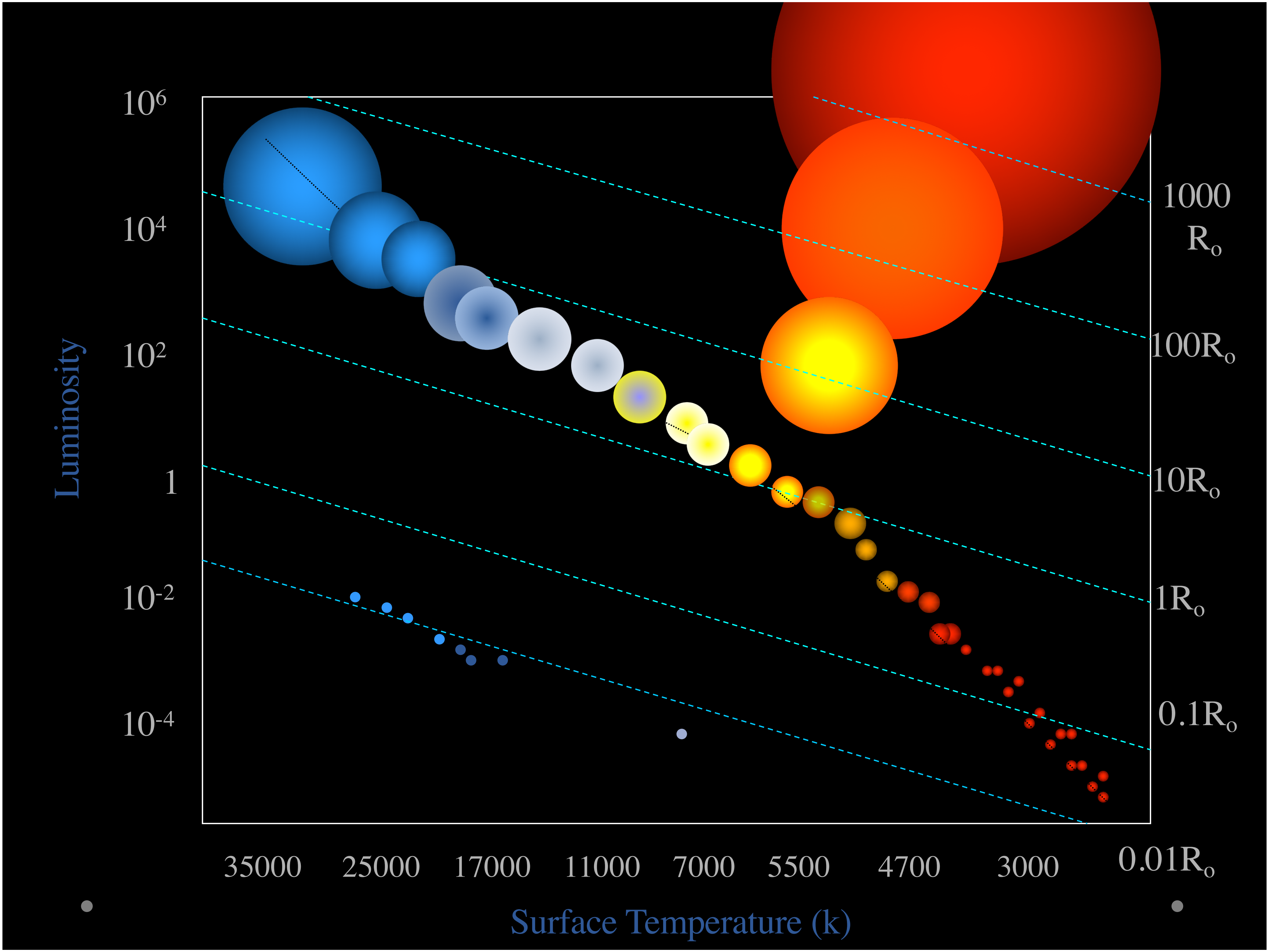
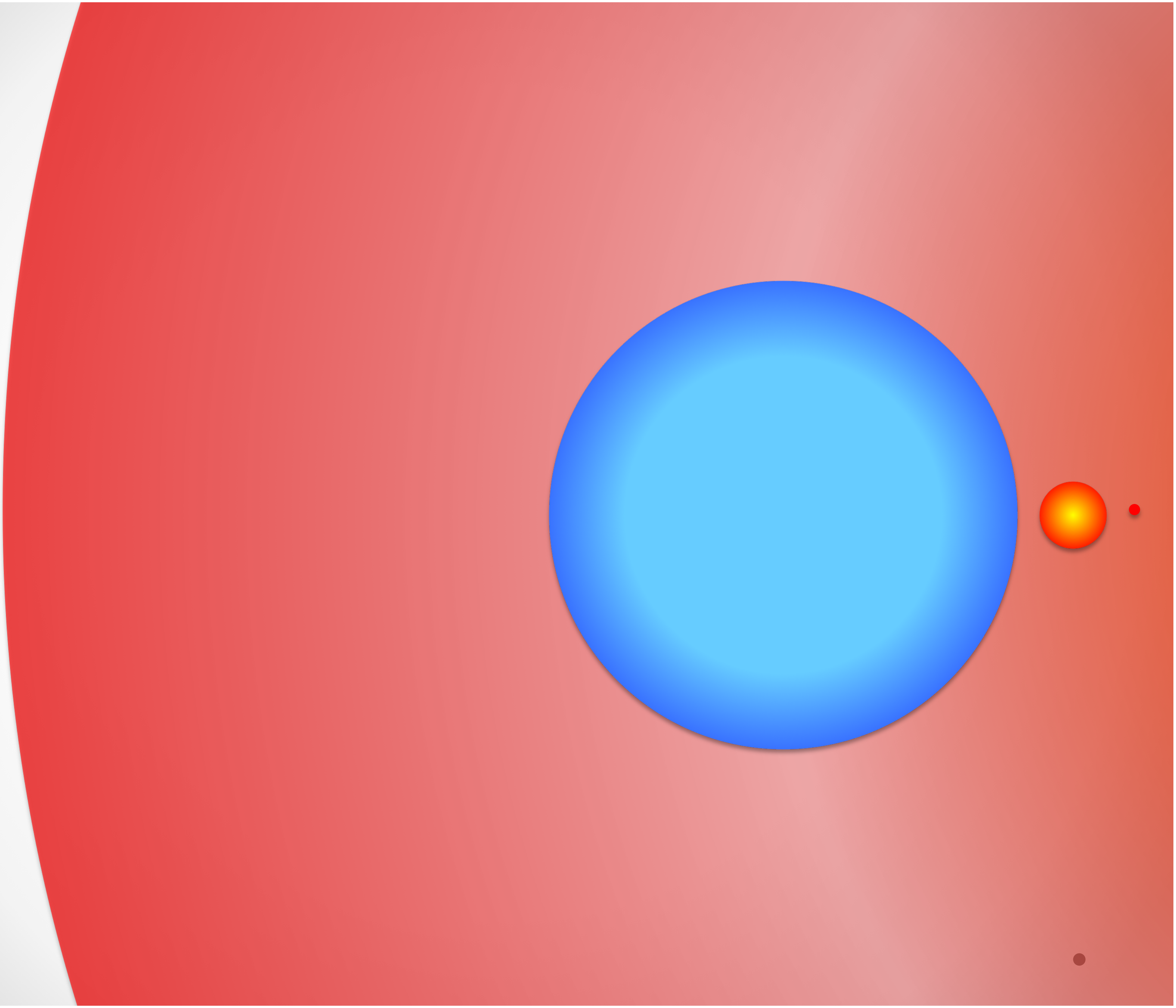
AY1 Announcements

- Quiz 2 will be Tuesday October 29
- Homework questions are available at the class website
  - Properties of stars
  - Stellar models, stellar energy source
  - Stellar evolution
  - Stellar endpoints

