (in the denominator) but not if there is a + or − sign between them.

$$a \cdot b = \frac{2}{\cancel{b}} \cdot \cancel{b} \Rightarrow a \cdot b = 2$$

$$\frac{a \cdot \cancel{b}}{\cancel{b}} = \frac{2}{b} \Rightarrow a = \frac{2}{b}$$

$$a \cdot b + \frac{1}{b} = 5$$

$$a \cdot b - \frac{5}{a} = 2$$

cannot cross out because the a/b on top and the a/b on bottom are separated by a + or − sign.

③ Multiplying or dividing multiple terms separated by + or − is the same as multiplying or dividing each term.

$$2 \cdot (a+b+c) = 2a + 2b + 2c$$

$$\frac{a+b+c}{3} = \frac{a}{3} + \frac{b}{3} + \frac{c}{3}$$

if every term has the same value in it, you can "pull" (factor) that value out

$$2a + 2b + 2c = 2(a+b+c)$$

$$2a + 3a + 5a = a(2+3+5) = 10a$$

$$\frac{a}{3} + \frac{b}{3} + \frac{c}{3} = \frac{(a+b+c)}{3}$$

$$a + 2a + \frac{a}{2} = a\left(1+2+\frac{1}{2}\right) = \frac{7}{2}a$$

(4) Dividing by a value is the same as multiplying by the inverse. Multiplying by a value is the same as dividing by the inverse. The inverse is 1/value

$$a \cdot \frac{1}{2} = \frac{a}{2}$$

$$a \cdot 3 = \frac{a}{1/3} = \frac{3}{1/a} = \frac{1}{1/3a}$$

$$\frac{3}{1/2} = 3 \cdot 2 = 6$$

$$\frac{a}{6} = a \cdot \frac{1}{6}$$

$$\frac{a}{5b} = \frac{a}{b} \cdot \frac{1}{5} = \frac{a}{5} \cdot \frac{1}{b} = a \cdot \frac{1}{b} \cdot \frac{1}{5}$$

$$\frac{a}{b} = \frac{1/b}{1/a}$$

5. You can replace any value, term, or set of terms with something it is equal to.

$a = 5b$

$2a = 10 \Rightarrow 2(5b) = 10$

$c = 5 + d$

$3c + 5 = 26 \Rightarrow 3(5+d) + 5 = 26$

$3f = 2g$

$15f + 6 = 40 \Rightarrow 5 \cdot 3f + 6 = 40 \Rightarrow 5 \cdot 2g + 6 = 40$

6. If you have two equations, you can multiply or divide each side of one equation by each side of the other, or add/subtract each side.

$$5a = 2b \quad , \quad 10c = 4d$$

multiply

$$5a \cdot 10c = 2b \cdot 4d$$

divide

$$\frac{5a}{10c} = \frac{2b}{4d}$$

add

$$5a + 10c = 2b + 4d$$

subtract

$$5a - 10c = 2b - 4d$$

7 Exponent Rules

base → a^b ← exponent

- if you have the same base and two different exponents multiplied together you can add the exponents

$$x^5 \cdot x^3 = x^{(5+3)} = x^8$$

$$x^6 \cdot x^{-2} = x^{(6-2)} = x^4$$

- if you have the same base and two different exponents divided by each other, you subtract the exponents

$$\frac{x^6}{x^4} = x^{(6-4)} = x^2$$

- a base to a negative exponent is the same as the inverse to the positive exponent and vice versa

$$x^{-5} = \frac{1}{x^5}$$

$$\frac{1}{x^{-6}} = x^6$$

(8) You can raise both sides of an equation to the same exponent, but make sure to put parenthesis around the whole side if there is more than one value

$$x = 2 \Rightarrow x^2 = 2^2$$

$$2a = 3b \Rightarrow (2a)^2 = (3b)^2$$

$$\frac{a}{2} = 5b \Rightarrow \left(\frac{a}{2}\right)^2 = (5b)^2$$

(9) To get Rid of an exponent, Raise both sides to the inverse of that exponent this is the (square, cube, etc) Root and "cancels out" the exponent

