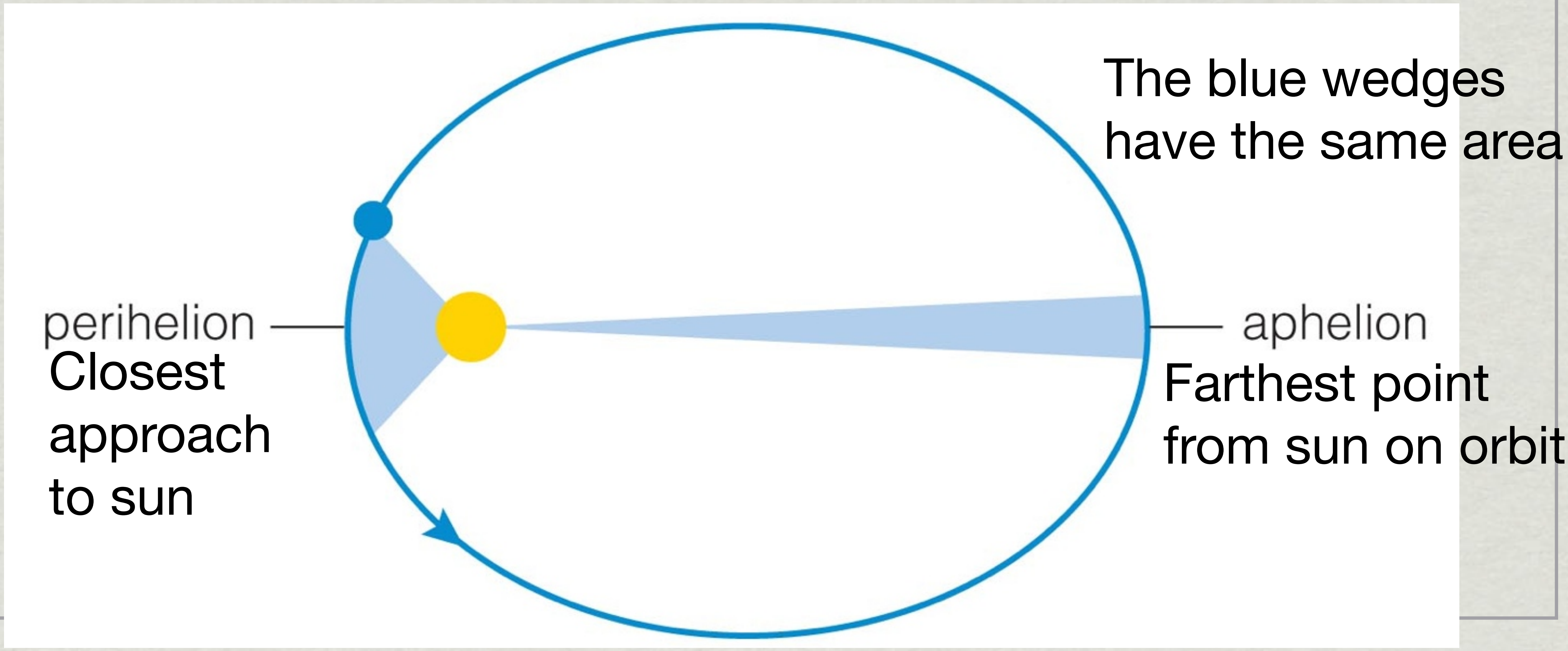
Announcements

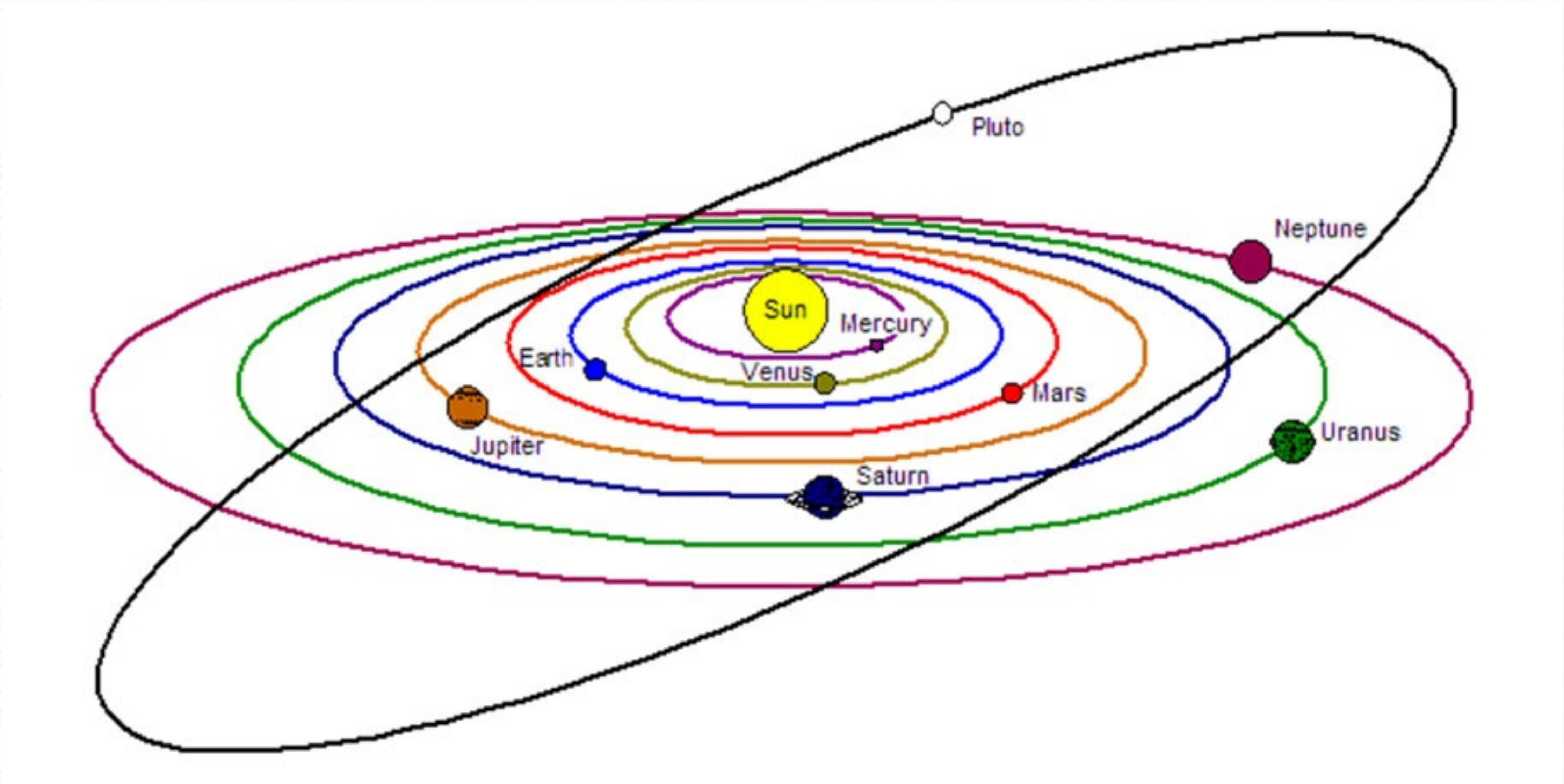
- Homework1 was due today
- Plato’s office hours: 3pm on Thursdays, ISB 165
- I’ll make the official homework due time at 5pm, after Plato’s office hours
- Remember; there will normally be reading due on Thursdays, too
- You should see a polling session active if you are using the REEF app. Make sure you are signed in
- I have two iClickers to loan out. First come, first served. For today’s lecture only

Kepler's Laws

- Kepler's Second Law: A planet moving along its orbit sweeps out equal area in equal time



Describing Motion



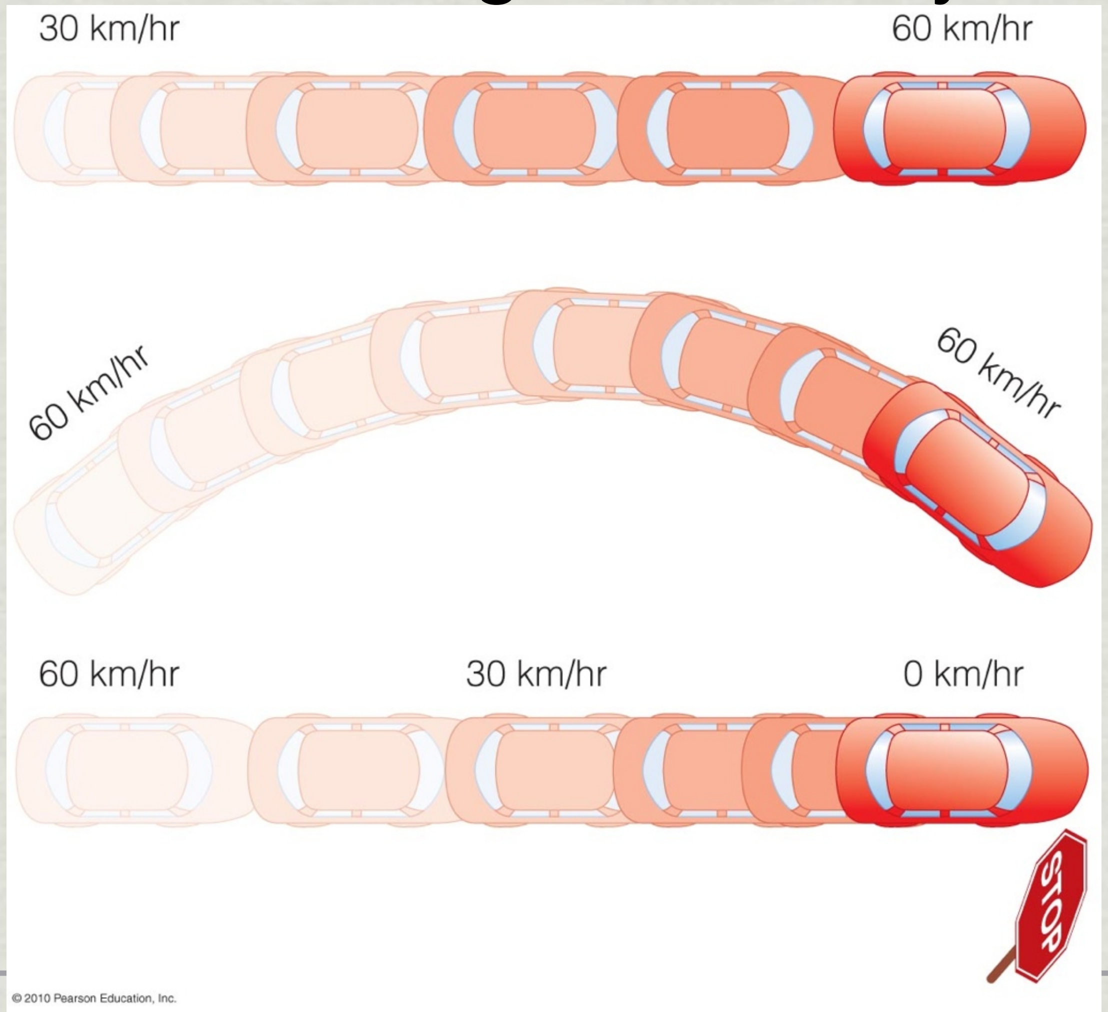
Describing Motion

Describing Motion

- **Speed**: $\frac{\text{Change in Position}}{\text{Change in Time}}$
  - units: meters/second (abbreviation: m/s)
  - You hike 12 miles in 4 hours. Your speed, in miles per hour?
    - this is your average speed over that 4 hours
- **Velocity**: a speed and a **direction** (m/s)

Describing Motion: Acceleration

- **Acceleration**: $\frac{\text{Change in Velocity}}{\text{Change in Time}}$
  - units: $\frac{\text{meters/second}}{\text{second}}$ = meters/second^2 (m/s^2)
  - change in velocity: change in speed or direction



All three motions shown here are accelerations

Describing Motion: Acceleration

- **Acceleration**: $\frac{\text{Change in Velocity}}{\text{Change in Time}}$

  - units: $\frac{\text{meters/second}}{\text{seconds}}$ = meters/second^2 (m/s^2)

Acceleration due to Earth's Gravity: 9.8 m/s^2

That's 22 miles/hour/second

What does that mean?
After 1 second: speed is 22 mph
After 2 seconds: speed is 44 mph
After 3 seconds: speed is 66 mph

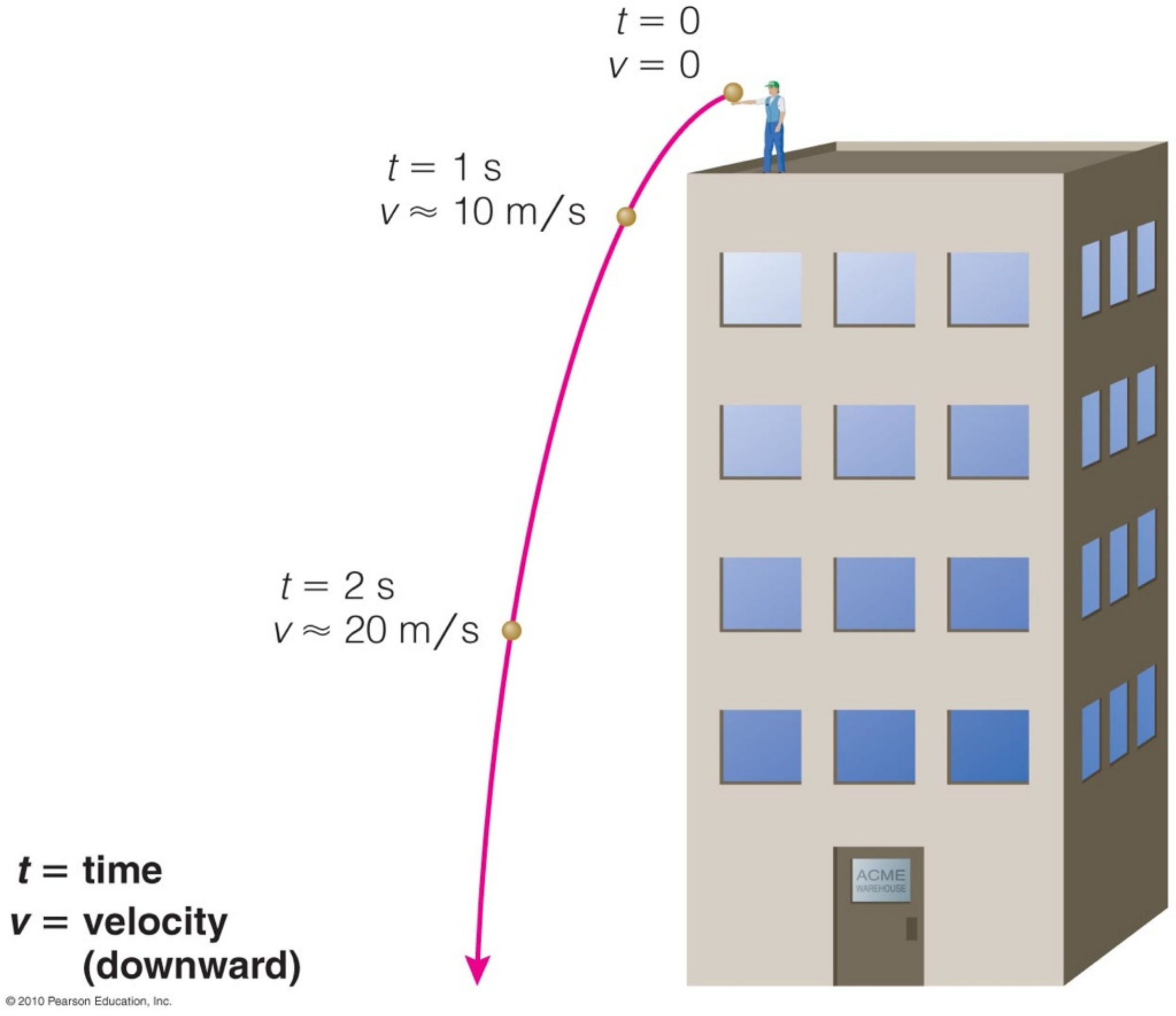
Cars: time to go from 0-60 mph
Sedans: about 10 sec. Sports cars: under 5 s

Describing Motion: Acceleration

- **Acceleration**: $\dfrac{\text{Change in Velocity}}{\text{Change in Time}}$

  - units: $\dfrac{\text{meters/second}}{\text{seconds}}$ = meters/second^2 (m/s^2)

Acceleration due to Earth's Gravity: 9.8 m/s^2



Motion away from building is due to horizontal speed you give the ball when you throw it.

If you just drop the ball: falls straight down at 9.8 m/s^2

Describing Motion

- **Mass**: amount of matter (stuff)
  - different from weight
  - units: kg
- **Momentum**: mass x velocity (kg m/s)
  - large momentum = large mass and/or large velocity
- Need a **force** to change momentum
  - if two objects experience the same change in velocity, the more massive object has a larger change in momentum
    - that means it experiences a larger force

Describing Motion

- **Mass**: amount of matter (stuff)
  - different from weight
  - units: kg
- **Momentum**: mass x velocity (kg m/s)
  - large momentum = large mass and/or large velocity
- Need a **force** to change momentum
  - for an object of some mass, like,  , a bigger change in velocity means a bigger change in momentum

Newton's First Law

Objects in motion stay in motion,

objects at rest stay at rest,

unless acted on by a force.