Stellar Properties

Property	Technique	Range of Values
Distance	Trig parallax	1.3pc - 100pc
Surface Temp.	Colors/Spec Type	3000K-50000K
Luminosity	Distance+brightness	$10^{-5}L_o$ - $10^{6}L_o$
Radius	Stephan’s Law	$0.01R_o$ - $800R_o$
Mass	Binary orbits	$0.08M_o$ - $80M_o$


Luminosity
10^6
10^4
10^2
1
10^-2
10^-4
1000 R_o
100R_o
10R_o
1R_o
0.1R_o
0.01R_o
35000
25000
17000
11000
7000
5500
4700
3000
Surface Temperature (k)



Next Section

- Central Temperature of Stars
- Stellar Energy Sources
- Stellar Lifetimes
- Stellar Evolution and Remnants
- Nucleosynthesis in stars

Stellar Structure and Central Temperature

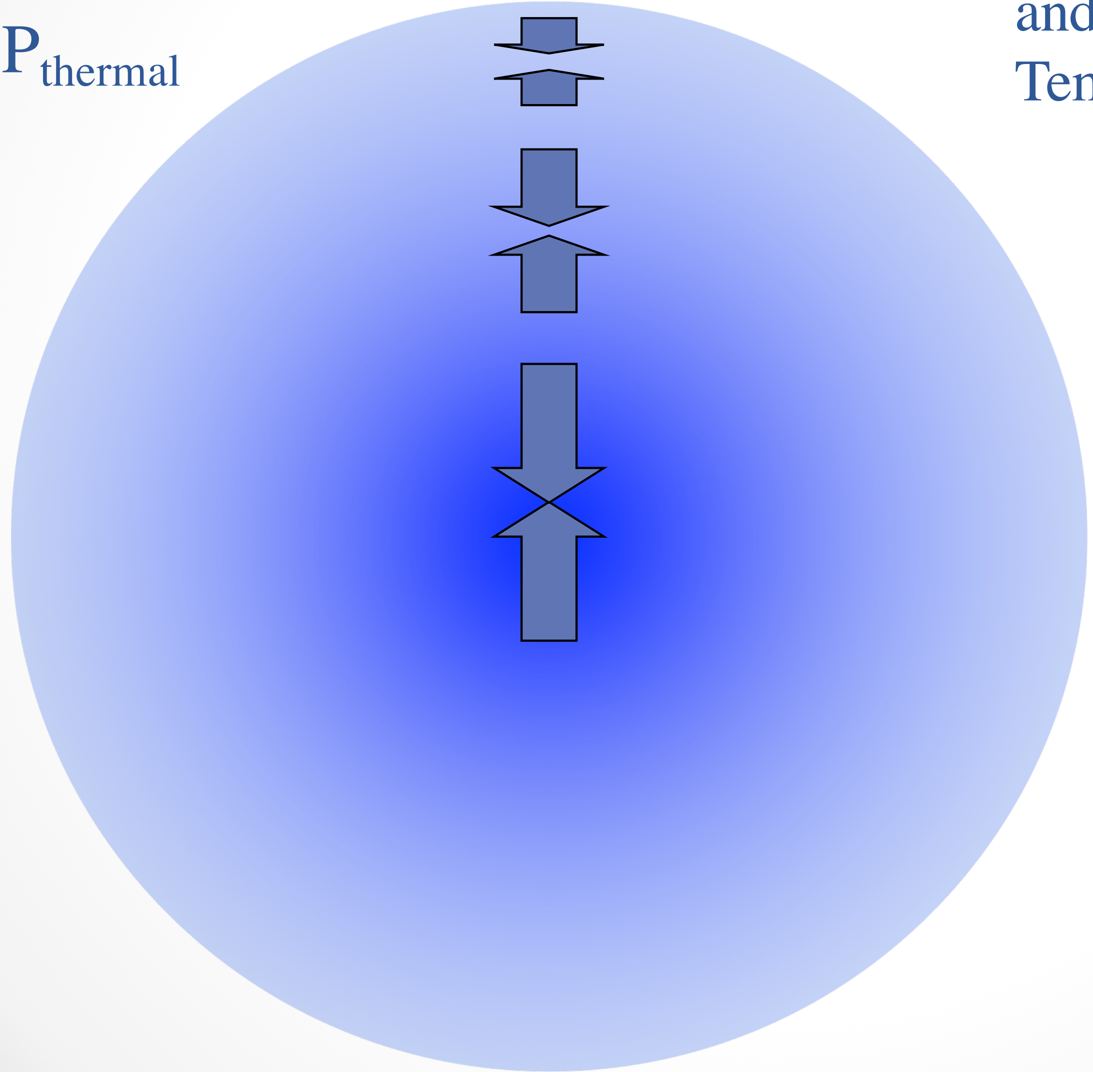
- We can determine another property of stars by using a model of Stellar Structure

- The basic principle is that stars are in

  Hydrostatic Equilibrium

Hydrostatic Equilibrium

At each radius
$P_{grav}=P_{thermal}$



$P_{thermal}$ is due to gas pressure and is proportional to Temperature

As the weight of overlying material goes up, the temperature needs to go up to keep pressure balance

What happens if a star is not in hydrostatic equilibrium?

The Structure of the Sun

- Build a model of the Sun in hydrostatic equilibrium and you will predict the *Temperature* and *Density* as a function of radius. You need to have a relationship between pressure, temperature and density -- this is called the *Equation of State*.

- The first stellar structure models were constructed in the late 1950s. Today you could build a stellar model and evolve the star on an iphone.

FUNDAMENTAL STELLAR STRUCTURE EQUATIONS (FSSE) IN TIME-INDEPENDENT (STATIC) FORM

$$\frac{dP}{dr} = -G\frac{M_r\rho}{r^2}$$

HYDROSTATIC EQUILIBRIUM

$$\frac{dM_r}{dr} = 4\pi r^2\rho$$

MASS CONSERVATION

$$\frac{dL_r}{dr} = 4\pi r^2\rho\epsilon$$

ENERGY EQUATION

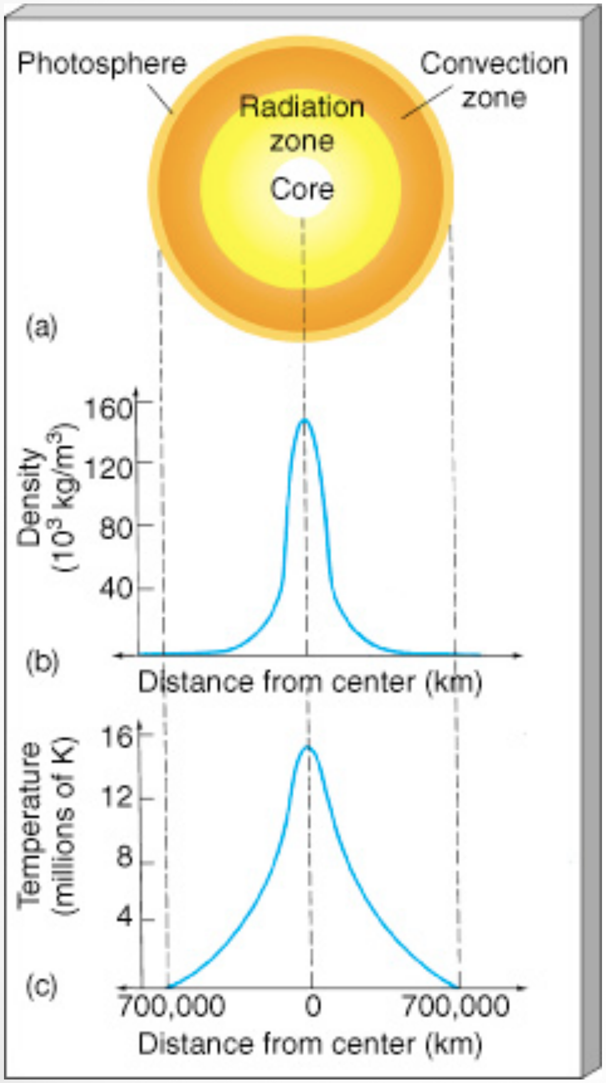
$$\left.\frac{dT}{dr}\right|_{rad} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