$x^{1/2} \equiv \sqrt{x}$ two ways of writing the same thing

$x^{1/3} \equiv \sqrt[3]{x}$

example

$x^2 = 4 \Rightarrow (\cancel{x^2})^{\cancel{1/2}} = 4^{1/2}$

$x = 4^{1/2} = \sqrt{4} = 2$

$x^3 = 27 \Rightarrow (\cancel{x^3})^{\cancel{1/3}} = 27^{1/3}$

$x = 27^{1/3} = \sqrt[3]{27} = 3$

$\sqrt{x} = 2 = x^{1/2} \Rightarrow (\sqrt{x})^2 = 2^2 = (x^{\cancel{1/2}})^{\cancel{2}} \Rightarrow x = 4$

$\sqrt[3]{x} = 4 = x^{1/3} \Rightarrow (\sqrt[3]{x})^3 = 4^3 = (x^{\cancel{1/3}})^{\cancel{3}}$

$\Rightarrow x = 64$

10 When you raise a term (with no + or - signs) to an exponent, that exponent gets distributed to every value in the term

$$(ab)^2 = a^2 b^2$$

$$\frac{1}{(2a)^2} = \frac{1}{2^2 a^2} = \frac{1}{4a^2}$$

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

if every term has the same exponent you can pull that exponent out

$$a^4 b^4 = (ab)^4$$

$$4a^2 = 2^2 a^2 = (2a)^2$$

$$\frac{16b^2}{a^2} = \frac{4^2 b^2}{a^2} = \left(\frac{4b}{a}\right)^2$$

Order of operations

"PEMDAS"

(parenteses, exponents, multiplication & division, addition & subtraction)

- multiplication & division can be done at the same time, same for addition and subtraction
- inside a parenthesis, follow same order
- numerator & denominator ("top" and "bottom") of a division sign have implied parenthesis

when solving an equation, do math steps in this order

when isolating a variable, do steps in opposite order

Examples on next page

Solving an equation

$$(2 \cdot 3)^2 + 5 \cdot 3 - 7 + \frac{2 \cdot 4}{10-6} = x$$

① parentheses (add parenthesis around top and bottom of division sign)

$$(2 \cdot 3)^2 + 5 \cdot 3 - 7 + \frac{(2 \cdot 4)}{(10-6)} = x$$

$$6^2 + 5 \cdot 3 - 7 + \frac{8}{4} = x$$

② exponents

$$6^2 + 5 \cdot 3 - 7 + \frac{8}{4} = x$$

$$36 + 5 \cdot 3 - 7 + \frac{8}{4} = x$$

③ multiplication & division

$$36 + 5 \cdot 3 - 7 + \frac{8}{4} = x$$

$$36 + 15 - 7 + 2 = x$$

④ addition & subtraction

$$36 + 15 - 7 + 2 = x$$

$$46 = x$$

SOLVED!

Isolating a variable

$$\frac{(x^2-4)}{3}+2=9$$

① addition & subtraction

$$\frac{(x^2-4)}{3}+2=9$$

$$\frac{(x^2-4)}{3}+2-2=9-2$$

$$\frac{(x^2-4)}{3}=7$$

② multiplication & division

$$\frac{(x^2-4)}{3}=7$$

$$\frac{(x^2-4)}{\cancel{3}}\cdot\cancel{3}=7\cdot 3$$

$$(x^2-4)=21$$

③ exponents

no exponents outside parenthesis

cont →

④ parenthesis

start order over once you can drop ( )

$(x^2-4) = 21$

$x^2-4 = 21$

back to ① addition & subtraction

$x^2-4 = 21$

$x^2-4+4 = 21+4$

$x^2 = 25$

② multiplication & division

none

③ exponents

$x^2 = 25$

$(x^2)^{1/2} = (25)^{1/2} = \sqrt{25}$

$x = 5$ solved!

