Problem Set #1

In[4]:=

1. (32) Draw a wave envelope diagram for the dispersion relation

In[16]:=

```
Omega[k_] := Sqrt[k^2 - k^4]
```

In[7]:=

Solution: The wave envelope diagram is the set of all lines one for each wave mode, whose equation is given by

In[17]:=

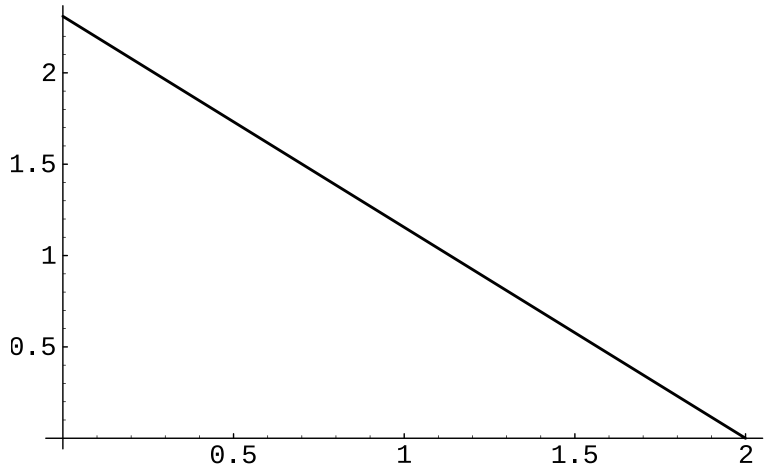
```
fn[k_] := (1. - k x)/Omega[k]
```

In[9]:=

To see what this means, look at a single line corresponding to a value of k = .5

In[18]:=

```
Plot[Evaluate[fn[.5]], {x,0,2}]
```



Out[18]=

```
-Graphics-
```

The x intercept is at 1/k and the t intercept is at 1/Omega. This is a wave going to the left, and the wavecrests are moving at a speed

In[19]:=

```
Omega[.5]/.5
```

Out[19]=

```
0.866025
```

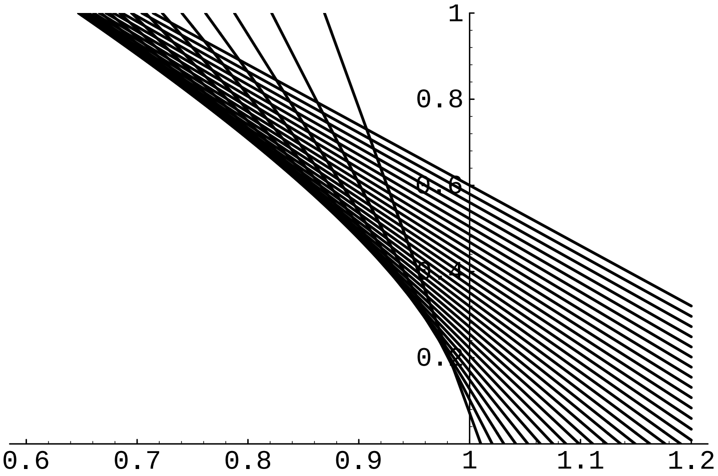
Now let us create a table with a large number of values of k, and plot the resulting family of lines

In[20]:=

```
ks = Table[ j, {j, .7, .99, .01}];
```

In[21]:=

```
Plot[Evaluate[fn[ks]], {x,.6,1.2}, PlotRange->{0,1}]
```



Out[21]=

```
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```

For k values greater than 1 there are no solutions to the dispersion relation. This means that this system is unstable for short wavelength perturbations. Imaginary Omega means that the solution functions are not sin or cos but exponentials. A system with instability on short wavelengths is rather unphysical, tending to crinkle up into very complicated ground states. Rather like biological materials.
I had intended to give you the system with the other sign

In[22]:=

```
Omega[k_] := Sqrt[k^4 - k^2]
```

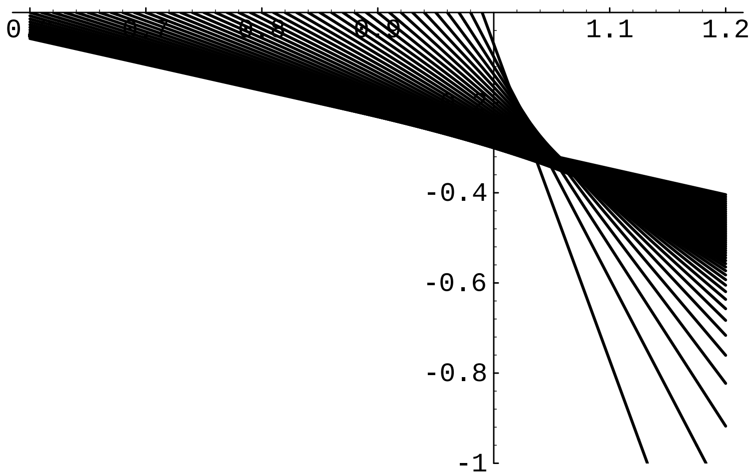
Now if we make a plot, choosing a different way of making the table so that we will have a better spacing for the lines

In[23]:=

```
ks = Table[ 1/j, {j, .5, .99, .01}];
```

In[24]:=

```
Plot[Evaluate[fn[ks]], {x,.6,1.2}, PlotRange->{-1,0}]
```



Out[24]=

```
-Graphics-
```

Now you see the system unstable for long wavelengths. This can be easily realized physically if the dimensions of the system are smaller than the smallest unstable mode. One interpretation of this dispersion relation is that it describes a system with compression and bending stiffness. Rather like putting a weight on top of a column. If the column is too long it will buckle.