RADIATIVE TRANSPORT

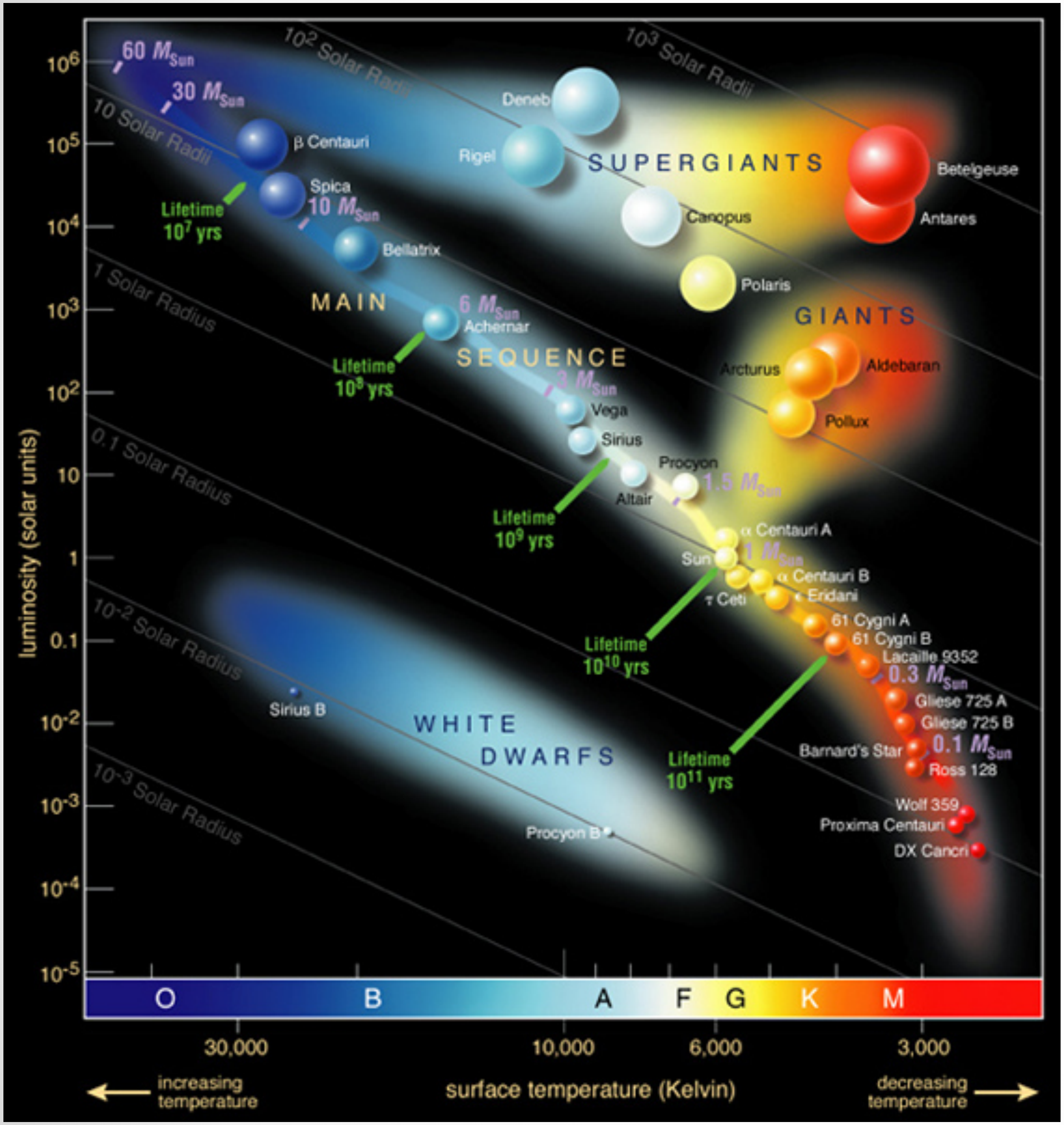
$$\left.\frac{dT}{dr}\right|_{\mathrm{ad}} = -\left(1-\frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2}$$

ADIABATIC CONVECTION

Solar Model



- Hydrostatic models for the Sun predict the central temperature to be about 16 x 10^6K.
- Some interesting things happen at this temperature! On Earth the only time this temperature has been reached is when *H-bombs were exploded.*



Stellar Models need to reproduce:

1) the H-R Diagram and
2) the mass-luminosity relationship

Energy Source for Stars

- A really good question is `how do stars produce all that luminous energy'
- The answer should also naturally explain the main-sequence and the mass-luminosity relation.
- Let's start with the Sun. Requirements are:

(1) $L_{SUN}=4x10^{33}$ ergs/sec

(2) for ~4.5 billion years

Coal or Wood Burning

What is "burning"? The conversion of molecular binding energy into heat.

Coal burning efficiency: 4 x 10^{12}ergs/gram

A 3000gram bucket of coal will generate

1.2 x 10^{16} ergs = 300 kilowatt-hrs

which would power a little space heater for about an hour.

Coal Burning

- Suppose all 2 x 10^{33} grams of the Sun are coal. The total energy you could generate would be:

$$\boxed{E_{total} = \left(4 \times 10^{12} \frac{ergs}{gram}\right) \times \left(2 \times 10^{33}\, grams\right) = 8 \times 10^{45}\, ergs}$$

- That's a lot of energy! At the rate of L_{SUN}, how long would the Sun last?

Coal Burning Lifetime

$$\boxed{t = \frac{8\times10^{45}\,\cancel{ergs}}{4\times10^{33}\,\frac{\cancel{ergs}}{\mathrm{sec}}} = 2\times10^{12}\,\mathrm{sec} = 63000\,\mathrm{years}}$$

- By the mid-1800s it was recognized that the Earth was at least *millions* of years old.
- Note: L_{SUN} is equivalent to 1 ton of coal burned per second per square foot of the solar surface.
- Coal/wood doesn't work!

Gravitational Potential Energy

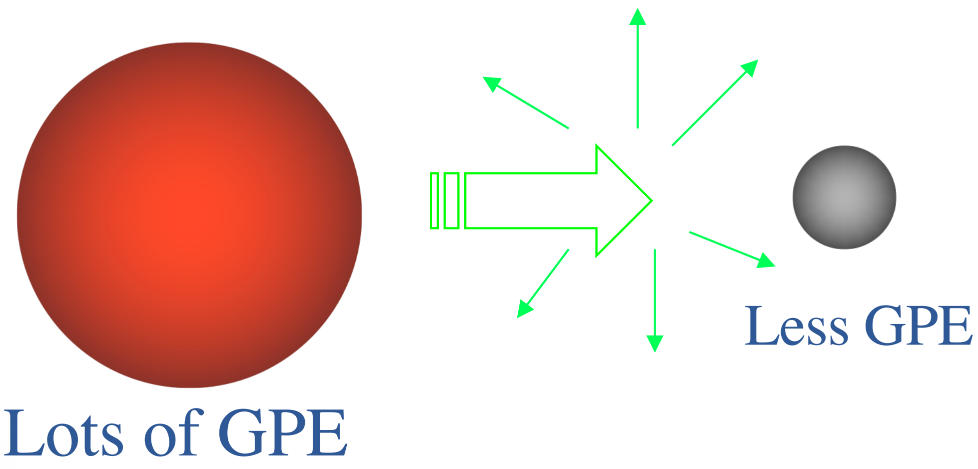
- Anytime you have a collection of mass, there is associated gravitational potential energy (GPE). For a sphere of gas (like a star for example) the GPE is given by:

$$\boxed{GPE = -\frac{5}{3} \times G \times \frac{M}{R^2}}$$

- Where R is the radius of the sphere, M is the mass and G is the universal gravitational constant.

