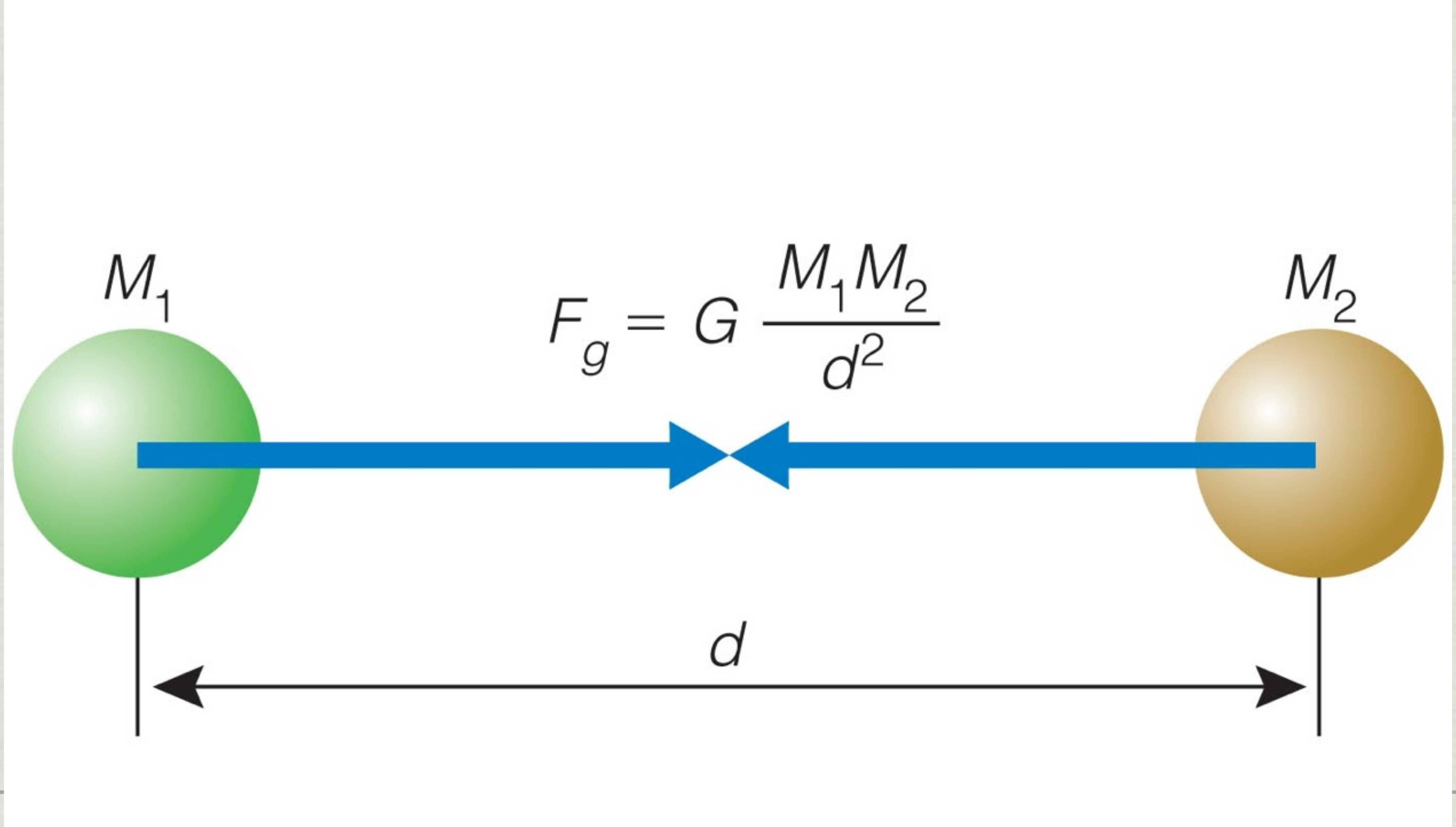
Announcements

- You should see a polling session active if you are using the REEF app. Make sure you are signed in
- I have two iClickers to loan out. First come, first served. For today's lecture only
- Added after class: I agree with you, the display you see of the reading and homework assignments is terrible. No wonder so many of you didn't see the reading due today. I'll give everyone an extension to Saturday, February 11th at 5pm. That's the Saturday before the midterm — you should complete the assignment by then to study for the exam, too.