- To change a velocity (acceleration) requires a force

Newton’s Second Law

- Force: $\frac{\text{Change in Momentum}}{\text{Change in Time}}$ Units: (kg m/s^2)

  - For something with mass **m**:
    change in momentum (mv) = change in velocity

A change in velocity is an acceleration: $\frac{\text{Change in Velocity}}{\text{Change in Time}}$

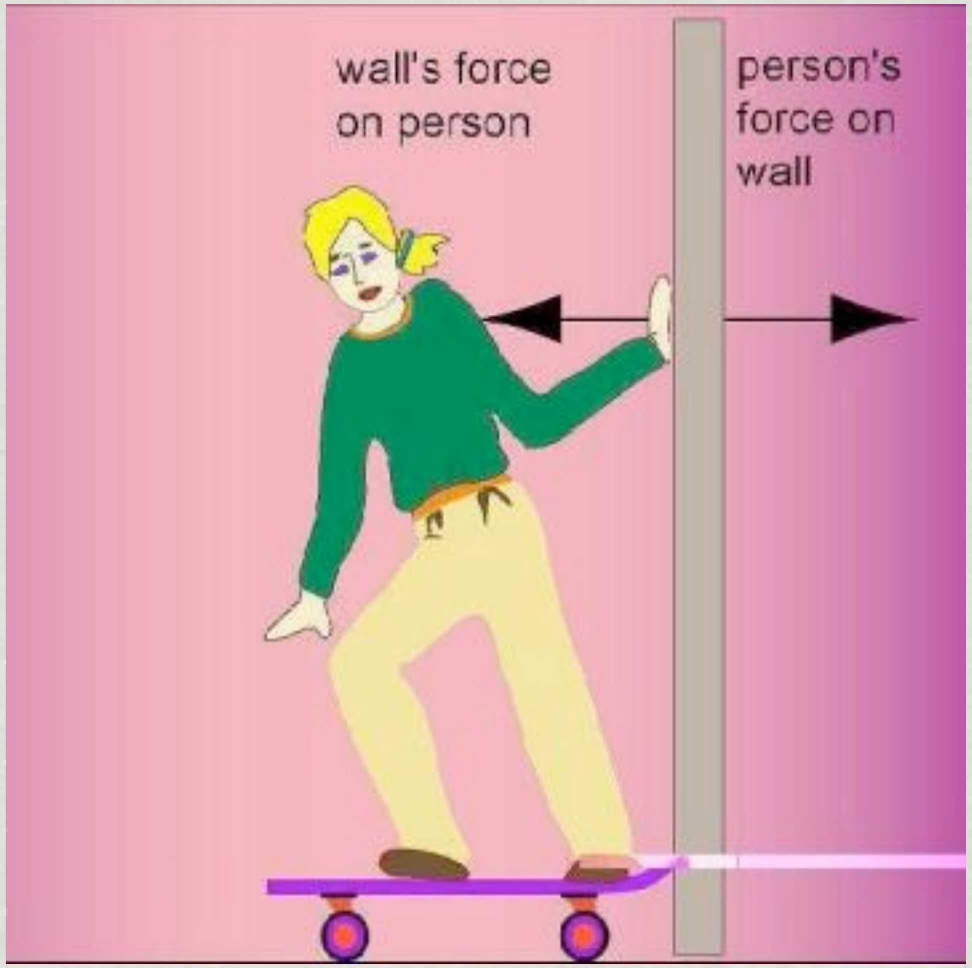
Big acceleration comes from a big change in velocity or
a fast change in velocity

➜ Force = mass x acceleration

familiar form: F = m a

Newton's Third Law

- For any force, there is an equal reactive force that works in the opposite direction



Newton's Third Law

- For any force, there is an equal reactive force that works in the opposite direction

$F_{boy \rightarrow girl} = F_{girl \rightarrow boy}$

$M_{boy}\ a_{boy} = M_{girl}\ a_{girl}$

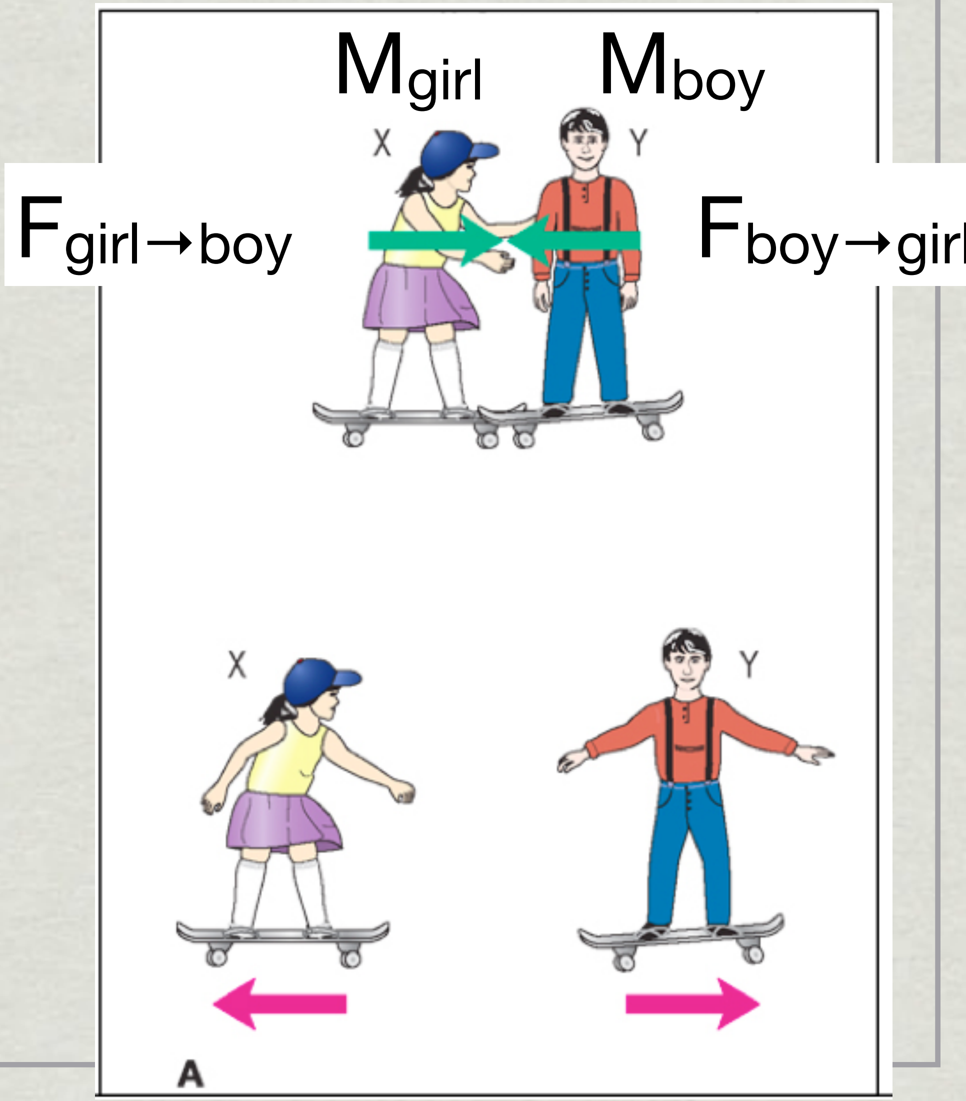
Remember: $F = m\ a$

Girl pushes off from boy. Boy stands still, hands at his side

If: $M_{boy} = 20$ kg
$M_{girl} = 20$ kg
$F_{girl \rightarrow boy} = 80$ kg m/s^2

What is $F_{boy \rightarrow girl}$?

A 40 kg m/s^2
B 20 kg m/s^2
C 80 kg m/s^2
D 160 kg m/s^2



Newton's Third Law

- For any force, there is an equal reactive force that works in the opposite direction

Remember: $F = m\,a$

$F_{boy \to girl} = F_{girl \to boy}$

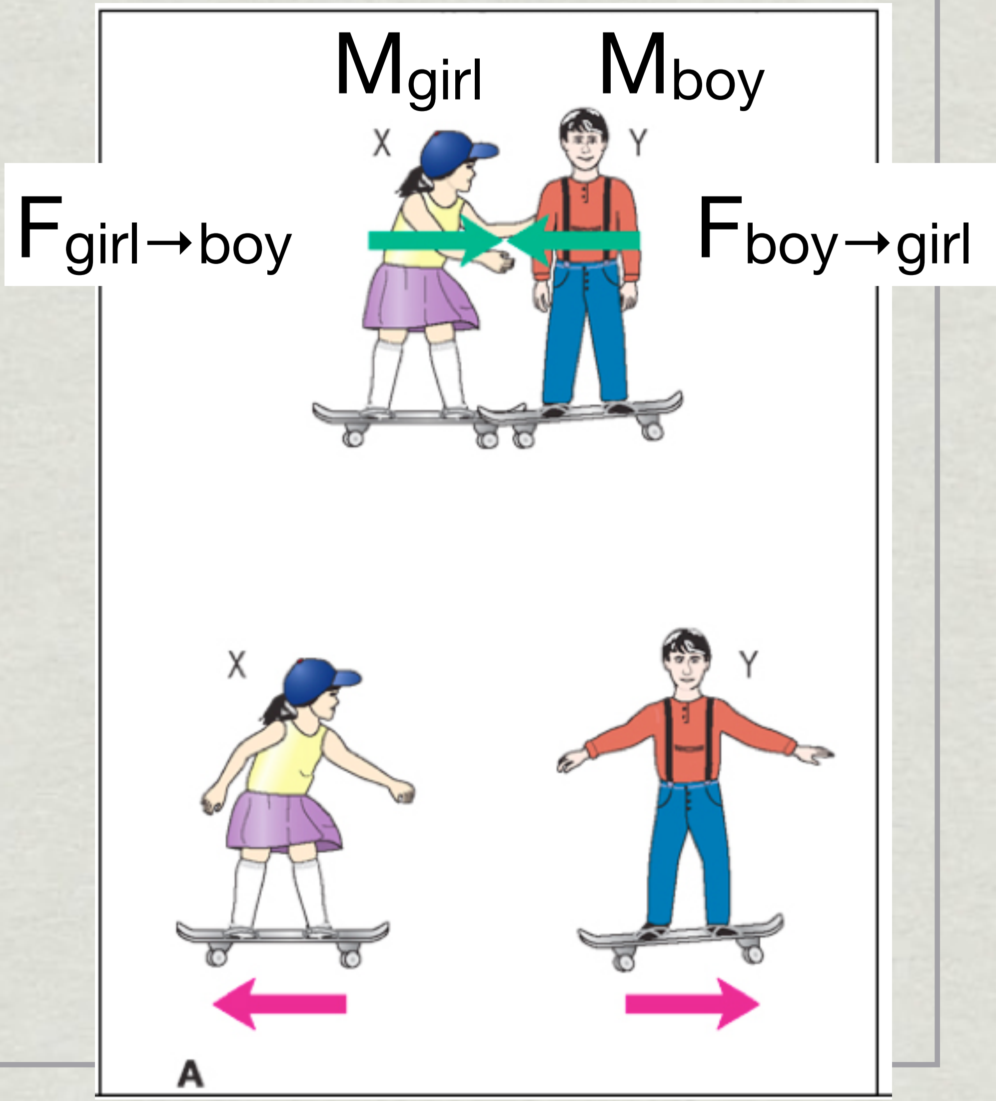
$M_{boy}\,a_{boy} = M_{girl}\,a_{girl}$

Girl pushes off from boy. Boy stands still, hands at his side

If: $M_{boy} = 20$ kg
$M_{girl} = 20$ kg
$F_{girl \to boy} = 80$ kg m/s^2

What is $F_{boy \to girl}$?

A 40 kg m/s^2
B 20 kg m/s^2
C 80 kg m/s^2
D 160 kg m/s^2



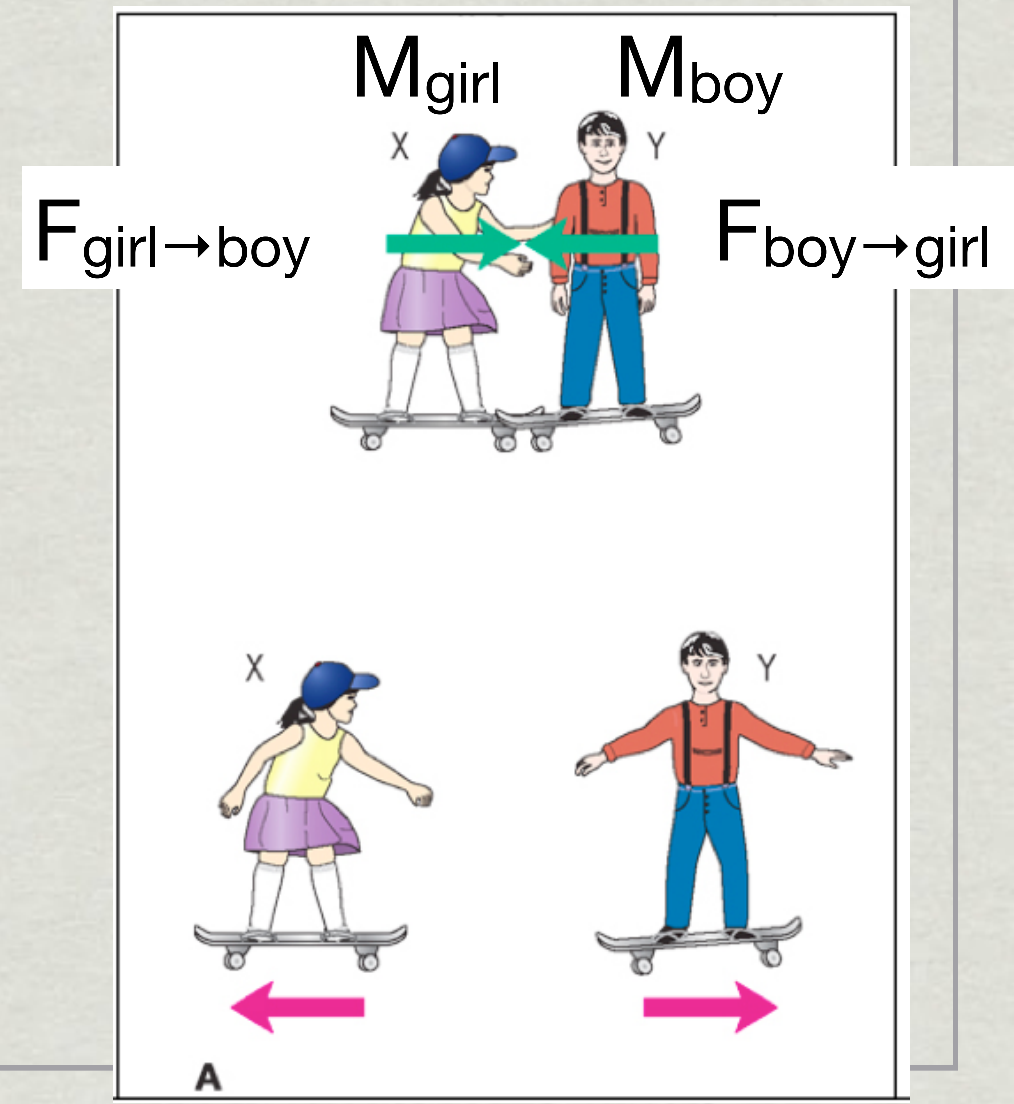
Newton's Third Law

- For any force, there is an equal reactive force that works in the opposite direction

$F_{boy \rightarrow girl} = F_{girl \rightarrow boy}$

$M_{boy}\ a_{boy} = M_{girl}\ a_{girl}$

Girl pushes off from boy. Boy stands still, hands at his side



If: $M_{boy} = 20$ kg
$M_{girl} = 20$ kg
$F_{girl \rightarrow boy} = 80$ kg m/s^2

What is $F_{boy \rightarrow girl}$? 80 kg m/s^2
What is a_{girl} ?

A 2 m/s^2
B 4 m/s^2
C 8 m/s^2
D 16 m/s^2

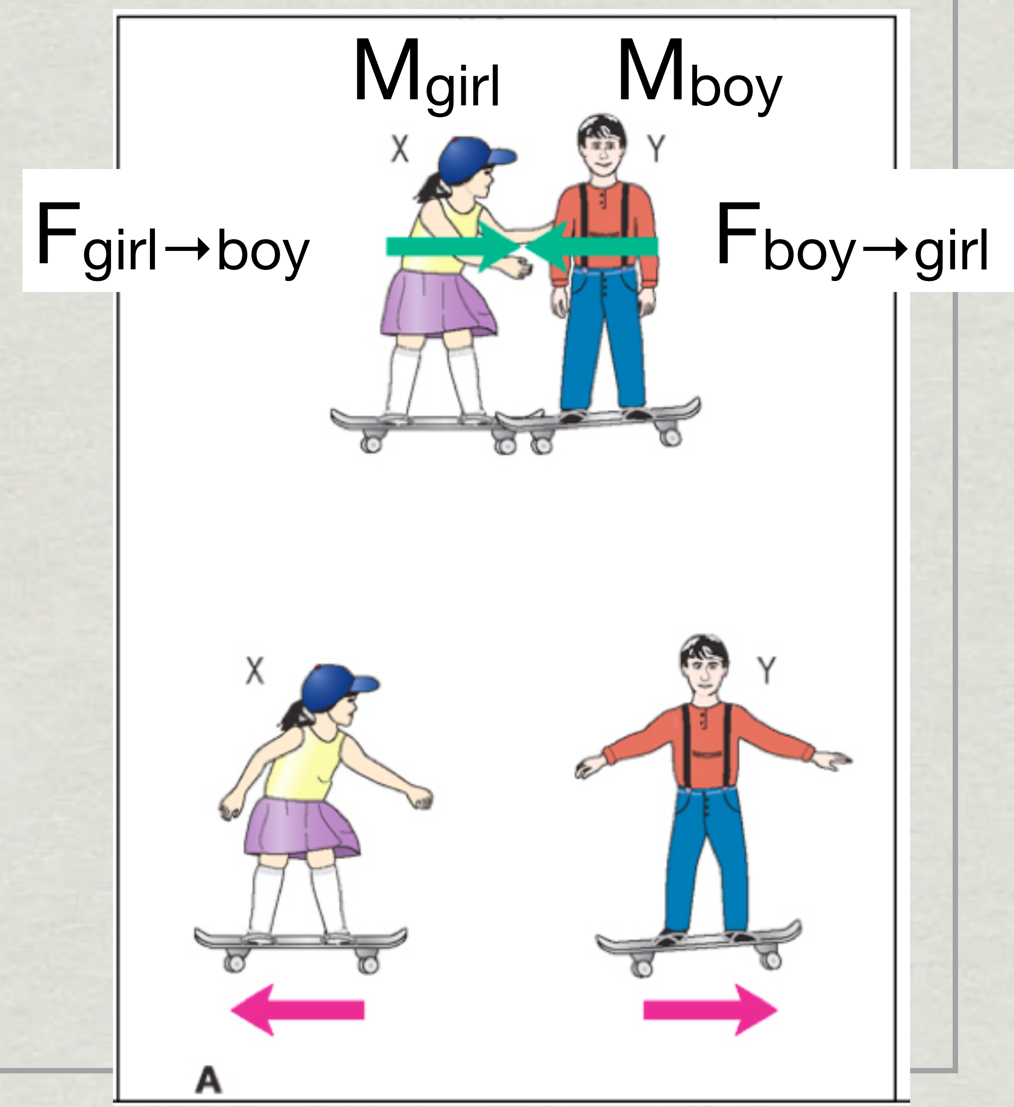
Newton's Third Law

- For any force, there is an equal reactive force that works in the opposite direction

$F_{boy \rightarrow girl} = F_{girl \rightarrow boy}$

$M_{boy}\ a_{boy} = M_{girl}\ a_{girl}$

Girl pushes off from boy. Boy stands still, hands at his side



If: $M_{boy} = 20$ kg
$M_{girl} = 20$ kg
$F_{girl \rightarrow boy} = 80$ kg m/s^2

What is $F_{boy \rightarrow girl}$? 80 kg m/s^2
What is a_{girl} ?