GPE

- Imagine a large gas cloud that shrank in radius at constant mass.



- Difference in GPE (actually 1/2 of it) must be released as E-M radiation during the shrinking.

GPE

- Note that we are familiar with this idea based on life on Earth. A piano on the edge of a building has more GPE than it has at the bottom of the building (it is at a larger R from the center of the Earth).
- Drop the piano and this GPE is converted into kinetic energy, then into sound waves, breaking molecular bonds and heat.

GPE efficiency

- We can calculate the GPE of the Sun from the formula a few slides back:

$$\boxed{GPE_{Sun} = 2 \times 10^{48}\, ergs}$$

- If we could extract this amount of energy for radiation (by letting the Sun shrink) it would yield:

$$\boxed{Eff\,(gpe) = \frac{2 \times 10^{48}\, ergs}{2 \times 10^{33}\, grams} = 10^{15}\, \frac{ergs}{gram}}$$

GPE

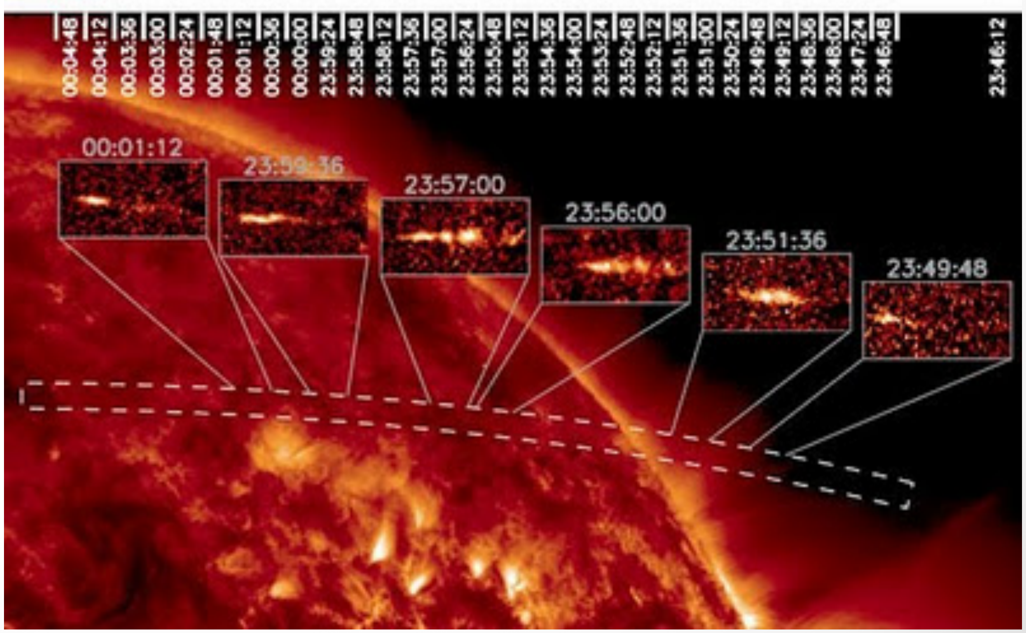
- This is 250 times more efficient than chemical burning, but if you do the lifetime calculation again:

$$\boxed{t = \frac{2\times 10^{48}\,ergs}{4\times 10^{33}\,ergs/\mathrm{sec}} = 5\times 10^{14}\ \mathrm{sec} = 16\times 10^{6}\,years}$$

- Still too short! (Plus the Sun would have to have been much larger in the past)

GPE

- A variant of this method was considered a serious possibility until the early 1900s. Comets falling into the Sun would make a good energy source. Only need about 1 Earth mass per year.
- The mass of the Sun would be increasing, and there is no evidence from changes in the Earth's orbit to support this idea.



Nuclear Power

- The answer for the power source of the Sun had to await the discovery of the atomic nucleus and the nuclear force.

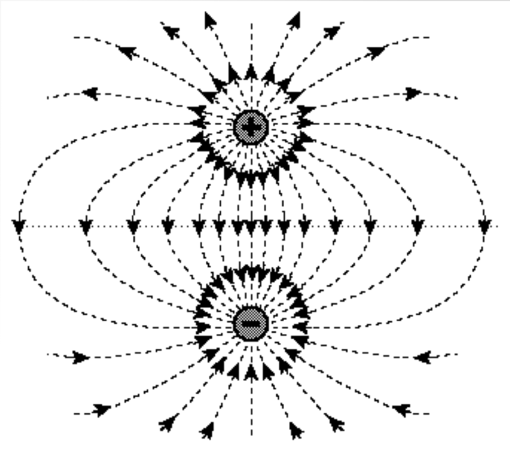
Forces of Nature

First, let's review the fundamental forces.

(1) Gravity is an attractive force between objects with mass. This keeps you in your seat, keeps the Earth in orbit around the Sun, prevents the hot gases in the Sun from expanding away into space and other useful stuff.

This is the weakest of the fundamental forces.

Forces of Nature



(2) <u>Electric Force</u> between electrically-charged objects (protons and electrons).

Like charges Repel, opposite charges Attract.

This is the force that holds atoms and molecules together and is useful for running nifty gadgets. It is also the reason that you don't fall through the floor and get cooked in the middle of the Earth.

Chemical burning is converting electrical forces into E-M radiation.

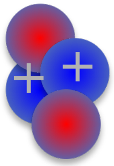
Forces of Nature

(3) Weak Nuclear Force. This is responsible for radioactive decay of some nuclei.

(4) Strong Nuclear Force. This is an attractive force between nucleons and it is what binds protons and neutrons together in the nuclei of atoms

Which force is stronger? (iclicker quiz)

A. electric
B. nuclear strong force



Must be the nuclear strong force! The protons in a nucleus repel one another, yet nuclei are not flying apart. In fact, the Nuclear strong force would right this minute be binding everything in the Universe into a tiny ball except for the fact that is only acts over <u>VERY TINY</u> distances.

The Four Forces

- Note that Gravity is MUCH weaker than the electric force or the strong nuclear force. Comparing the electric repulsion of two protons to their gravitational attraction, gravity is weaker by a factor of 10^{36}.
- So, why is gravity the force we see dominating the Universe?