Solving for Ratios

Earth's radius is about twice that of Mars. How many times larger is Earth's surface area than Mars's? $A = 4\pi r^2$

r_E = radius of Earth
r_M = radius of Mars
A_E = surface area of Earth
A_M = surface area of Mars

We know: $r_E = 2r_M$ (radius of Earth = 2x radius of Mars)

$A_E = 4\pi r_E^2$, $A_M = 4\pi r_M^2$

2 methods $\longrightarrow$

$r_E = 2r_M$, $A_E = 2\pi r_E^2$, $A_M = 2\pi r_M^2$

Method 1 → Division

① Divide one equation by the other (math rule #6)

$$\frac{A_E}{A_M} = \frac{2\pi r_E^2}{2\pi r_M^2}$$

② Replace a term with something that is equal to it (rule 5)

$$\frac{A_E}{A_M} = \frac{2\pi r_E^2}{2\pi r_M^2} = \frac{2\pi (2r_M)^2}{2\pi r_M^2}$$

③ Distribute any exponents (rule 10)

$$\frac{A_E}{A_M} = \frac{2\pi (2r_M)^2}{2\pi r_M^2} = \frac{2\pi \cdot 2^2 \cdot r_M^2}{2\pi r_M^2} = \frac{2\pi \cdot 4 \cdot r_M^2}{2\pi r_M^2}$$

④ cancel out any terms in top & bottom (rule 2)

$$\frac{A_E}{A_M} = \frac{\cancel{2\pi} \cdot 4 \cdot \cancel{r_M^2}}{\cancel{2\pi} \cancel{r_M^2}} = 4$$

⑤ if you want, multiply both sides to rearrange (rule 1)

$$\frac{A_E}{A_M} = 4 \Rightarrow \frac{A_E}{\cancel{A_M}} \cdot \cancel{A_M} = 4 \cdot A_M \Rightarrow \boxed{A_E = 4A_M}$$

$r_E = 2r_M$, $A_E = 2\pi r_E^2$, $A_M = 2\pi r_M^2$

Method 2 → substitution First

① Replace a term in one equation with a term that is equal to it (Rule 5)

$$A_E = 2\pi r_E^2 = 2\pi (2r_M)^2$$

② Distribute any exponents (Rule 10)

$$A_E = 2\pi (2r_M)^2 = 2\pi \cdot 2^2 \cdot r_M^2 = 2\pi \cdot 4 \cdot r_M^2$$

③ Rearrange equation until you get a term that is in one of your other equations

$$A_E = 2\pi \cdot 4 \cdot r_M^2 = 4 \cdot 2\pi r_M^2$$

④ Replace that term with something it is equal to

$$A_E = 4 \cdot 2\pi r_M^2 = 4 \cdot A_M$$

$$\boxed{A_E = 4A_M}$$

other ways of writing this: $A_M = \frac{1}{4} A_E$ OR $\frac{A_E}{A_M} = 4$

FROM Q1:

FOR a solid at 2900 K at what wavelength is the peak of the Plank Radiation curve?

$$T\,(\text{in K}) = \frac{.29}{\lambda_p\,(\text{in cm})}$$

① multiply both sides by λ_p (Rule 1) and cancel out the one in top and bottom (Rule 2)

$$T \times \lambda_p = \frac{.29}{\cancel{\lambda_p}} \times \cancel{\lambda_p} = .29$$

② Divide both sides by T and cancel out any on top and bottom (Rules 1 & 2)

$$\frac{\cancel{T} \times \lambda_p}{\cancel{T}} = \frac{.29}{T} \Rightarrow \lambda_p\,(\text{in cm}) = \frac{.29}{T\,(\text{in K})}$$

③ plug in your value for T

$$\lambda_p\,(\text{in cm}) = \frac{.29}{2900}\ \text{cm}$$ ← good enough for full credit

④ convert to scientific notation to solve

$$\lambda_p = \frac{.29}{2900}\ \text{cm} = \frac{2.9 \times 10^{-1}}{2.9 \times 10^{3}}\ \text{cm} = \frac{2.9}{2.9} \times 10^{(-1-3)}\ \text{cm} = \boxed{1 \times 10^{-4}\ \text{cm}}$$

Divide coefficient, subtract exponents