A 2 m/s^2
B 4 m/s^2
C 8 m/s^2
D 16 m/s^2

Newton's Third Law

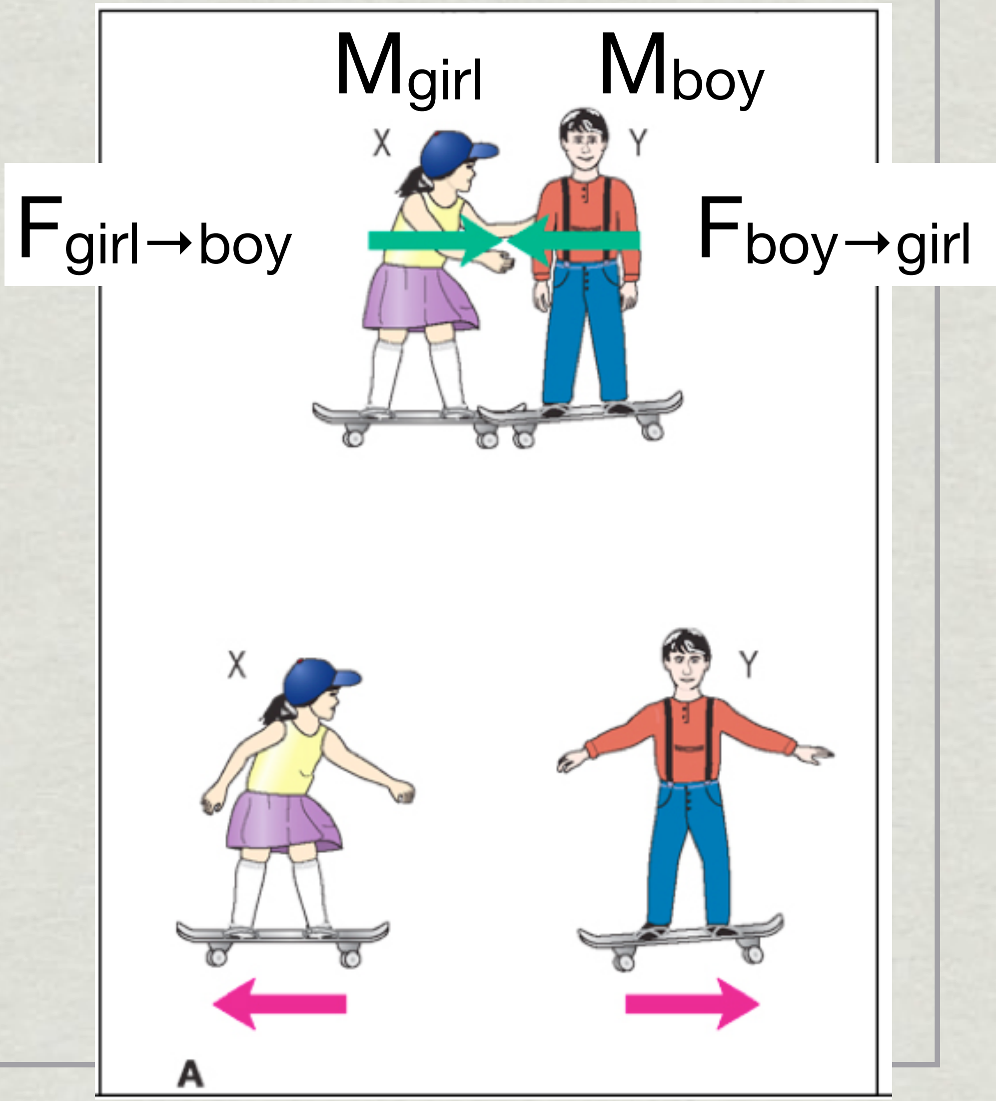
- For any force, there is an equal reactive force that works in the opposite direction

$F_{boy \rightarrow girl} = F_{girl \rightarrow boy}$

$M_{boy}\, a_{boy} = M_{girl}\, a_{girl}$

Girl pushes off from boy. Boy stands still, hands at his side

If: $M_{boy} = 20$ kg
$M_{girl} = 20$ kg
$F_{girl \rightarrow boy} = 80$ kg m/s^2



What is $F_{boy \rightarrow girl}$? 80 kg m/s^2
What is a_{girl} ? 4 m/s^2
What is a_{boy} ?

A 2 m/s^2
B 4 m/s^2
C 8 m/s^2
D 16 m/s^2

Newton's Third Law

- For any force, there is an equal reactive force that works in the opposite direction

$F_{boy \to girl} = F_{girl \to boy}$

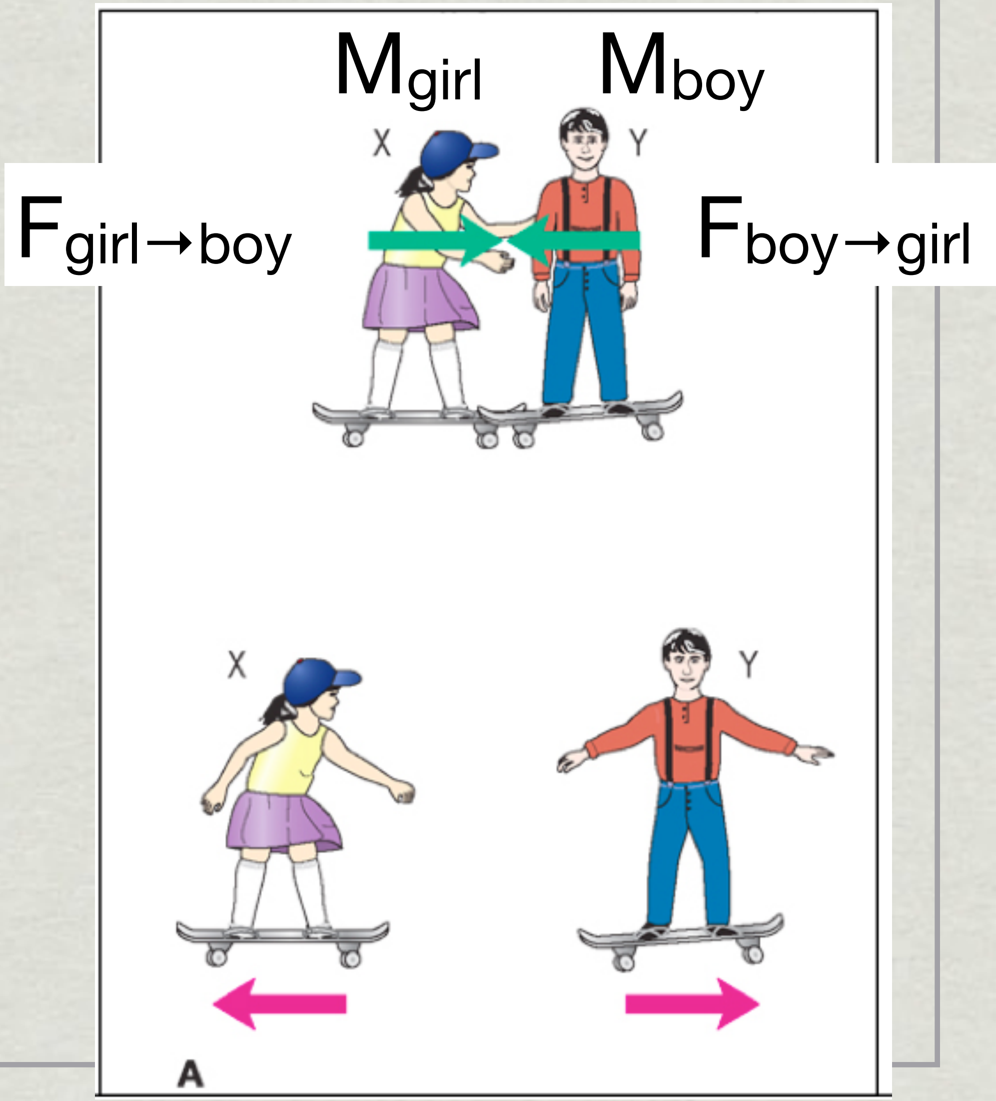
$M_{boy}\, a_{boy} = M_{girl}\, a_{girl}$

If: $M_{boy} = 20$ kg
$M_{girl} = 20$ kg
$F_{girl \to boy} = 80$ kg m/s^2

What is $F_{boy \to girl}$? 80 kg m/s^2
What is a_{girl} ? 4 m/s^2
What is a_{boy} ?

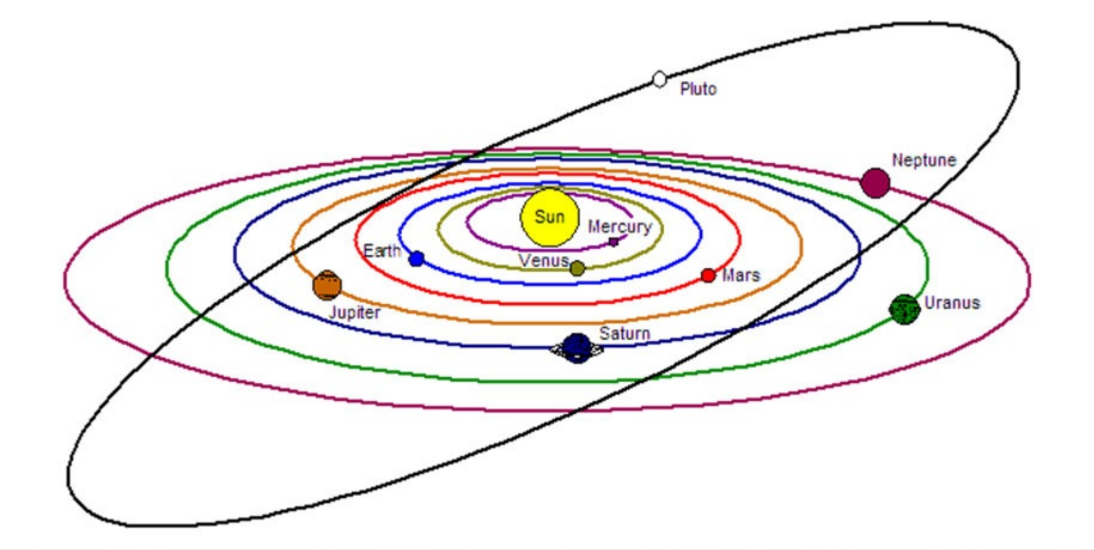
A 2 m/s^2
B 4 m/s^2
C 8 m/s^2
D 16 m/s^2

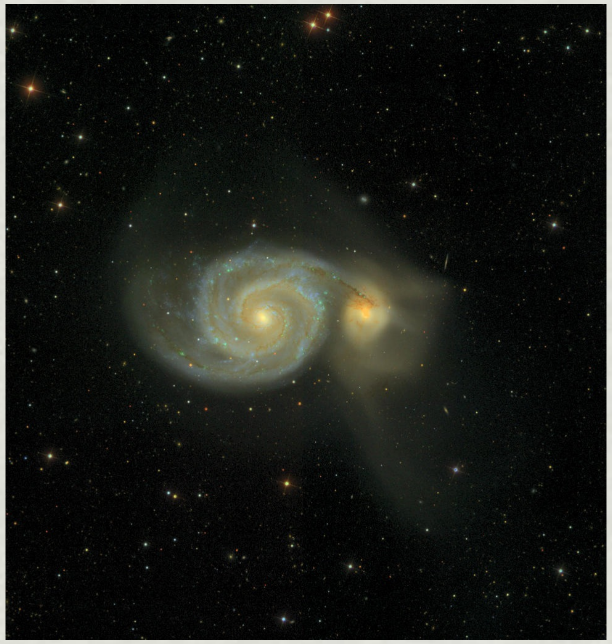
Girl pushes off from boy. Boy stands still, hands at his side

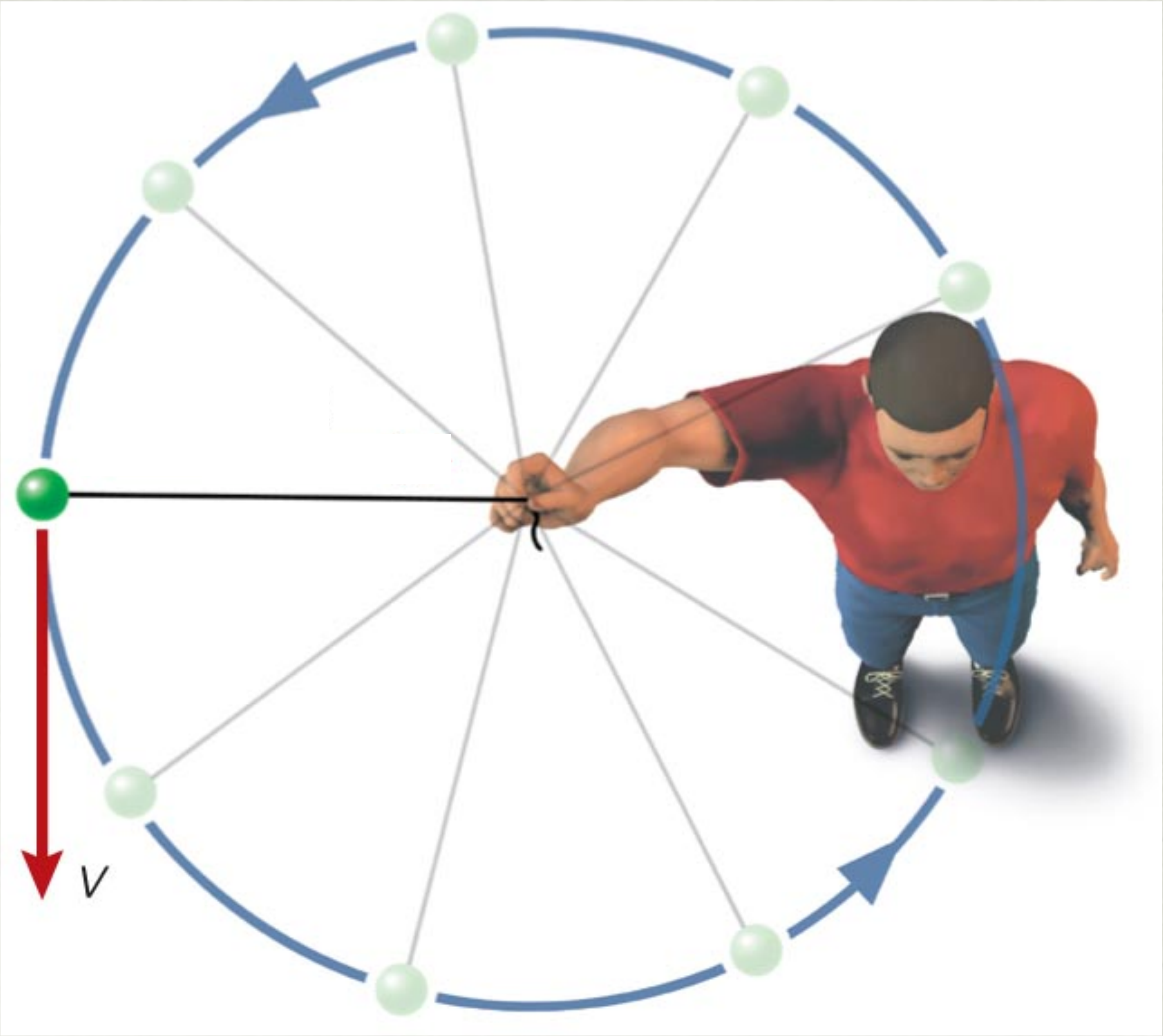


Circular Motion

- Is there a force acting on the ball?
  - is it accelerating?