Energy from the Nuclear Force

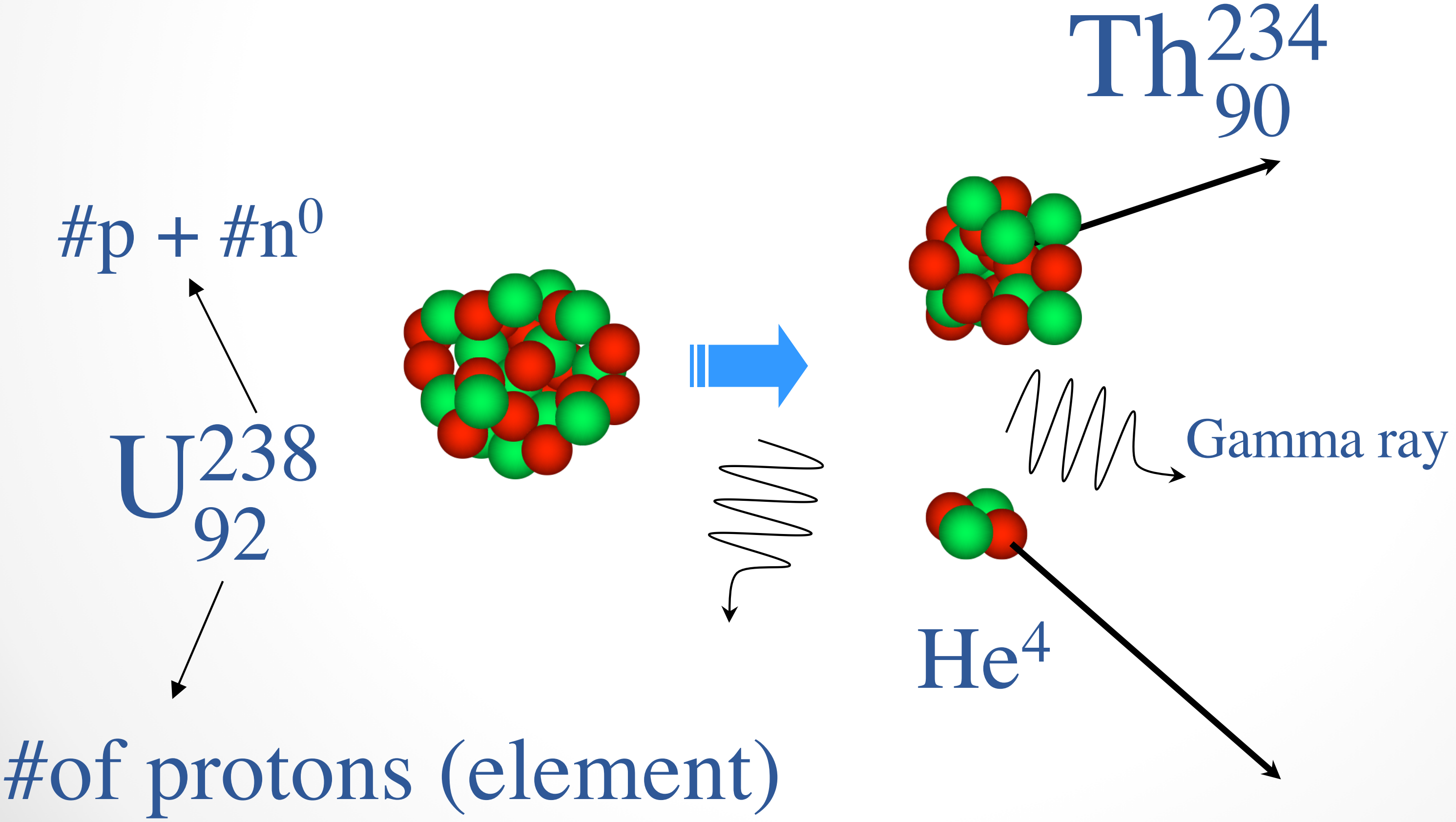
- There are two paths to deriving energy from nuclear reactions (a nuclear reaction is adding or subtracting a proton or neutron from a nucleus).

(1) Nuclear Fission

(2) Nuclear Fusion

Nuclear Fission

- There are some elements in the Periodic Table that spontaneously fall apart. These are the radioactive elements.



Th^{234}_{90}

$\#p + \#n^0$

U^{238}_{92}

Gamma ray

He^4

#of protons (element)

Nuclear Fission

$$\boxed{U^{238} \longrightarrow Th^{234} + He^{4}}$$

There is a remarkable thing!

$$\mathrm{Mass}(U^{238}) > \mathrm{Mass}(Th^{234}+He^{4})$$

Mass Defect

- Even though there are the same number of nucleons before and after, a little mass disappeared in the reaction. This mass is associated with the difference in Nuclear Binding energy before and after.
- The missing mass has been converted into energy via everyone's favorite physics equation!

$$\boxed{E = mc^2}$$

In this application, `m’ is the mass that went missing and `E’ is the energy released in the form of gamma rays. For some fission reactions, the amount of energy released per gram of missing mass is large.

Nuclear Fission

Fission can release as much as

10^{18} ergs/gram

Recall, coal burning released

4×10^{12} ergs/gram

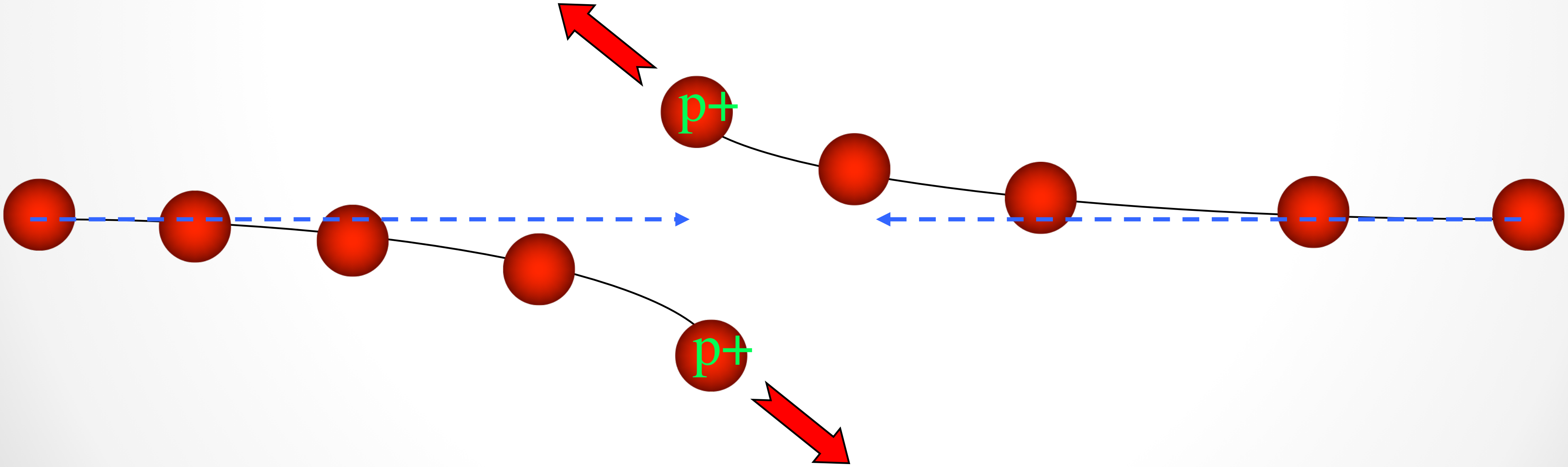
- Nuclear fission could produce enough energy to fuel the Sun for ~6 x 10^{10} years *if* the Sun were composed completely of fissionable material.
- It is not! For example, the Uranium abundance in the Sun is less than a millionth of the solar mass.

Nuclear *Fusion*

- Imagine a gas of protons (p+). At `low' temperatures the electrical repulsion force prevents the close approach of two p+
- But as T increases, the minimum approach distance decreases and some p+ get close enough (10^{-15}m) for the short-range, but VERY STRONG nuclear force to bind them together.

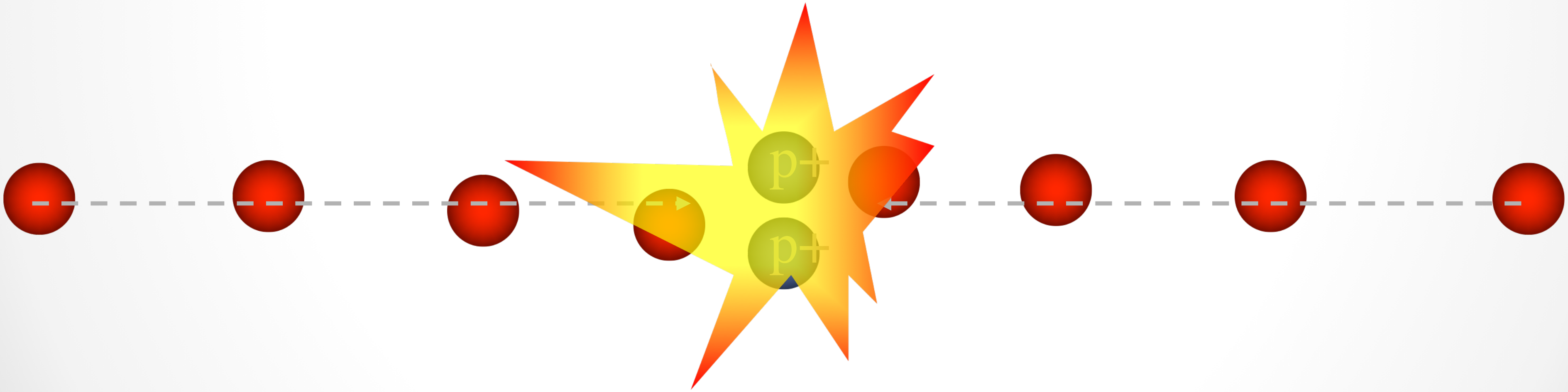
Hydrogen (proton) fusion

Like electrical charges repel. So, protons in a gas *avoid* `collisions’



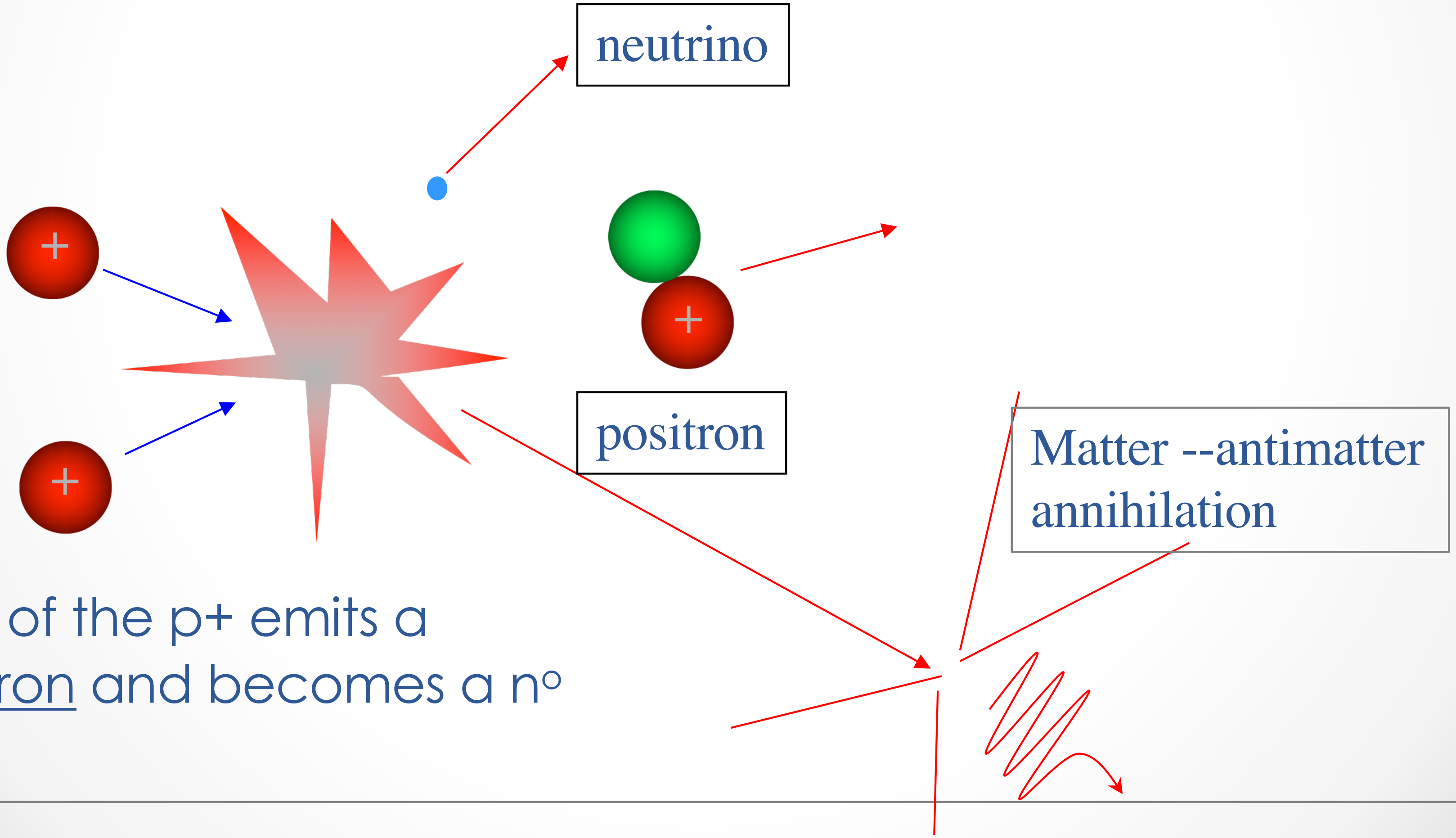
Hydrogen (proton) fusion

However, as a gas temperature goes up, the average speed of the particles goes up and the protons get closer before repelling one another. If the proton get very close, the short-range *nuclear force* fuses them together.



Hydrogen Fusion

- Proton fusion involves some interesting ideas.



One of the p+ emits a
Positron and becomes a n°

Hydrogen Fusion

When two protons fuse, almost immediately one turns into a neutron by emitting a positively charged electron (known as a positron and `beta-decay'). The e+ is antimatter (!) When it comes into contact with its matter partner (e-) it annihilates entirely into energy.

What element has a nucleus with 1 proton and 1 neutron? Deuterium (heavy hydrogen)