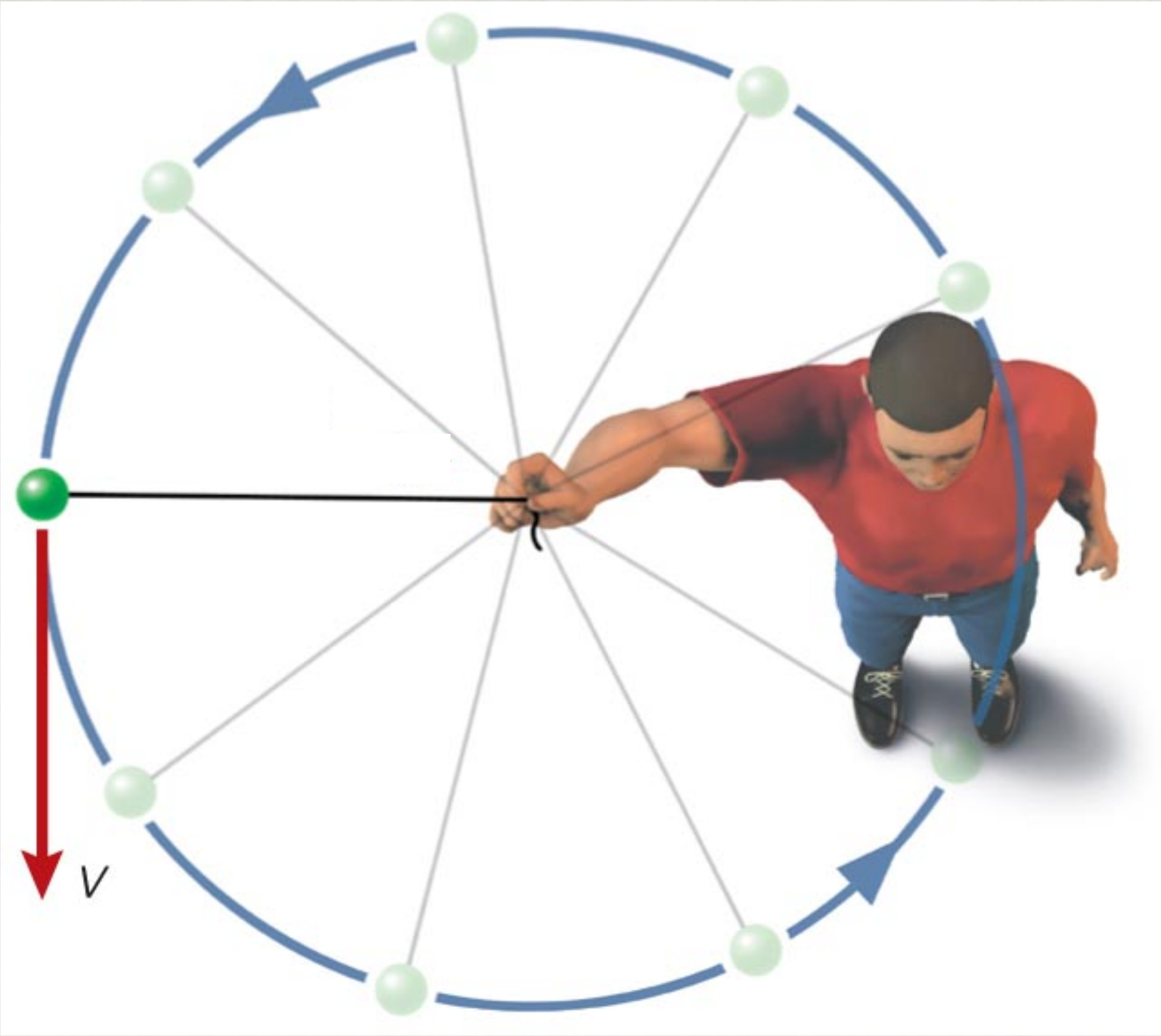






Circular Motion

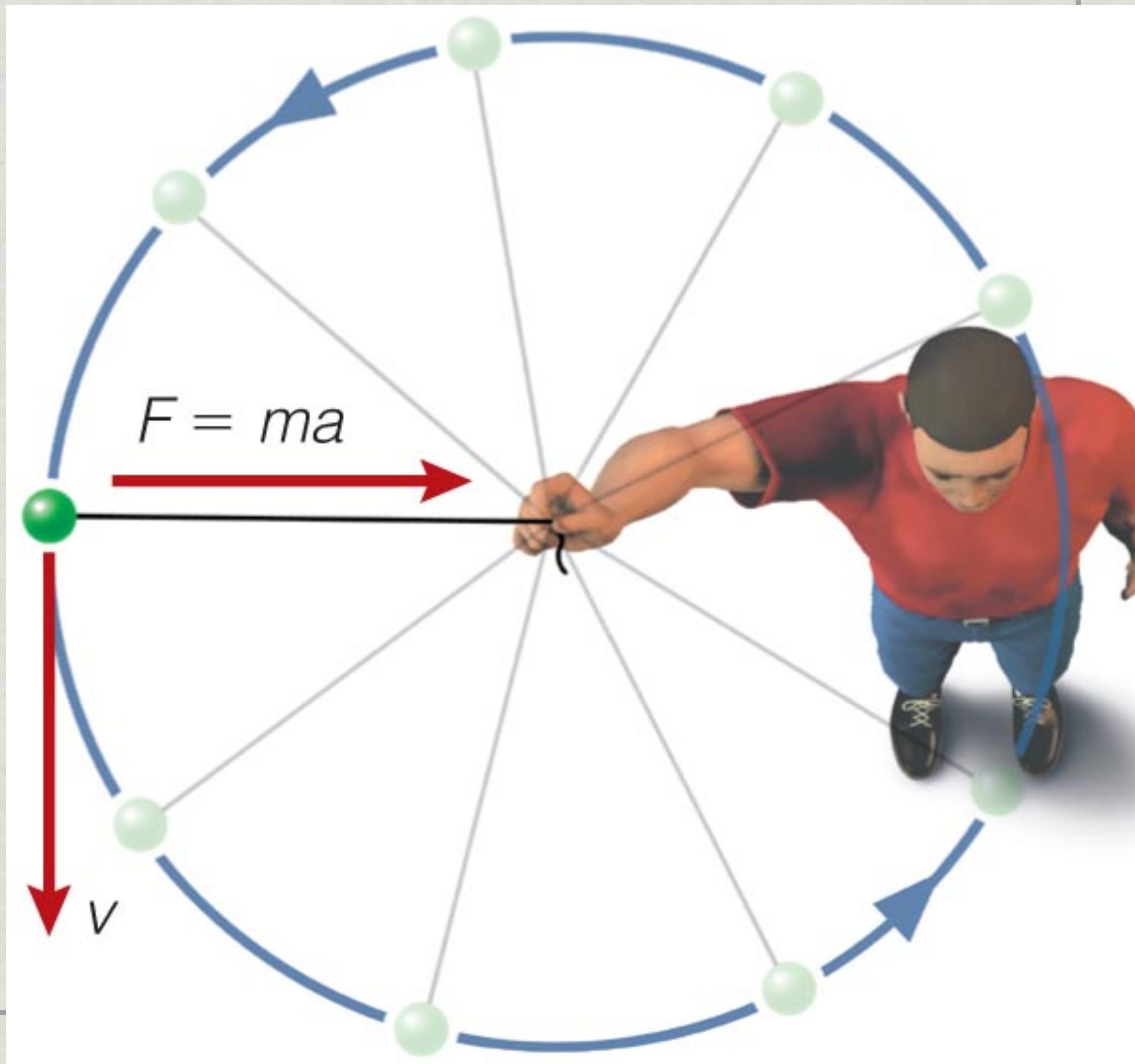
- Is there a force acting on the ball?
  - is it accelerating?
  - where (how) is the force applied to cause that change in velocity?
  - hint: what happens if the string breaks?



Circular Motion

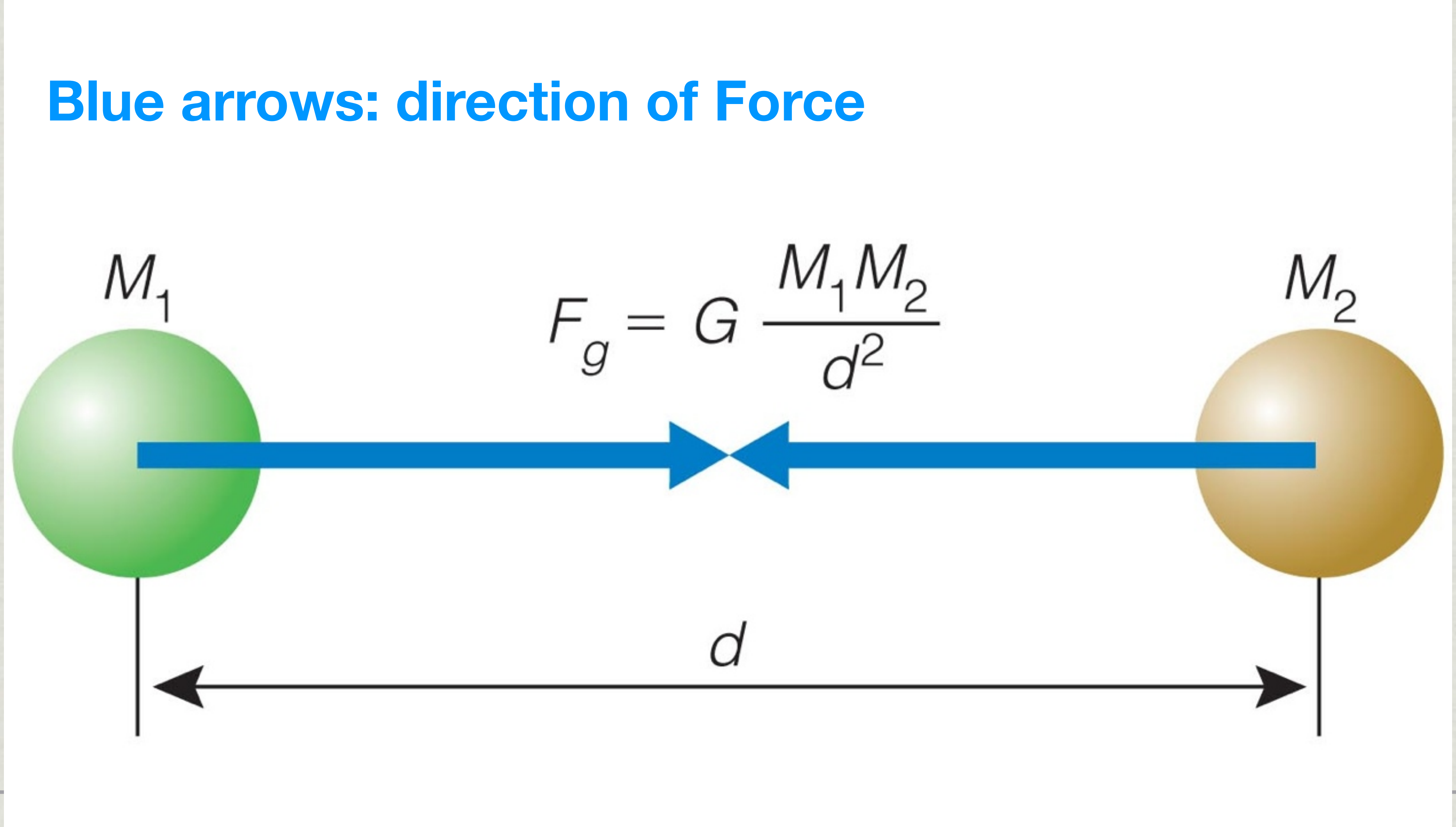
- Is there a force acting on the ball?
  - is it accelerating?
  - where (how) is the force applied to cause that change in velocity?
  - hint: what happens if the string breaks?

- Same principle, a force pulling the ball to the center of the circle, explains why planets orbit the sun, the moon orbits the Earth, ...



Gravity

- The force that holds you onto the Earth, the moon moving in orbit around the earth, the planets moving in their orbits around the sun, is Gravity

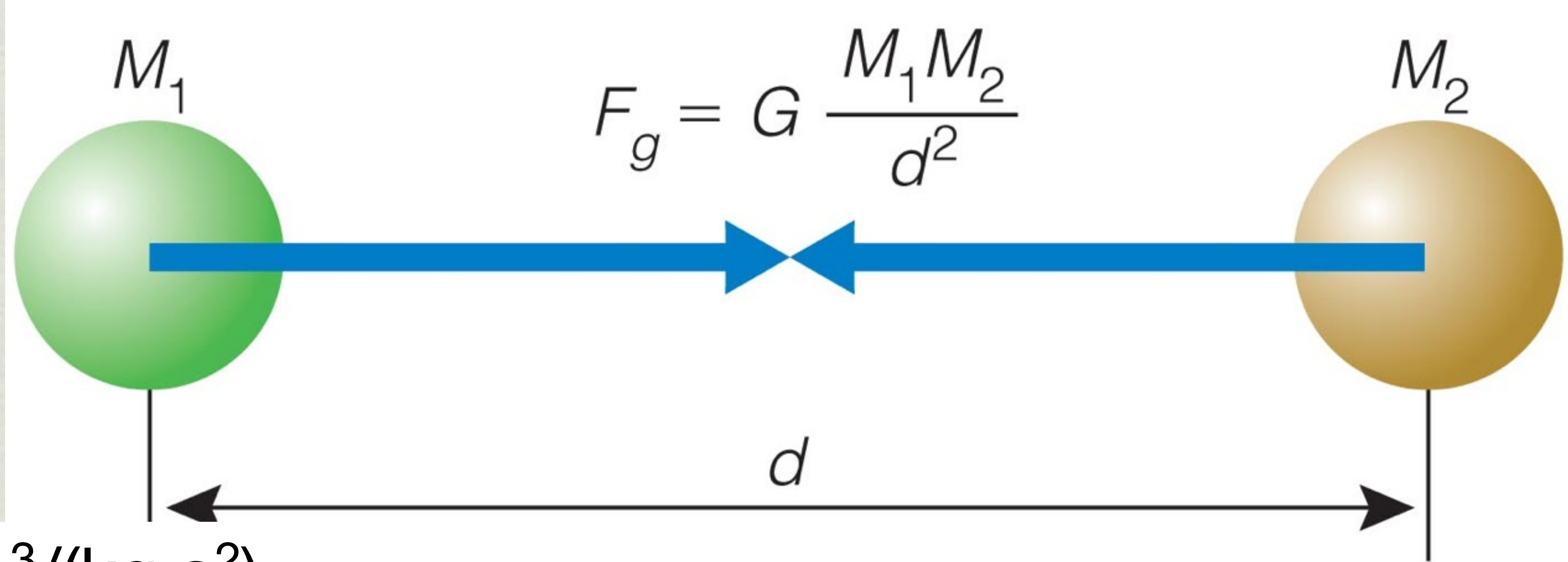


Gravity

- The force that holds you onto the Earth, the moon moving in orbit around the earth, the planets moving in their orbits around the sun, is Gravity

Force from gravity:

- Force of **M1** on **M2** = Force of **M2** on **M1** (Newton's 3rd law)
- Force weaker for larger d
- Force stronger for larger **M1** and/or **M2**



$G = 6.67 \times 10^{-11}\ m^3/(kg\ s^2)$

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

Gravity: $F = ma = \dfrac{G\,M\,m}{d^2}$

Force, Acceleration, Gravity

Newton's Law: $F = m\,a \;=\; \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

Gravity: $F = \mathbf{m}a = \dfrac{G\,M\,\mathbf{m}}{d^2}$

In this picture, what mass is m ?

What mass is M?

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which of the two rocks has greater acceleration?

A The big one
B The small one
C They have the same acceleration

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which of the two rocks has greater acceleration?

$$F_{big} = m_{big}\,a = \frac{G\,M\,m_{big}}{d^2}$$

$$F_{small} = m_{small}\,a = \frac{G\,M\,m_{small}}{d^2}$$

Force, Acceleration, Gravity

Newton's Law: $F = m\, a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\, M\, m}{d^2}$$

Which of the two rocks has greater acceleration?

$$F_{big} = \cancel{m}_{big}\, a = \frac{G\, M\, \cancel{m}_{big}}{d^2}$$

$$a = \frac{G\, M}{d^2}$$

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which of the two rocks has greater acceleration?

A The big one
B The small one
C They have the same acceleration

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which of the two rocks has greater acceleration?

A The big one
B The small one
C They have the same acceleration

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which rock feels the larger force when it hits the ground?

$$F_{big} = m_{big}\,a = \frac{G\,M\,m_{big}}{d^2}$$

$$F_{small} = m_{small}\,a = \frac{G\,M\,m_{small}}{d^2}$$

A The big one
B The small one
C They feel the same force

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which rock feels the larger force when it hits the ground?

Hint: we just said both rocks have the same acceleration as they fall. Do they have the same mass?

$$F_{big} = m_{big}\,a = \frac{G\,M\,m_{big}}{d^2}$$

$$F_{small} = m_{small}\,a = \frac{G\,M\,m_{small}}{d^2}$$

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which rock feels the larger force when it hits the ground?

$$F_{big} = m_{big}\,a = \frac{G\,M\,m_{big}}{d^2}$$

$$F_{small} = m_{small}\,a = \frac{G\,M\,m_{small}}{d^2}$$

A The big one
B The small one
C They feel the same force

Force, Acceleration, Gravity

Newton's Law: $F = m\,a = \dfrac{\text{Change in Momentum}}{\text{Change in Time}}$

$$F = ma = \frac{G\,M\,m}{d^2}$$

Which rock feels the larger force when it hits the ground?

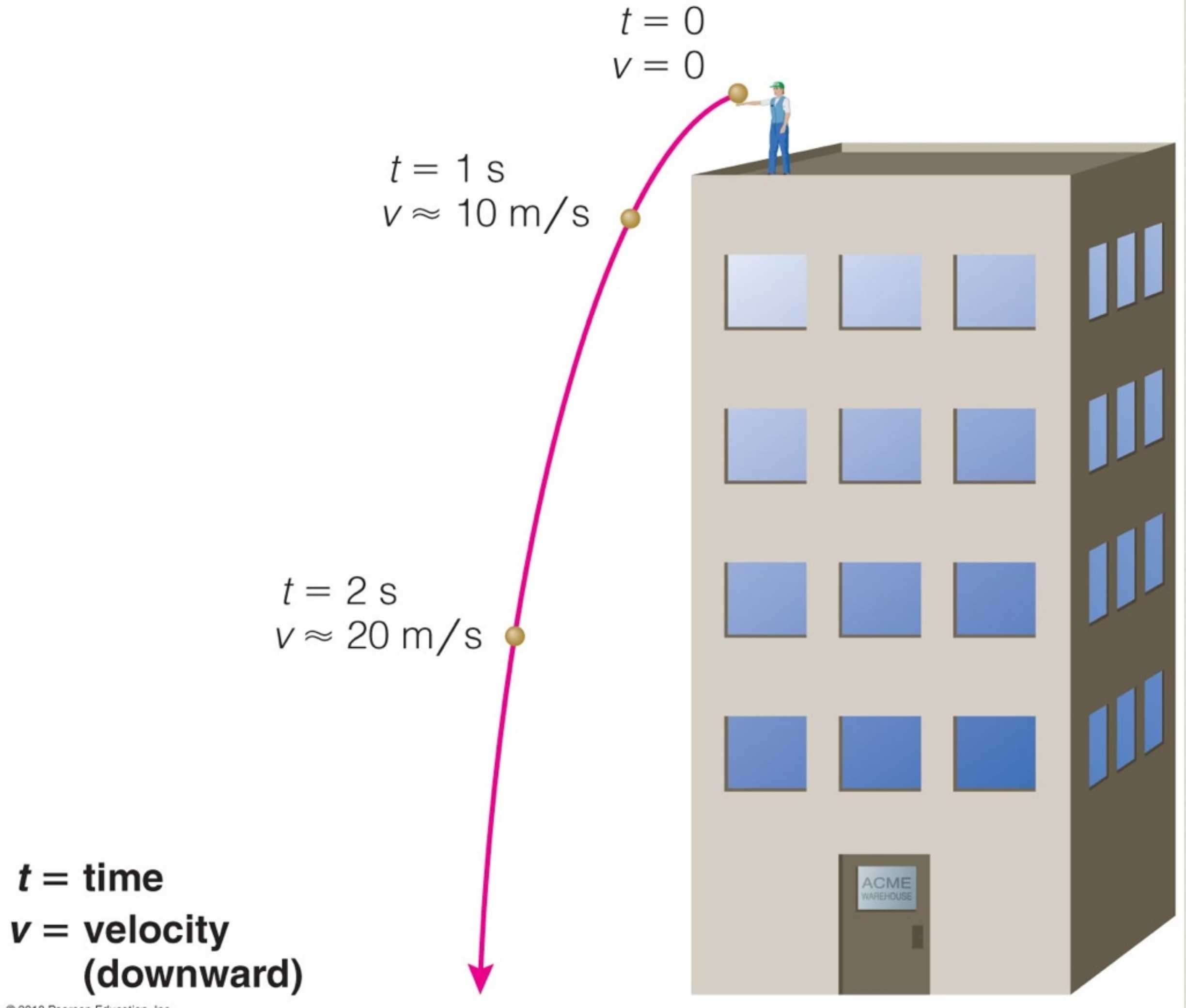
$$F_{big} = m_{big}\,a = \frac{G\,M\,m_{big}}{d^2}$$