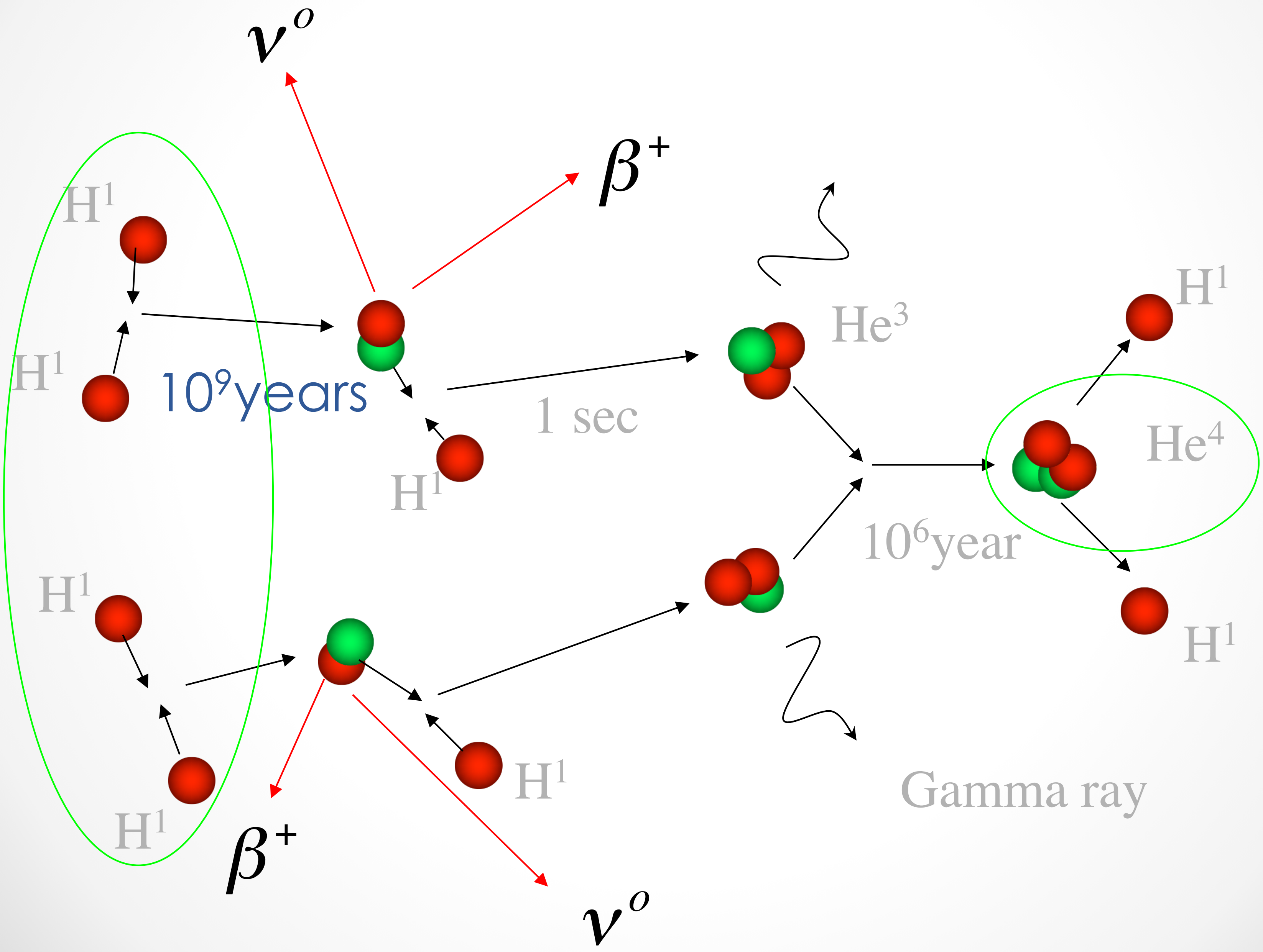
Neutrinos

- Another curious particle that flies out of the reaction is called a `neutrino' (little neutron).
- This is a chargeless, nearly massless particle which has a tiny crossection for interaction with other types of matter. The mean free path in lead is five light years.
- Neutrinos were first postulated in 1932 to account for missing angular momentum and energy in beta-decay reactions (when a proton becomes a neutron and emits a positron).
- Neutrinos are leptons - subatomic particles that participate in weak interactions, but not the strong nuclear force.

Neutrinos

- The Sun emits about 10^{38} neutrinos/sec.
- Humans emit about 3×10^{8} neutrinos/day (radioactive potassium).
- Every second we have many neutrinos pass through our bodies: 400,000 billion from the Sun, 50 billion from the Earth and 100 billion from nuclear power plants.

P-P Chain



ν^o
β^+
H^1
H^1
10^9years
H^1
1 sec
He^3
H^1
He^4
10^6year
H^1
H^1
H^1
H^1
β^+
ν^o
Gamma ray

P-P Chain

- The net result is

$$4H^1 \text{ --> } He^4 + \text{energy} + 2 \text{ neutrinos}$$

where the released energy is in the form of gamma rays.

The source of the energy is again a tiny bit of mass that goes missing:

$\text{Mass}(4H) = 6.6943 \times 10^{-24}$ grams

$\text{Mass}(He^4) = 6.6466 \times 10^{-24}$ grams

P-P Chain

- The amount of missing mass is:

$$\boxed{\Delta mass = 0.048 \times 10^{-24}\ grams}$$

- The energy generated is:

$$\boxed{E = \Delta mc^2 = 4.3 \times 10^{-5}\ ergs}$$

- This much energy is released by $4H^1$ with a total mass of 6.6943×10^{-24} grams. The efficiency of hydrogen fusion is therefore:

$$\boxed{6.4 \times 10^{18}\ \text{ergs/gram}}$$

Sun's Lifetime with H-fusion

- Total energy available:

$$\boxed{6.4\times10^{18}\,\frac{ergs}{gram}\times\left(2\times10^{33}\,grams\right)=12.8\times10^{51}\,ergs}$$

- Lifetime of the fusion-powered Sun

$$\boxed{\frac{12.8\times10^{51}\,ergs}{4\times10^{33}\,\dfrac{ergs}{\text{sec}}}=3.2\times10^{18}\,\text{sec}=10^{11}\,\text{years}}$$

- This is promising! Is hydrogen fusion for real? Yes, remember Bikini Atoll.

Requirements for a Fusion-powered Sun

(1) Need lots of protons (H nuclei). ☑

Element	Abundance (#)
Hydrogen	92.0%
Helium	8.3%
Oxygen	0.06%
Carbon	0.02%
Nitrogen	0.01%
The rest	…

Requirements for Fusion-powered Sun

(2) Need a high temperature. For the p-p chain the gas temperature needs to be $>10^7$K.

- Recall the discussion about stellar structure. In the center of the Sun, hydrostatic equilibrium requires the temperature to be around 15 million K.
- In the center 10% of the Sun $T > 10^7$K ☑

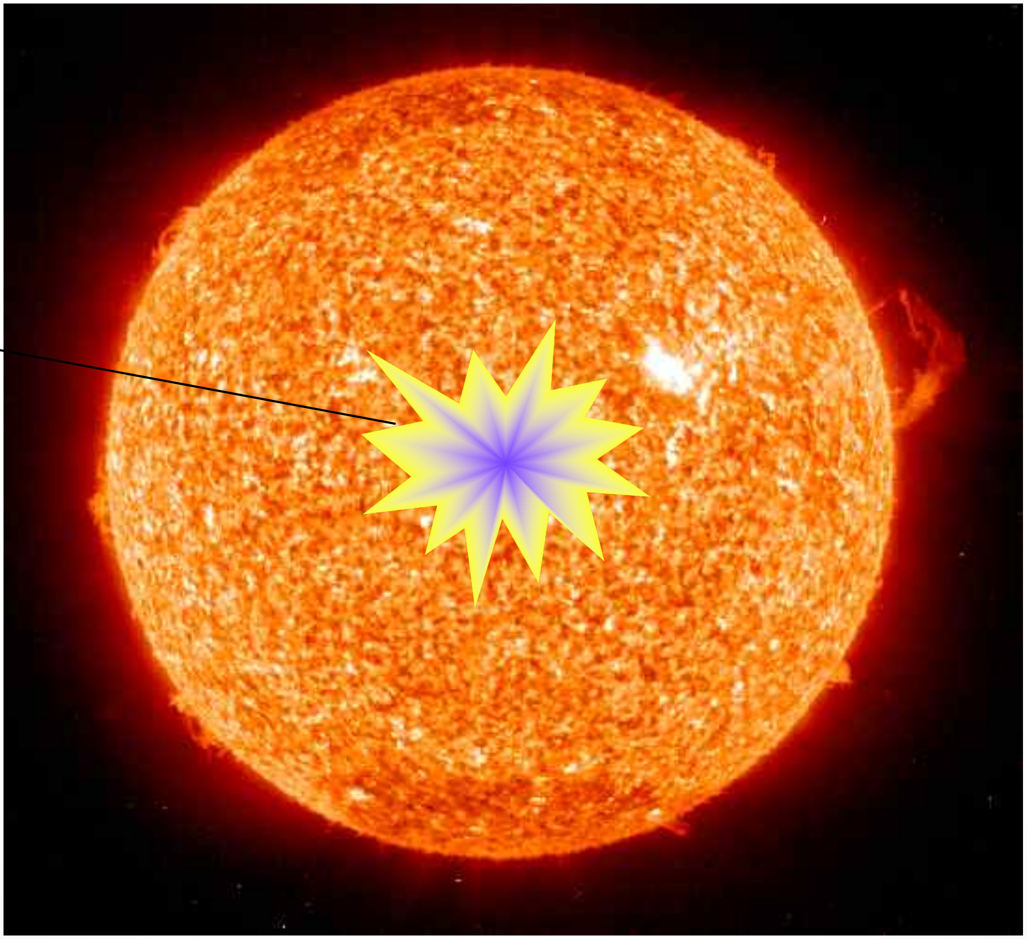
Fusion-powered Sun

1) Lots of <u>fuel</u>

2) Conditions are <u>right</u> in the central 10% for the P-P cycle to run

3) P-P fusion is <u>efficient</u> enough to power the Sun for billions of years.

Looks like the Answer!



H -->He

Note a small detail

- In the Sun, the protons don’t really get close enough for the nuclear force to be stronger than the electric repulsive force.
- Fusion in the Sun (and life on Earth) is the result of a very odd property of atomic matter called <u>Quantum Tunneling</u>.
- An analogy is me running full speed and attempting to leap over a 30-foot-tall wall. Most attempts, I crash into the wall and fall to the ground. But, once in awhile, even though I don’t leap 30’ into the air, I am successful and end up on the other side of the wall.

Fusion-powered Sun

- An interesting aside. To produce the solar luminosity, the Sun is converting matter into energy:

$$\boxed{(4\times 10^{33}\,\frac{ergs}{\sec})\times(\frac{1}{6.4\times 10^{18}\,\frac{ergs}{gram}}) = 6.25\times 10^{14}\,\frac{grams}{\sec}}$$

- This is the mass of H --> He per second. The amount of matter converted to energy is:

<u>4.3 million tons per second</u>

Q. Why do thermonuclear reactions only occur in the Sun’s core? (iclicker quiz)

A. That is the only place in the Sun it is hot enough
B. That is the only place in the Sun where there is hydrogen
C. That is the only place in the Sun where there is Uranium
D. That is the only place in the Sun where there is anti-matter

The principal reason we have ruled out nuclear fission as the source of energy for the Sun is:

A. The spectrum of the Sun is much too cool
B. The Sun has far too little fissionable material
C. The radioactivity of the Sun would have made life on Earth impossible
D. Even if the Sun were made completely of fissionable material (e.g. uranium) it would only last around 10 million years at its current luminosity

The Main Sequence

Once the energy source of stars has been identified, it is easy to understand the main sequence in the H-R Diagram.

(1) More massive stars require higher central temperatures (hydrostatic eqm.)

(2) The P-P fusion rate and luminosity is proportional to T^4

Therefore, more massive stars will have higher central temperature and higher Luminosity. This is what is seen along the H-R Diagram main sequence.

Stellar Lifetimes

- The Sun (and all stars) will eventually run out of fuel (hydrogen in regions where it is hot enough for fusion).
- If all the hydrogen in the Sun could fuse to helium, the Sun's lifetime would be 100 billion years.
- But, by the time about 10% of the Sun's H has been converted into He the solar structure will be changed and it will not be a main-sequence star.

Stellar Lifetimes

- The Sun has a main-sequence lifetime of 10 billion years. What about the other stars?

  (1) The <u>fuel</u> for stars is mass

  (2) The <u>fuel consumption rate</u> is Luminosity

  So, it's easy!

$$\boxed{Life_{m-s} \propto \frac{Mass}{Luminosity}}$$

Example Stellar Lifetime

Suppose you have a $15M_{SUN}$ star with a luminosity of $L=10{,}000L_{SUN}$. How long will this star spend on the main sequence?

$$\boxed{\text{Lifetime}(15M_{SUN}) = \frac{15}{10000} \times \text{Lifetime}(1M_{SUN})}$$

15 times as much fuel extends the life of the star

10,000 times L decreases the lifetime

Stellar Lifetimes

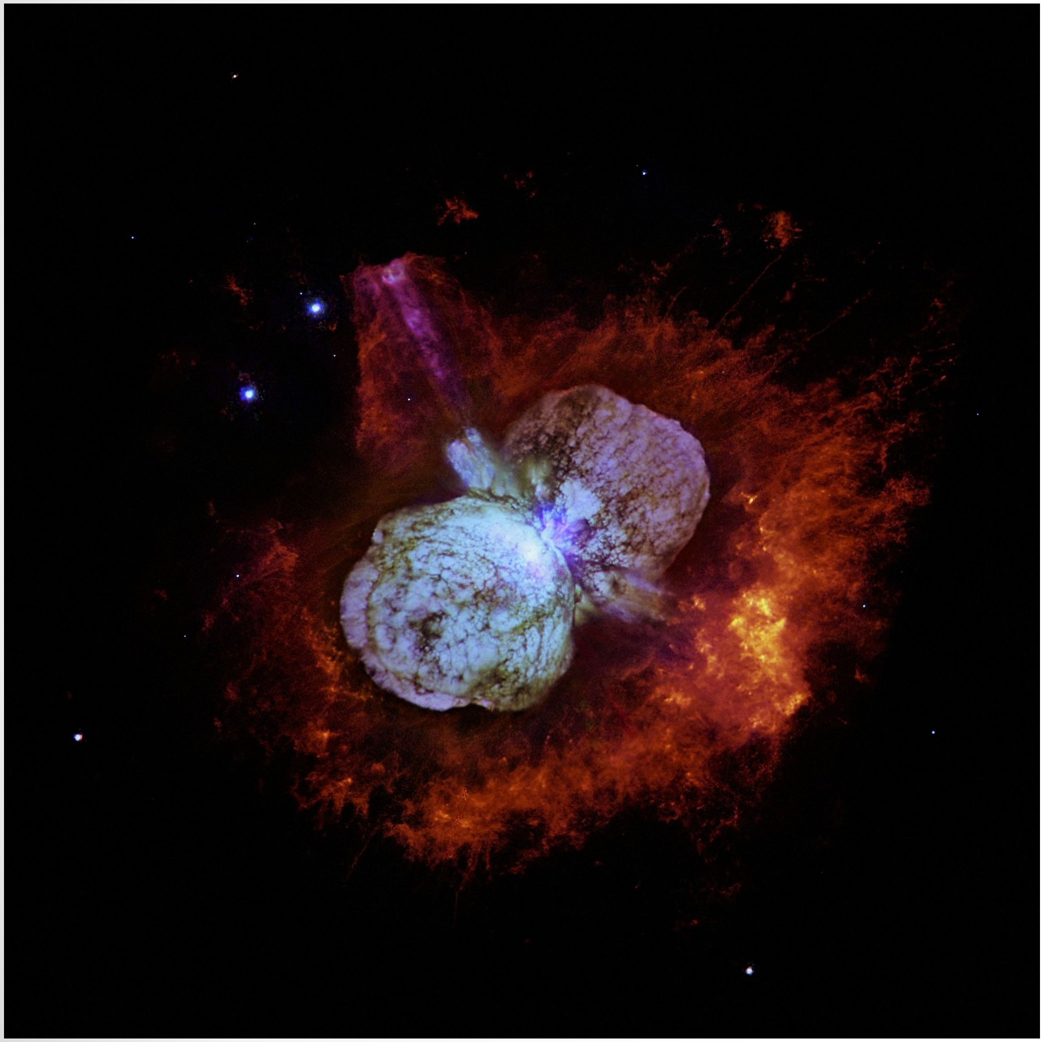
- So based on the extra fuel, you expect this star to live longer than the Sun, but this is more than counteracted by the high rate of using the fuel. This is the general trend.

Massive stars are like like gas-guzzling SUVs
Low-mass stars are like the Toyota Prius.

Lower Mass Limit for Stars

- We now can see why there is a lower limit for the mass of a star of about $0.08M_o$.
- For decreasing mass, the central temperature of a star decreases. At $0.08M_o$ the central temperature drops below that required for P-P fusion
- Objects below this mass are called Brown Dwarfs or planets

Upper Stellar Mass Limit



Upper limit for stellar mass is set by the luminosity pressure

The “Eddington Limit”

Eta Carina is a star of almost 100 solar masses.

Radiation pressure is blasting off the outer parts.

Philosophical Side Trip

- There is an interesting implication of our understanding of how the Sun and other stars produce energy:

The Universe is evolving:

- H ->He
- Stars do not live forever

Q. If the thermonuclear fusion in the Sun were suddenly to stop, what would happen to the radius of the Sun? (iclicker quiz)

A. It would grow larger and incinerate Mercury, Venus and the Earth
B. It would decrease as gravity gained the upper hand
C. It would not change at all because the fusion is occurring so deep inside of the Sun

Main-sequence lifetime

An $0.5M_{Sun}$ star generates a luminosity of $1/10\ L_{Sun}$. How long does this star spend on the main-sequence of the H-R Diagram? The main-sequence lifetime of the Sun is 10^{10} years. (Iclicker quiz)

A. $0.5 \times 0.1 \times 10^{10} = 0.5 \times 10^{9}$ years
B. $(0.5/0.1) \times 10^{10} = 0.5 \times 10^{11}$ years
C. $0.5 \times 0.1 / 10^{10} = 0.5 \times 10^{-12}$ years
D. Trick question: you need to have more information