$$F_{small} = m_{small}\,a = \frac{G\,M\,m_{small}}{d^2}$$

A The big one
B The small one
C They feel the same force

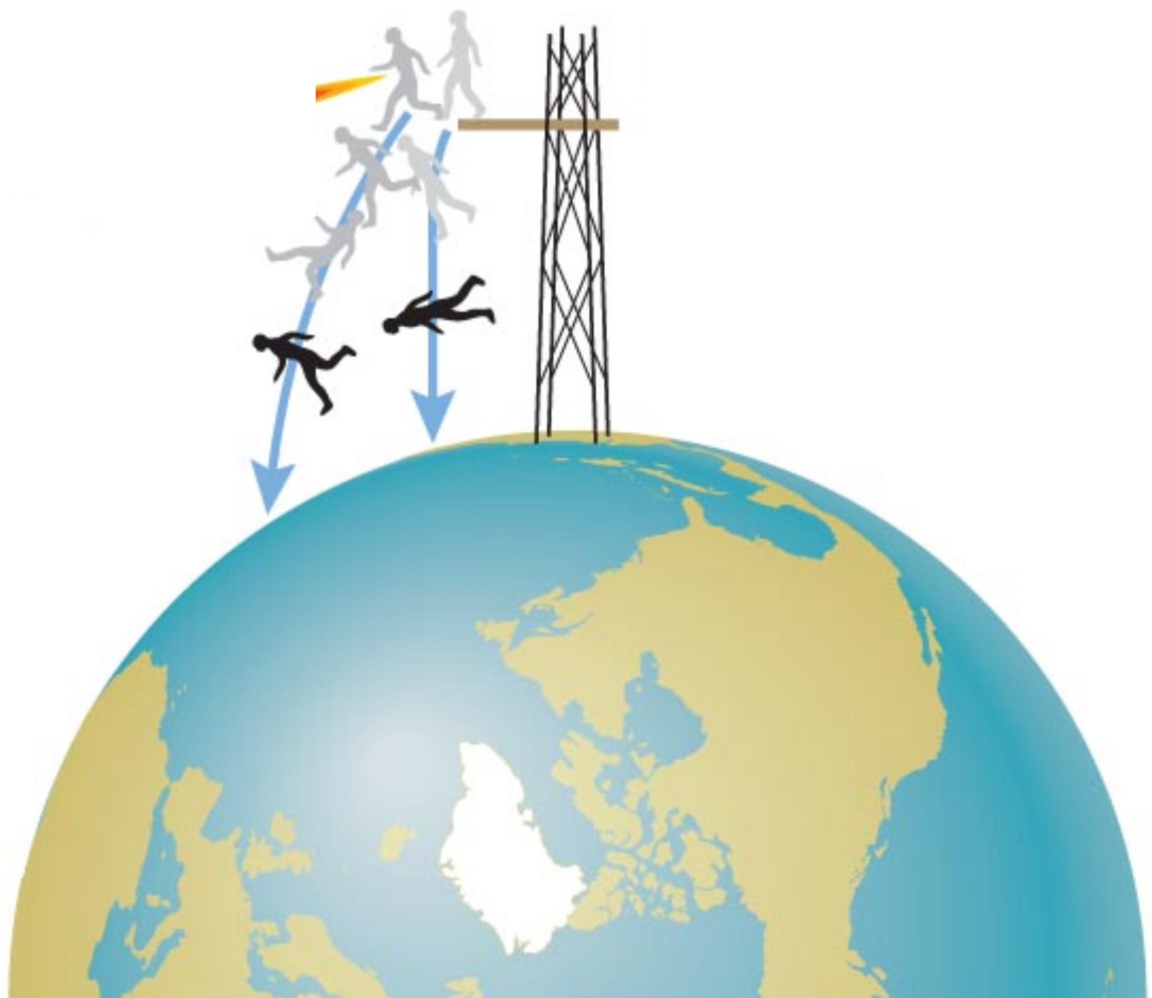
Force, Mass and Weight

- Weightlessness: no reaction force to push against the force of gravity.
  - you still have mass x acceleration = "weight"
  - Newton's 3rd law: you are still pulling on the earth with the same force it is pulling on you
- better description: free-fall
  - accelerating freely due to force of gravity



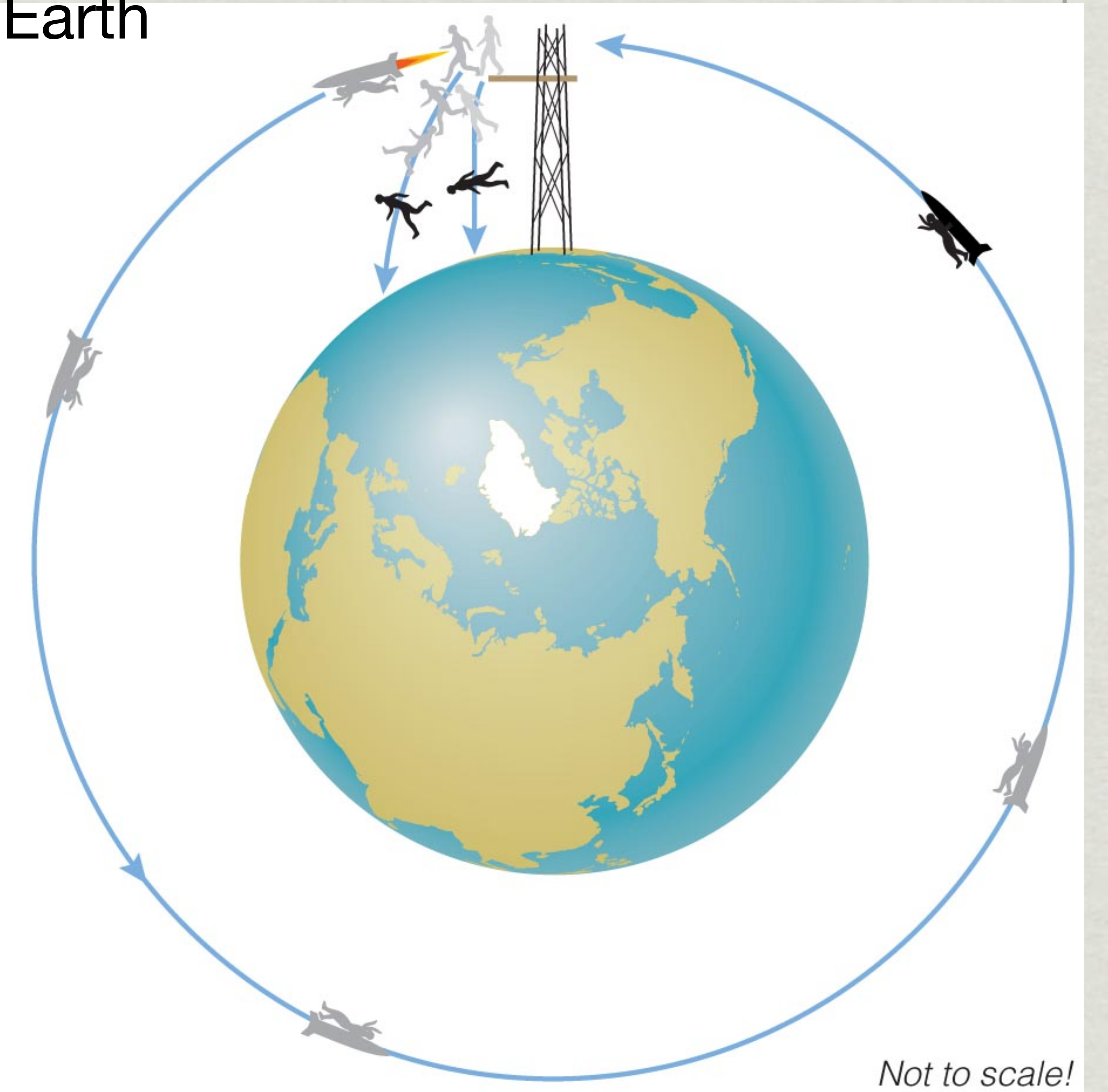
Gravity and Orbits

- Free-fall: accelerating freely due to the force of gravity
  - If you jump off the tower, you hit the ground
  - If you run up before you jump, you hit the ground a little bit away from the tower



Gravity and Orbits

- But if you get a really big running start (with the help of your super-hero rocket-pack) you never hit the ground: "free-fall"
- You are in orbit around the Earth
- Just like the astronauts in orbit, you are "weightless" when you are in free-fall
- Your velocity is so fast, you miss the ground when you fall, just keep going
- We'll come back to weight, mass, free-fall and orbits

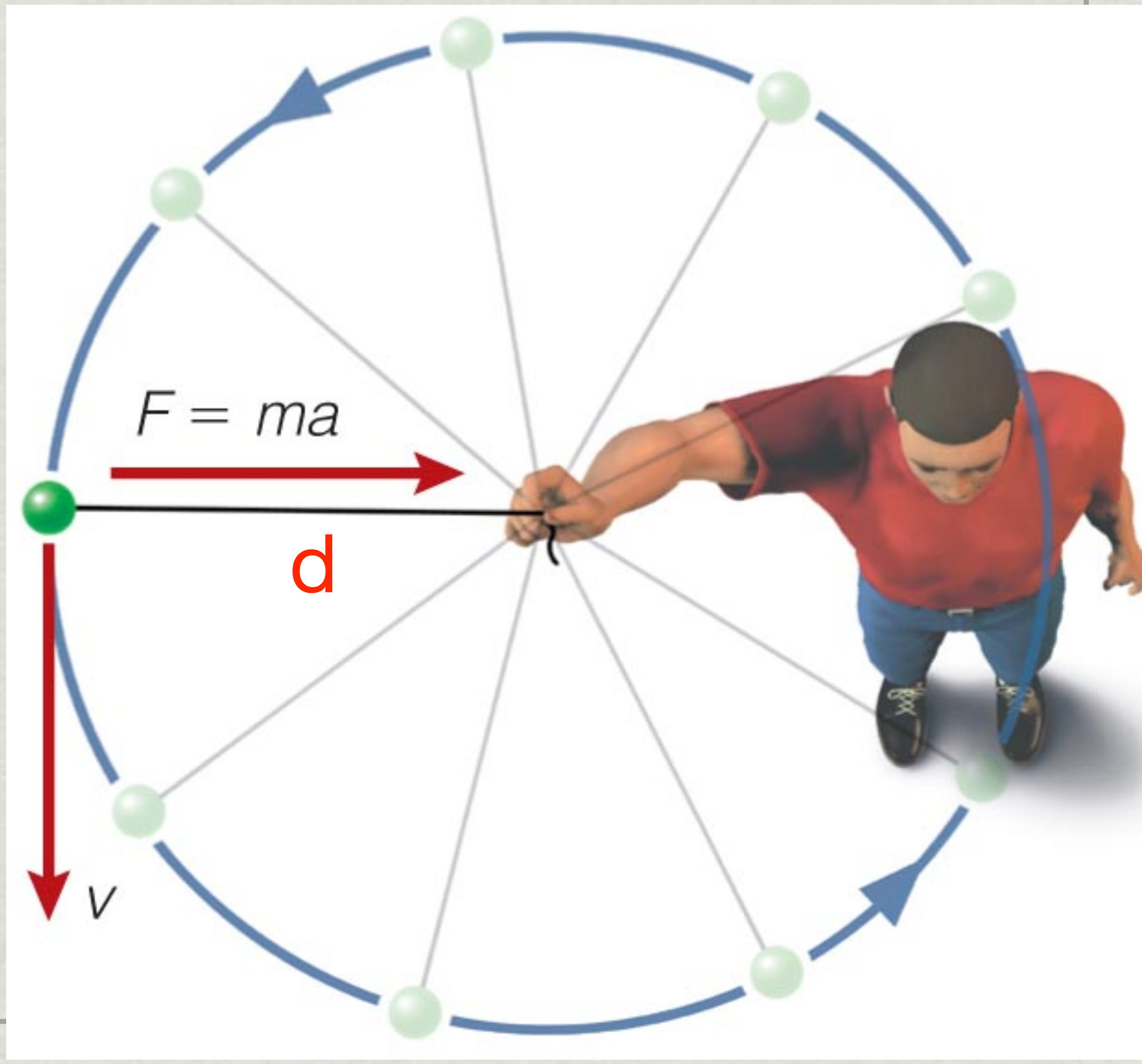


Orbits

- Fact: acceleration for circular motion is: $a = \frac{v^2}{d}$

v = velocity, d = distance to center of the circle

- acceleration larger for smaller d
  - change direction faster on a small circle
- acceleration larger for larger v
  - change direction faster if moving faster around the circle

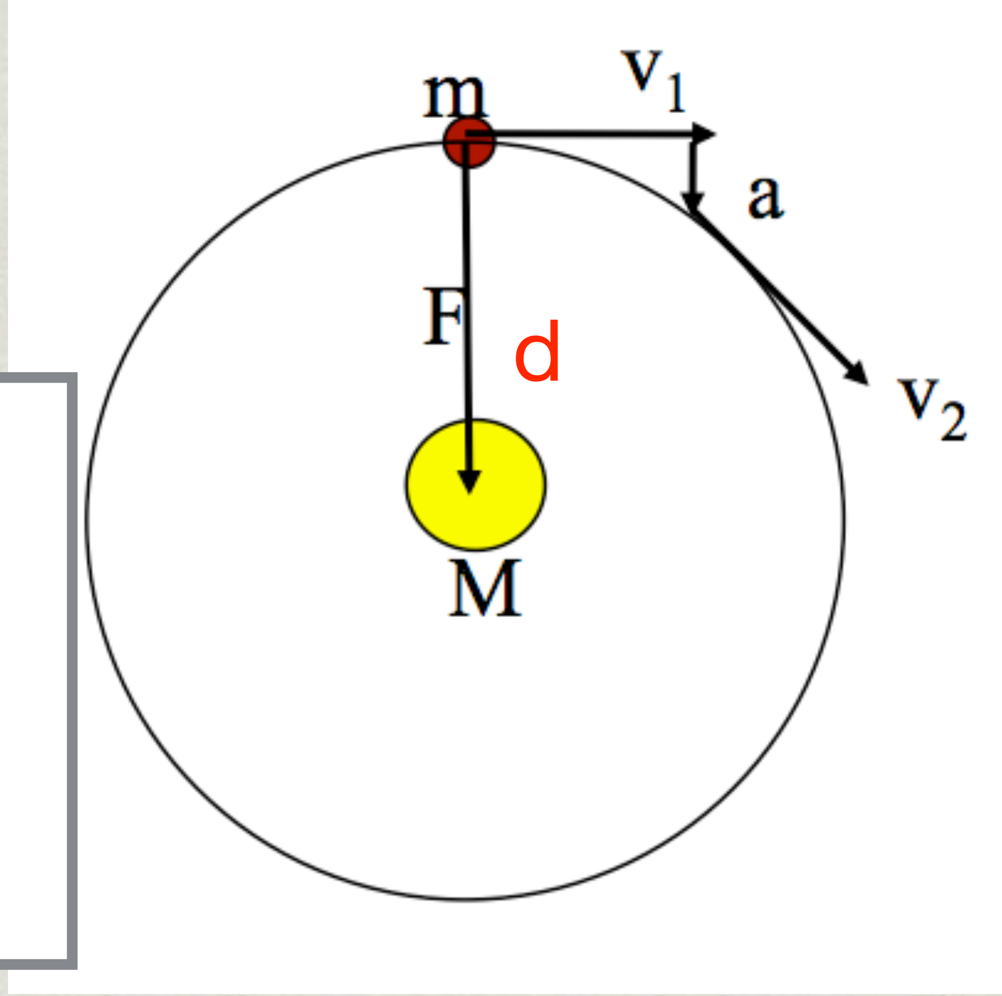


Orbits

- acceleration for circular motion: $a = \frac{v^2}{d}$

- Force from gravity: $F = ma = \frac{G\,M\,m}{d^2}$

This picture: a thing of mass m, like a planet (maybe Earth), in orbit around a thing of mass M, like the sun.



Planet orbits are elliptical (Kepler's 1st law!) but not very different from circles (not very eccentric).

We'll use the equation for motion in a circle because it is easier to work with

Orbits

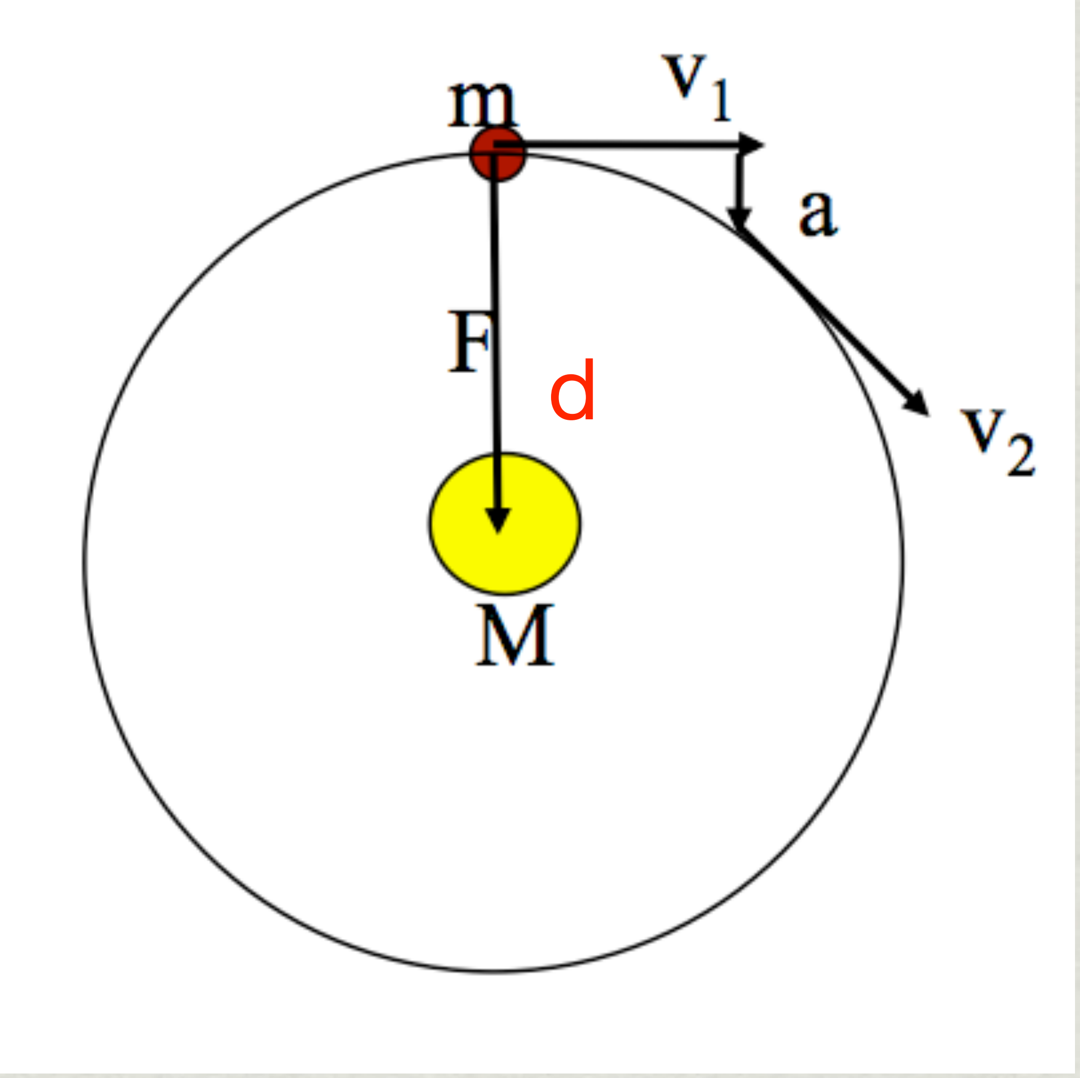
1. acceleration for circular motion: $a = \frac{v^2}{d}$

2. Force from gravity: $F = ma = \frac{G M m}{d^2}$

- acceleration from gravity:

$$F = \cancel{m}a = \frac{G M \cancel{m}}{d^2}$$

$$a = \frac{G M}{d^2}$$



Orbits

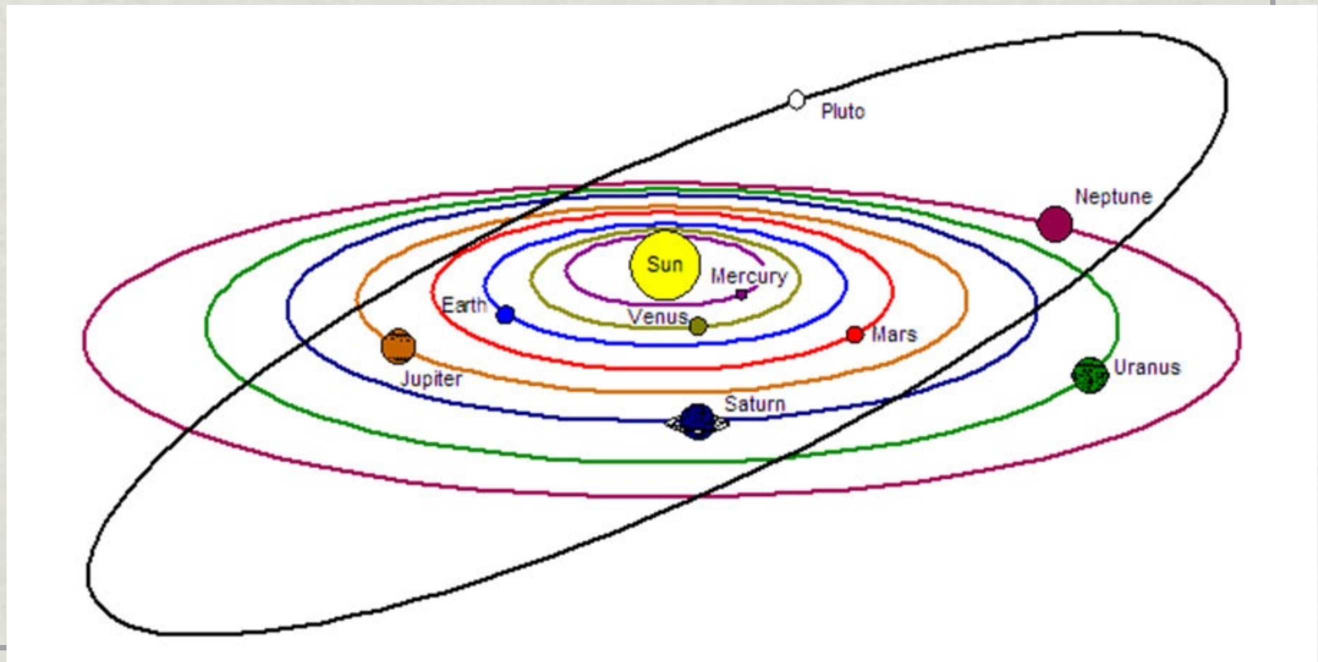
1. acceleration for circular motion: $a = \frac{v^2}{d}$

2. Force from gravity: $F = ma = \frac{G\,M\,m}{d^2}$

- acceleration from gravity:

$$F = \cancel{m}a = \frac{G\,M\,\cancel{m}}{d^2}$$

$$a = \frac{G\,M}{d^2}$$



Orbits

- For an object in a circular orbit:

  acceleration for circular motion = acceleration from Gravity

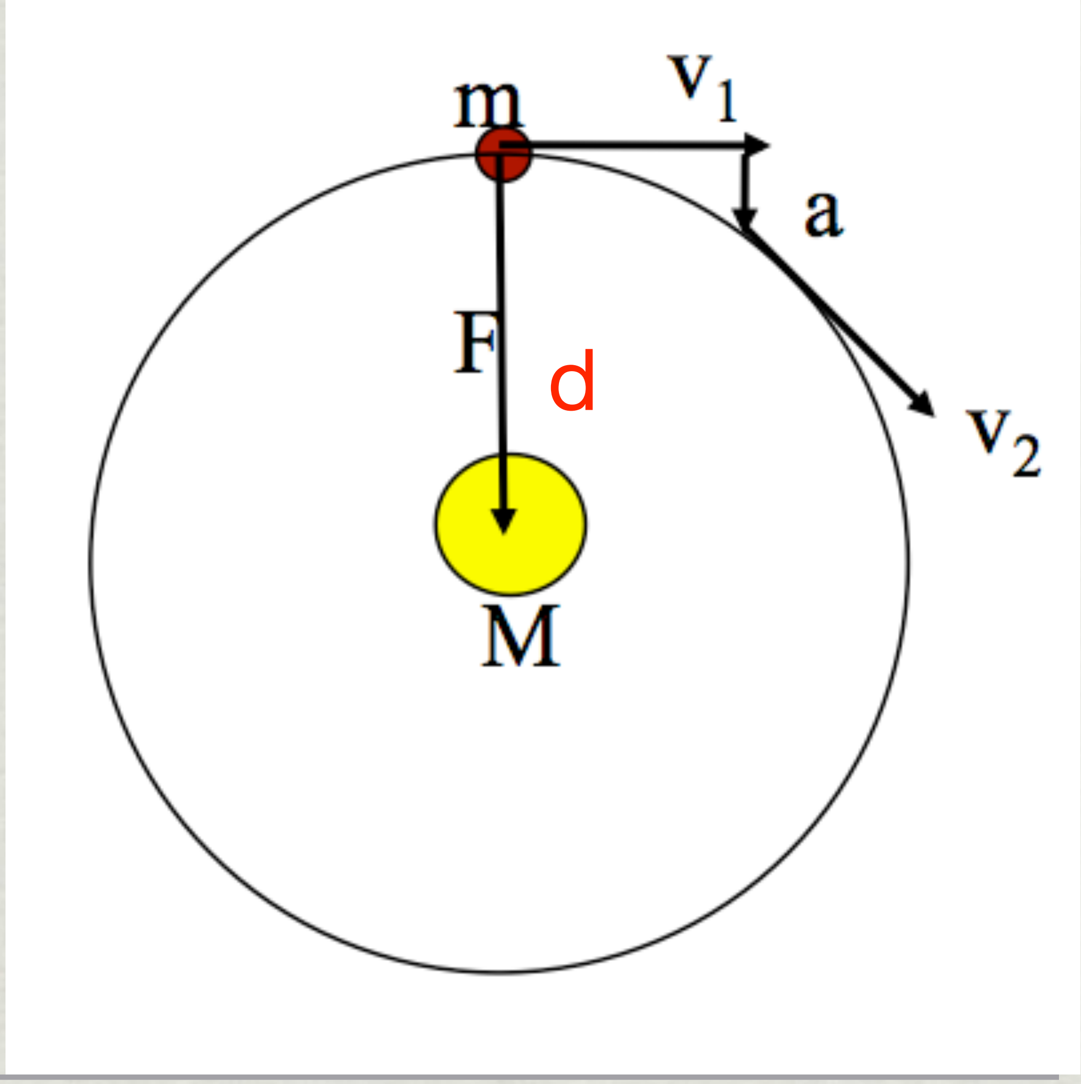
$$\frac{v^2}{d} = \frac{G\,M}{d^2}$$

$$\frac{v^2}{\cancel{d}} = \frac{G\,M}{d^{\cancel{2}}}$$

Now take square root:

$$v = \sqrt{\frac{G\,M}{d}}$$

- depends on M and d, not m



Orbits

- For an object in a circular orbit:

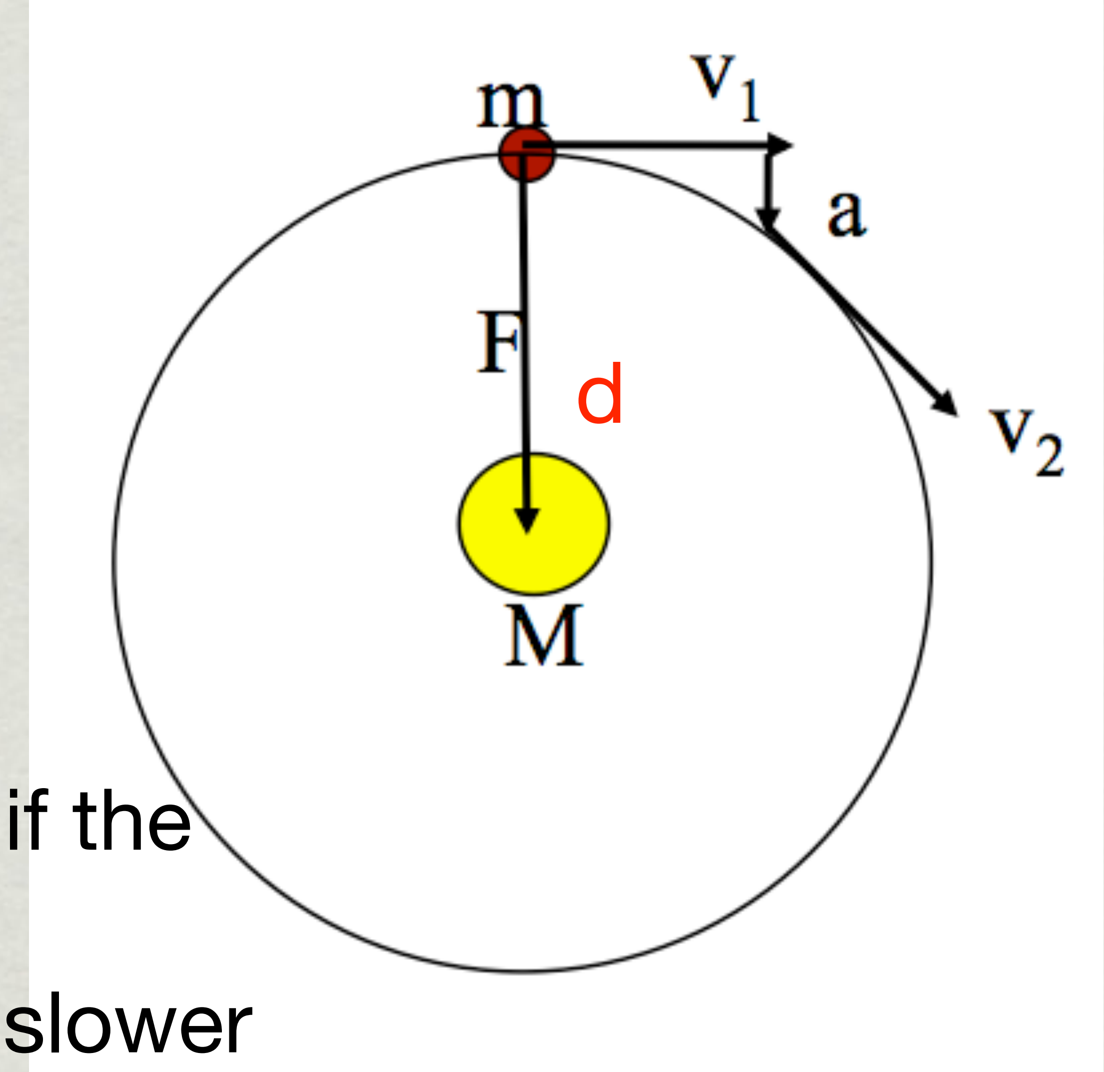
  acceleration for circular motion = acceleration from Gravity

$$\frac{v^2}{d} = \frac{G\,M}{d^2}$$

$$\frac{v^2}{\cancel{d}} = \frac{G\,M}{d^{\cancel{2}}}$$

Now take square root:

$$v = \sqrt{\frac{G\,M}{d}}$$



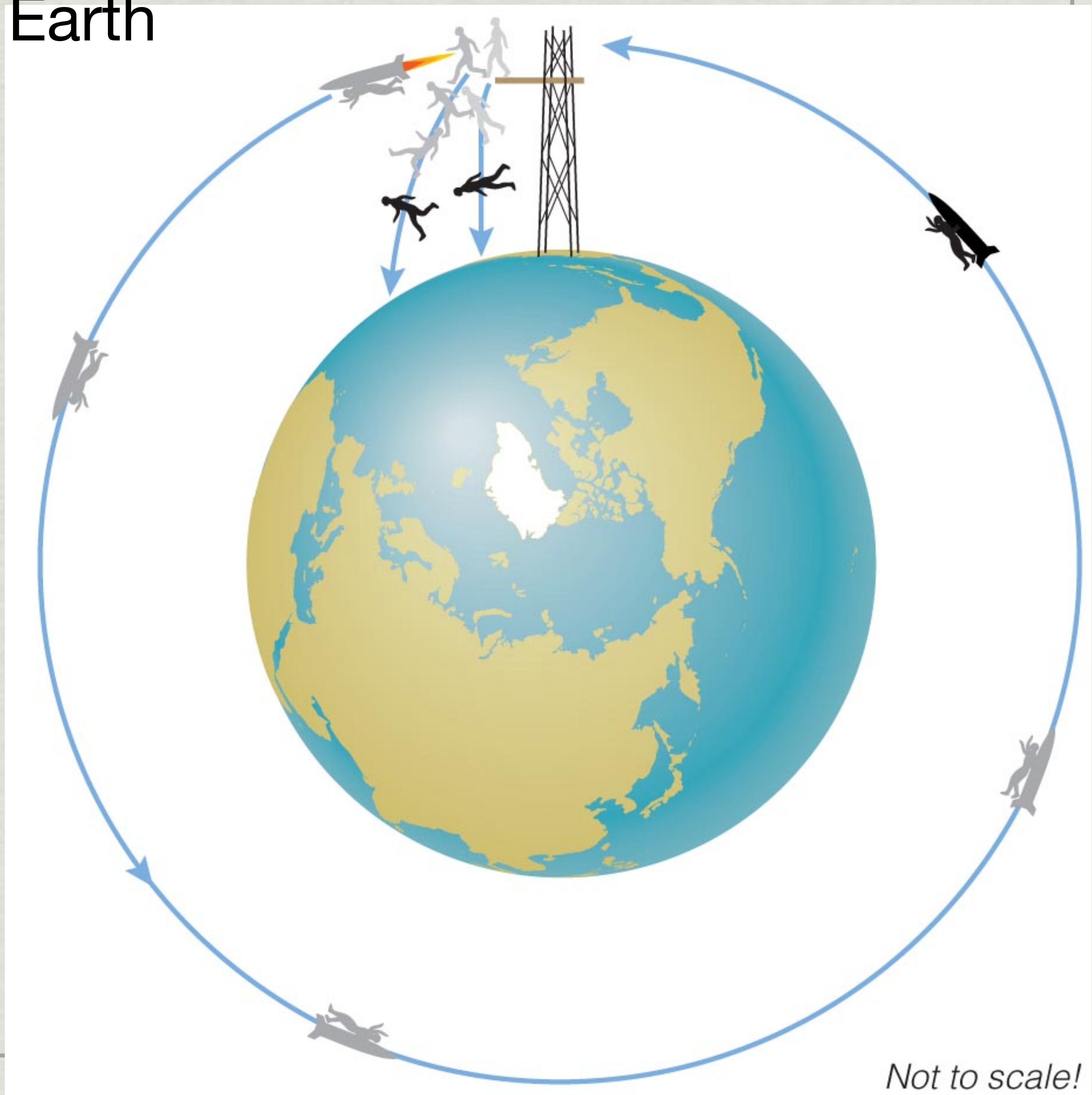
- depends on M and d, not m
- at distance d, an object moves faster if the central object M is more massive
- if m is farther away (d larger), it orbits slower

Gravity and Orbits

- But if you get a really big running start (with the help of your super-hero rocket-pack) you never hit the ground: “free-fall”
- You are in orbit around the Earth
- For an object in a circular orbit:

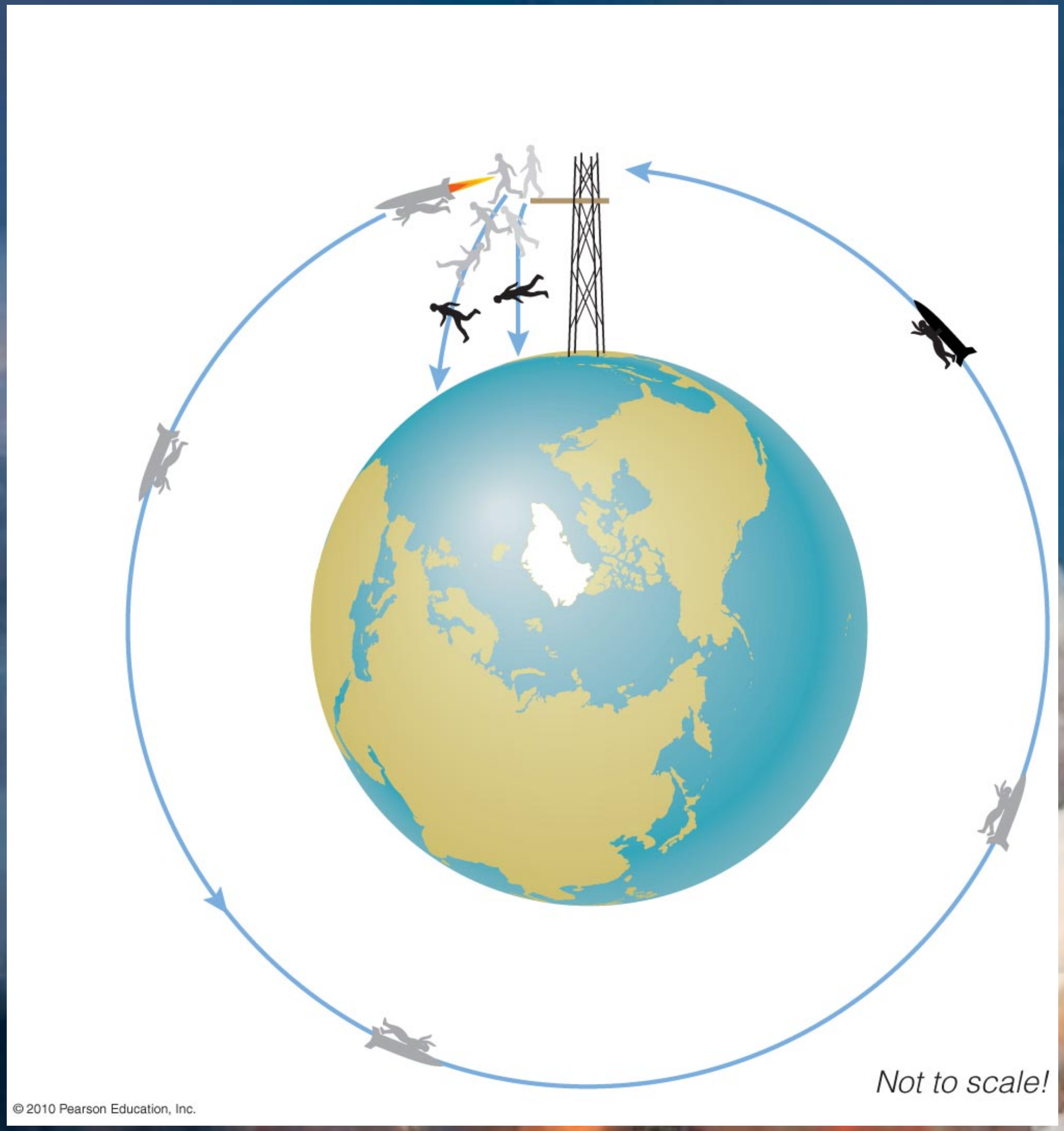
$$v = \sqrt{\frac{G\,M}{d}}$$

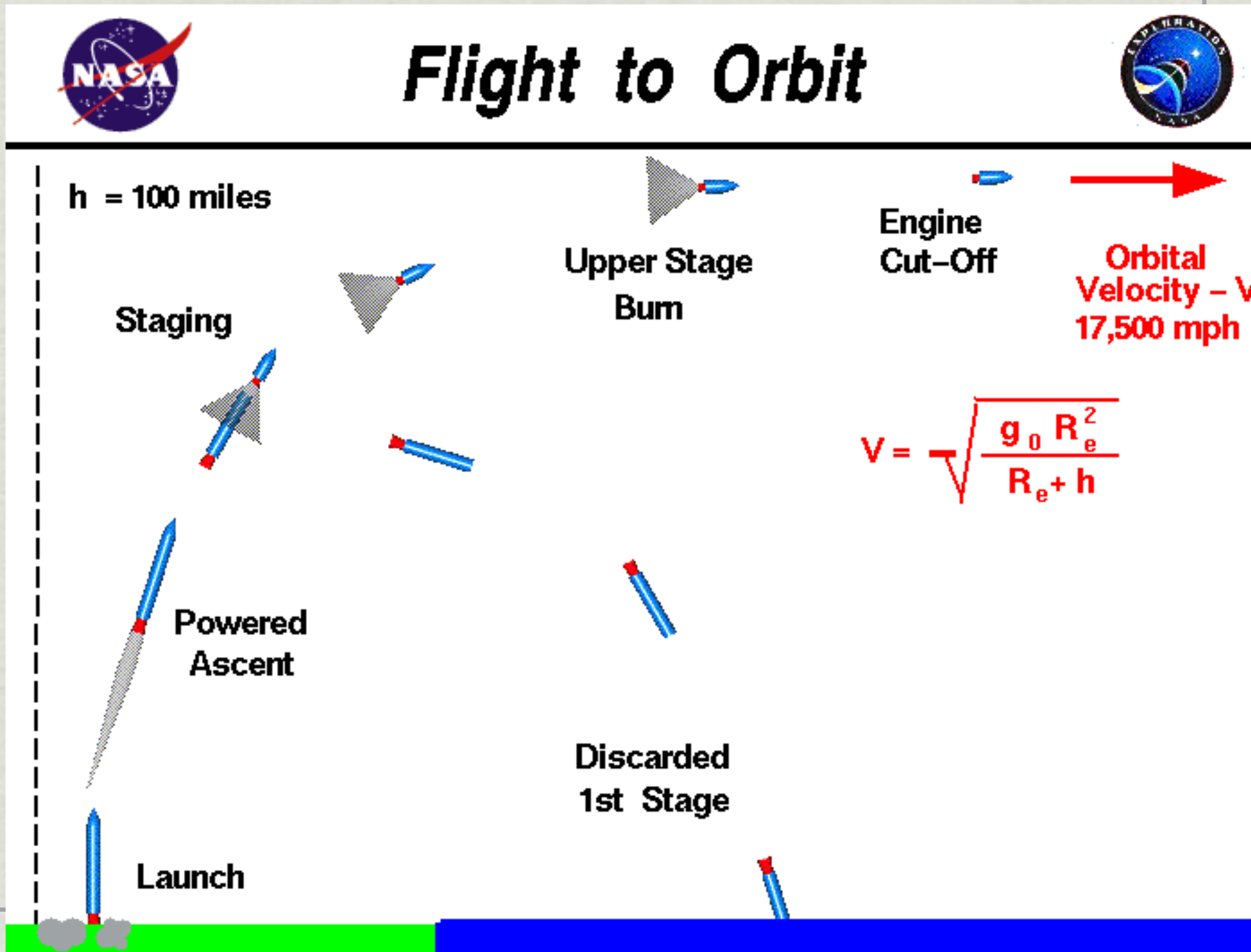
- if d is bigger, v can be smaller



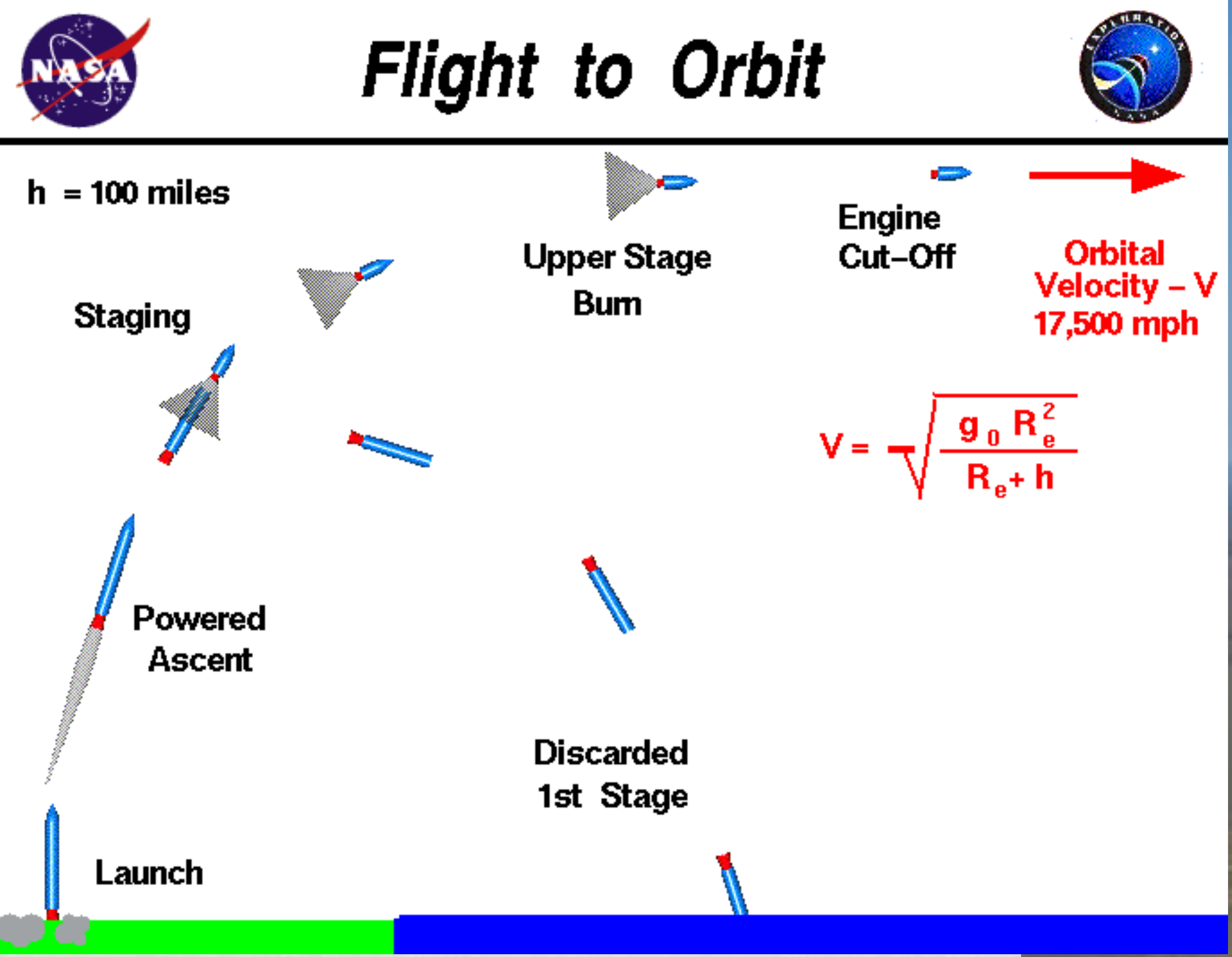
Balloon vs. Rocket

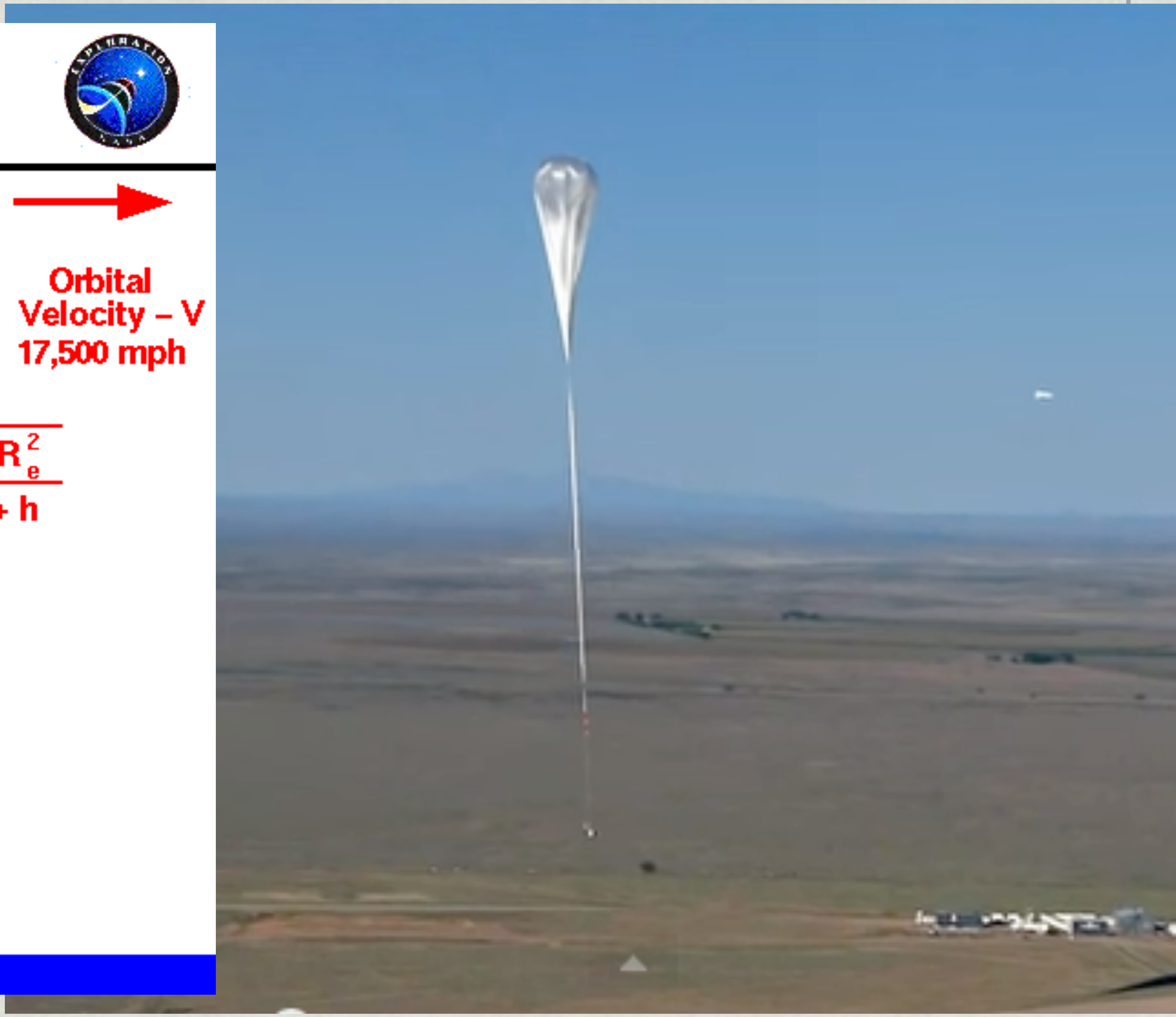
Rocket starts out vertical, moving away from earth due to force (thrust) generated by the engine.
Later the rocket turns so it gets horizontal acceleration to reach high enough velocity for circular orbit.





Balloon vs. Rocket





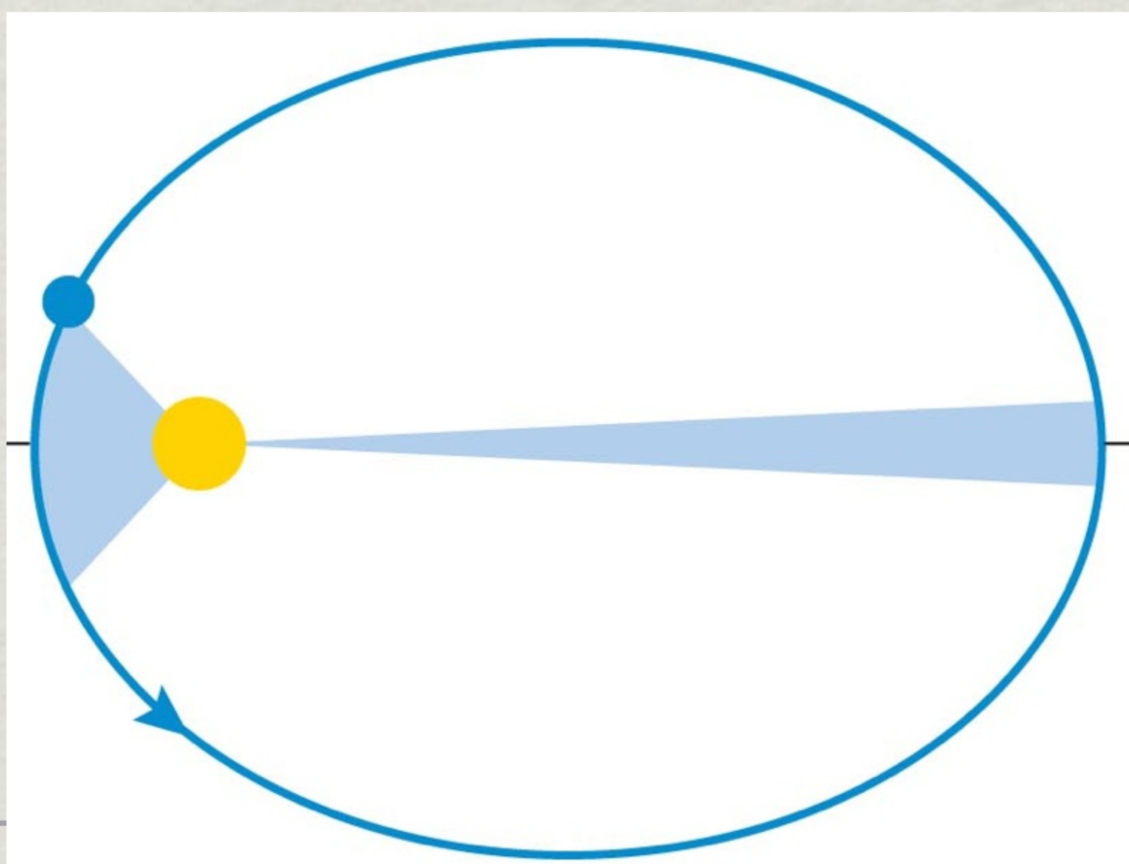
Orbits, Kepler's 3rd Law and Gravity

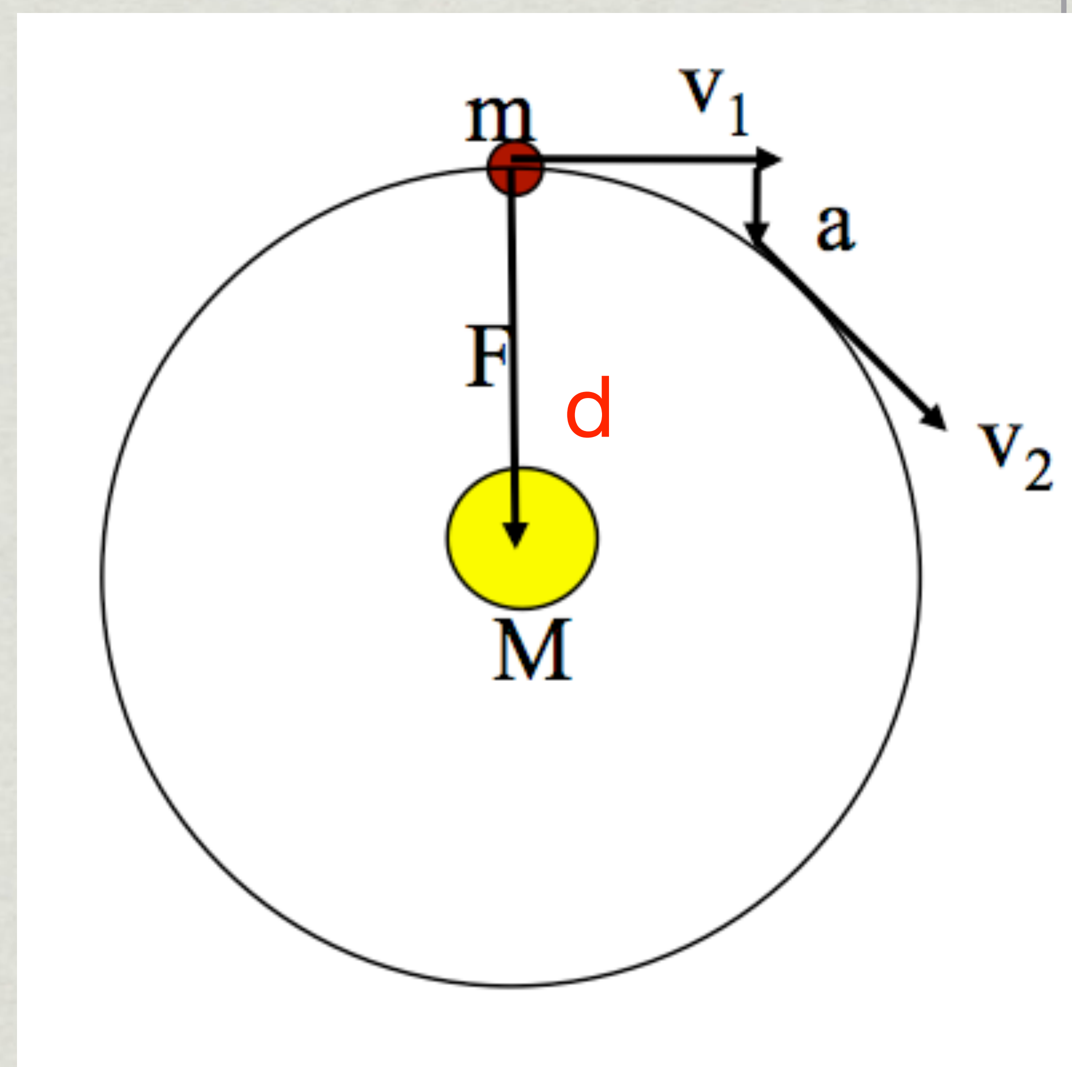
- #1 Kepler's third law: The ratio of:

(average distance from the sun)^3 to (orbital period)^2 $\frac{A^3}{P^2}$

is constant for all the planets.

Rewrite as $\frac{d^3}{P^2}$





Orbits, Kepler's 3rd Law and Gravity

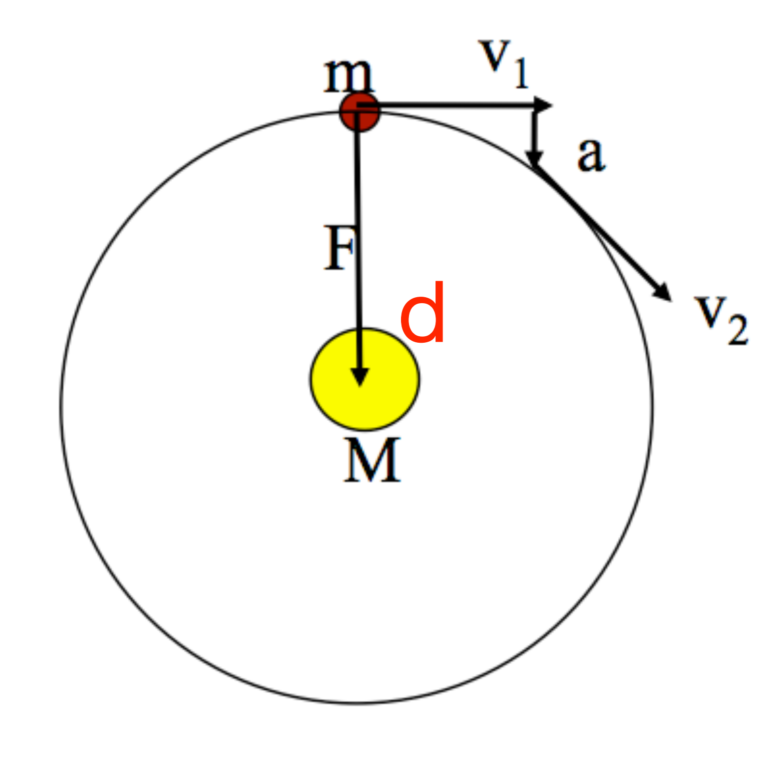
- #1 Kepler's third law: The ratio of (average distance from the sun)^3 to (orbital period)^2 is constant for all the planets. $\frac{d^3}{P^2}$

  #2 Orbital speed: $v = \frac{\text{change in distance}}{\text{change in time}}$

  $= \frac{\text{Circumference of orbit circle}}{\text{Time to travel once around the circle: orbit period P}} = \frac{2\pi d}{P}$

Circumference of Circle = $2\pi d$

Orbit period: P



Orbits, Kepler's 3rd Law and Gravity

- #1 Kepler's third law: The ratio of (average distance from the sun)^3 to (orbital period)^2 is constant for all the planets.

$$\frac{d^3}{P^2}$$

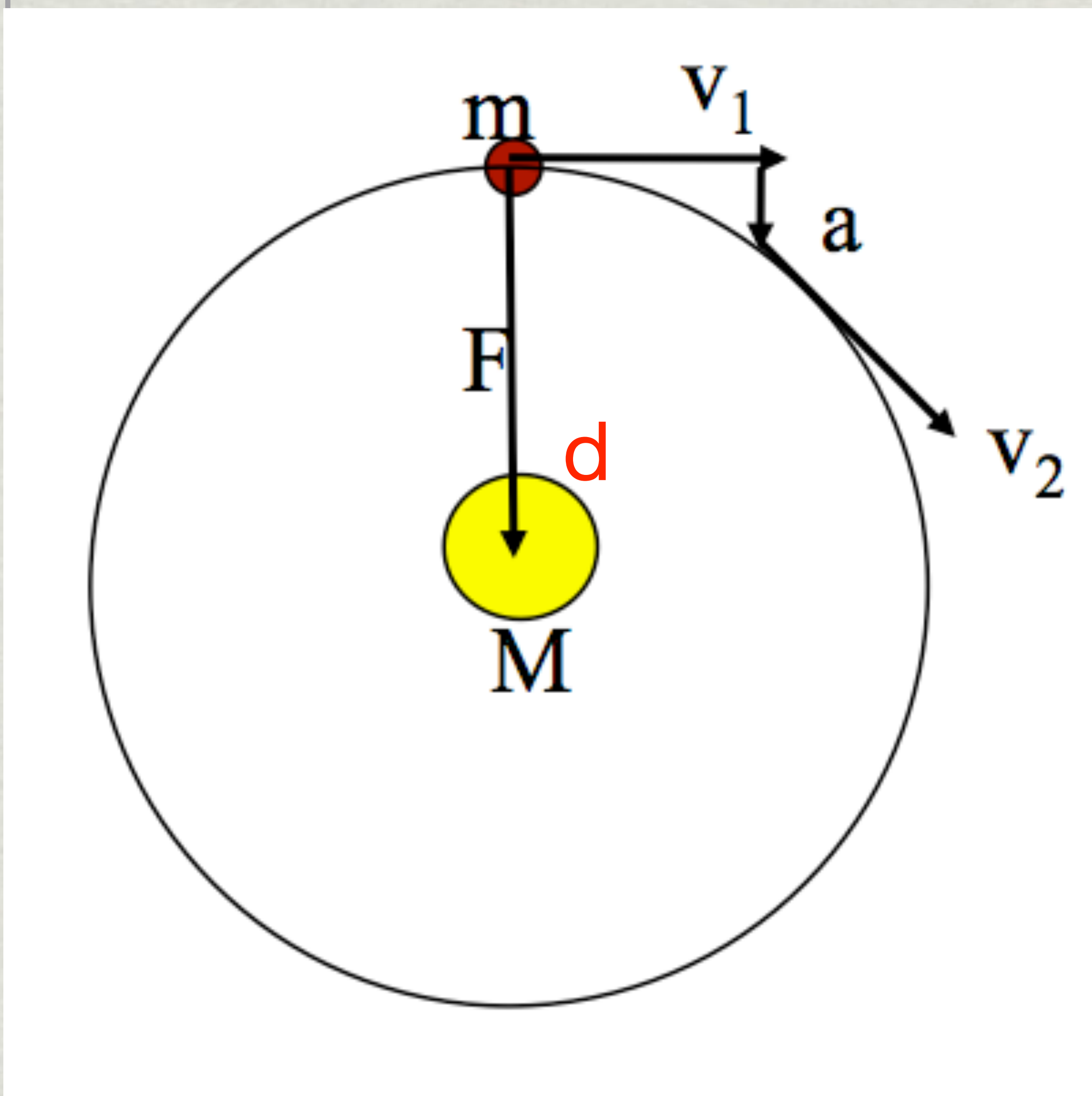
#2 Orbital speed: $v = \dfrac{\text{change in distance}}{\text{change in time}}$

$$= \frac{\text{Circumference of orbit circle}}{\text{Time to travel once around the circle: orbit period P}} = \frac{2\pi d}{P}$$

- #3 Orbital speed for circular motion: $v = \sqrt{\dfrac{G\,M}{d}}$

Square v: $v^2 = \dfrac{G\,M}{d}$

Orbits, Kepler's 3rd Law and Gravity



Remember:
M = mass of sun
d = distance to sun, radius of the circle of the orbit
v = speed of planet on its orbit

...nce of orbit circle / ...und the circle: orbit period P $= \frac{2\pi d}{P}$

V related to P, orbital period

Square v: $v^2 = \frac{G\,M}{d}$ ← This is a relation between v and d

Looking familiar: Kepler's 3rd law: $\frac{d^3}{P^2}$

Orbits, Kepler's 3rd Law and Gravity

- #1 Kepler's third law: The ratio of (average distance from the sun)^3 to (orbital period)^2 is constant for all the planets. $\frac{d^3}{P^2}$

  #2 Orbital speed: $v = \frac{\text{change in distance}}{\text{change in time}}$

  $$= \frac{\text{Circumference of orbit circle}}{\text{Time to travel once around the circle: orbit period P}} = \frac{2\pi d}{P}$$

- #3 Orbital speed for circular motion: $v = \sqrt{\frac{G\,M}{d}}$

Square v: $v^2 = \frac{G\,M}{d}$

Now rearrange: → $M = \frac{v^2\,d}{G}$

Orbits, Kepler's 3rd Law and Gravity

#1 Kepler's third law: The ratio of

(average distance from the sun)^3 to (orbital period)^2 $\frac{d^3}{P^2}$

is constant for all the planets.

#2 Orbital speed: $v = \frac{\text{change in distance}}{\text{change in time}}$

$= \frac{\text{Circumference of orbit circle}}{\text{Time to travel once around the circle: orbit period P}}$ $v = \frac{2\pi d}{P}$

#3 Orbital speed for circular motion: → $M = \frac{v^2 d}{G}$

#4 Substitute $\frac{2\pi d}{P}$ for v^2 : $M = \frac{d\, v^2}{G} = \frac{d \left(\frac{2\pi d}{P}\right)^2}{G}$

Then rearrange → $\frac{d^3}{P^2} = \frac{G\,M}{(2\pi)^2}$

Orbits, Kepler's 3rd Law and Gravity

#1 Kepler's third law: The ratio of

(average distance from the sun)^3 to (orbital period)^2 $\frac{d^3}{P^2}$

is constant for all the planets.

#2 Orbital speed: $v = \frac{\text{change in distance}}{\text{change in time}}$

$= \frac{\text{Circumference of orbit circle}}{\text{Time to travel once around the circle: orbit period P}}$ $v = \frac{2\pi d}{P}$

#3 Orbital speed for circular motion: → $M = \frac{v^2 d}{G}$

#4 Substitute $\frac{2\pi d}{P}$ for v^2 : $M = \frac{d\, v^2}{G} = \frac{d \left(\frac{2\pi d}{P}\right)^2}{G}$

Then rearrange → $\frac{d^3}{P^2} = \boxed{\frac{G\, M}{(2\pi)^2}}$

Kepler's 3rd law for any planet orbiting any star with mass M

Orbits, Kepler's 3rd Law and Gravity

- Kepler's Laws: describe motion, make predictions

- But Kepler's laws don't tell us why planets move in ellipses, why they change speed along their orbits, or why $A^3 = P^2$

- Gravity tells why the orbits of the planets follow Kepler's Laws:

$$\frac{d^3}{P^2} = \boxed{\frac{G\,M}{(2\pi)^2}}$$

Relation between d and P depends on the mass M of the star the planets are orbiting.

If our sun were a different mass, our year would be a different length

$$\frac{d^3}{P^2} = \boxed{\frac{G\,M}{(2\pi)^2}}$$

If our sun were much heavier, but the earth was still at a distance of 1AU from the sun, would the orbital period (our year) be longer or shorter?

A Longer
B Shorter

$$\frac{d^3}{P^2} = \boxed{\frac{G\,M}{(2\pi)^2}}$$

If our sun were much heavier, but the earth was still at a distance of 1AU from the sun, would the orbital period (our year) be longer or shorter?

A Longer
B Shorter

If M is larger, $\frac{d^3}{P^2}$ must also be larger.

If d in the numerator stays the same, then P in the denominator must get smaller. P, our year, must get shorter.

Fun with Gravity

- Gravity can be used to weigh stuff we can't put on a scale, like stars and planets

$$\frac{d^3}{P^2} = \boxed{\frac{G\,M}{(2\pi)^2}}$$

- We can even use it to weigh stuff we can't see, and prove that it is there
- A neat example: finding planets around other stars

Finding Planets Around Other Stars

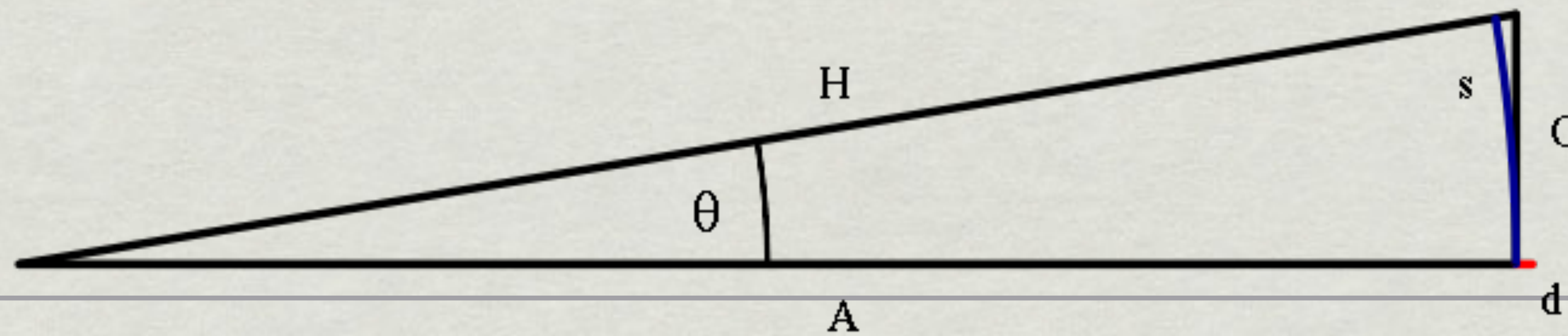
Why do we need gravity to do this?

- A sun-like star is about 1 billion times brighter than the light reflected from its planets.
  - the difference between staring directly at the sun and ambient moonlight is about a factor of 10 billion in brightness

Finding Planets Around Other Stars

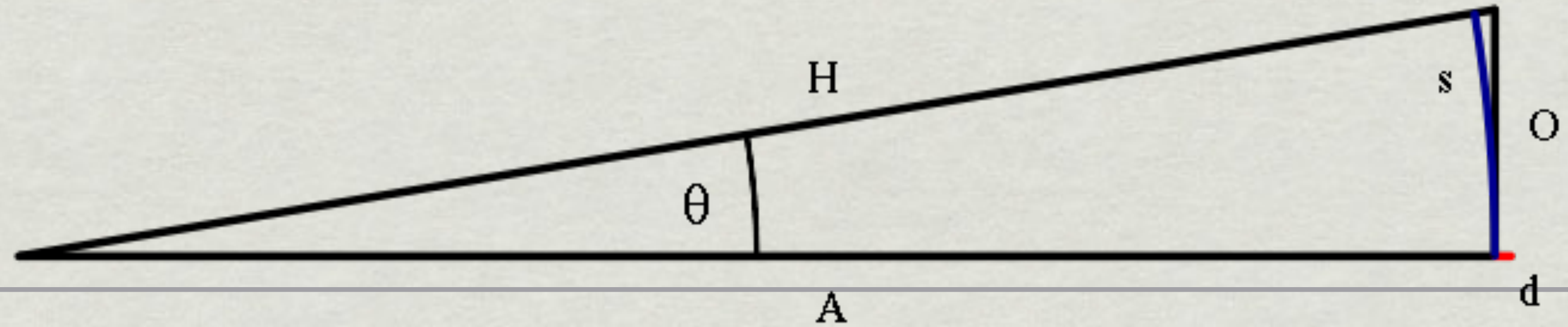
Why do we need gravity to do this?

- Even the most nearby stars are very far away
- Planets are ***much*** closer to their stars than the nearest stars are to us
- Angular-size distance relation: the angular separations we measure between the stars and their planets are tiny. Even something as big as a solar system is only a tiny angular size.
- Like being in San Francisco and trying to see the head of a pin 15 meters away from a grapefruit in Washington, DC



Finding Planets Around Other Stars

- So this is not easy:
  - planets around other stars are much fainter than their stars
  - they are also very, very close to their stars, so very difficult to identify the faint planet light



Nearest Star

- The nearest sun-like star, Tau Ceti, is 12 light-years away. That's 1.1×10^{17} m
- Jupiter is 7.8×10^{11} m away from our sun
- For an alien on Tau Ceti, what is the angular separation between Jupiter and the Sun?

$$\frac{\alpha}{360°} = \frac{s}{2\pi d}$$

$s = 7.8 \times 10^{11}$m

$d = 1.1 \times 10^{17}$ m

- Solve for α :0.0004 degrees = 1.4 arcseconds (1/3600 of circle)

Common size we can resolve from the ground: 1 arcsec

But remember, the sun is 1 billion times brighter that Jupiter, too