Gravity

Recap from last time

- The force that holds you onto the Earth, the moon moving in orbit around the earth, the planets moving in their orbits around the sun, is Gravity



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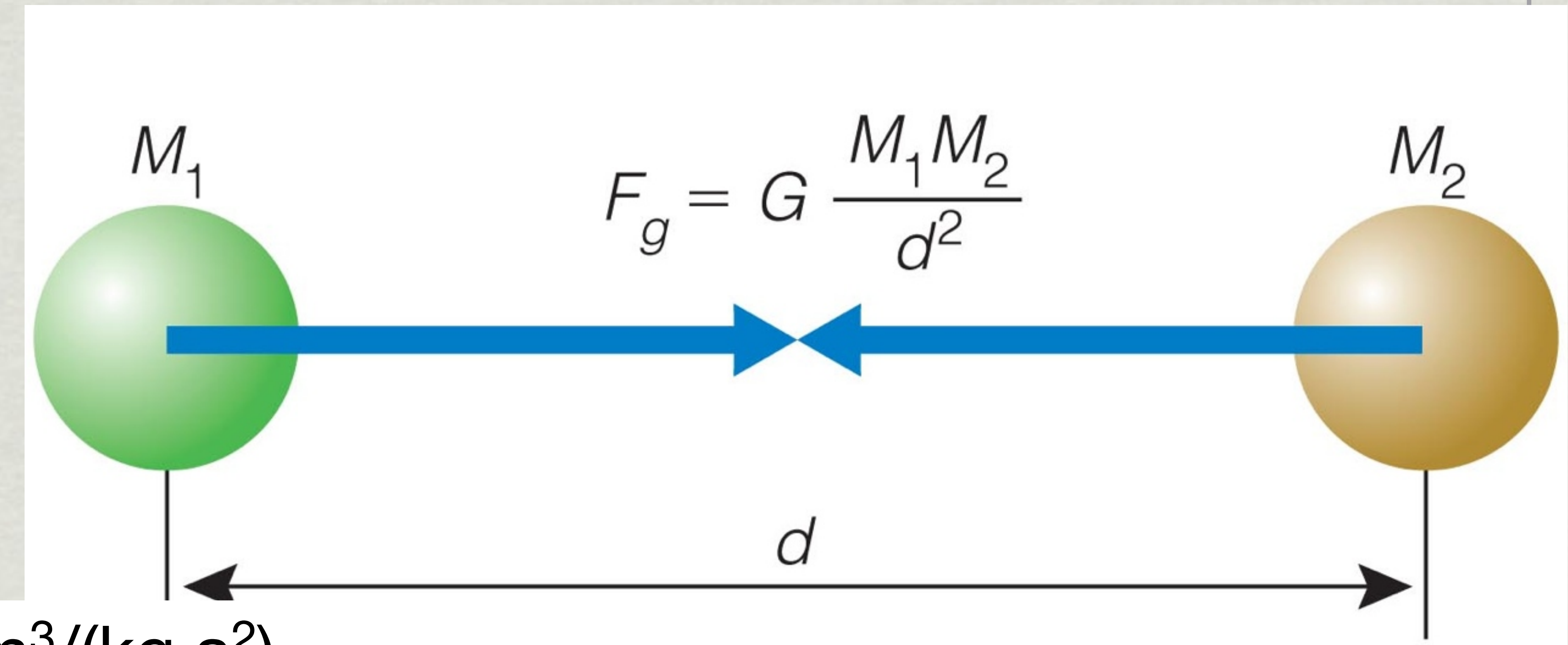
Force from gravity:
- Force of **M1** on **M2** and **M2** on **M1** (Newton's 3rd law)
- stronger for smaller d
- stronger for larger **M1** and/or **M2**



$G = 6.67 \times 10^{-11}\ m^3/(kg\ s^2)$

Fun with Gravity

Recap from last time

- Gravity can be used to weigh stuff we can't put on a scale, like stars and planets

$$\frac{d^3}{P^2} = \boxed{\frac{G\,M}{(2\pi)^2}}$$

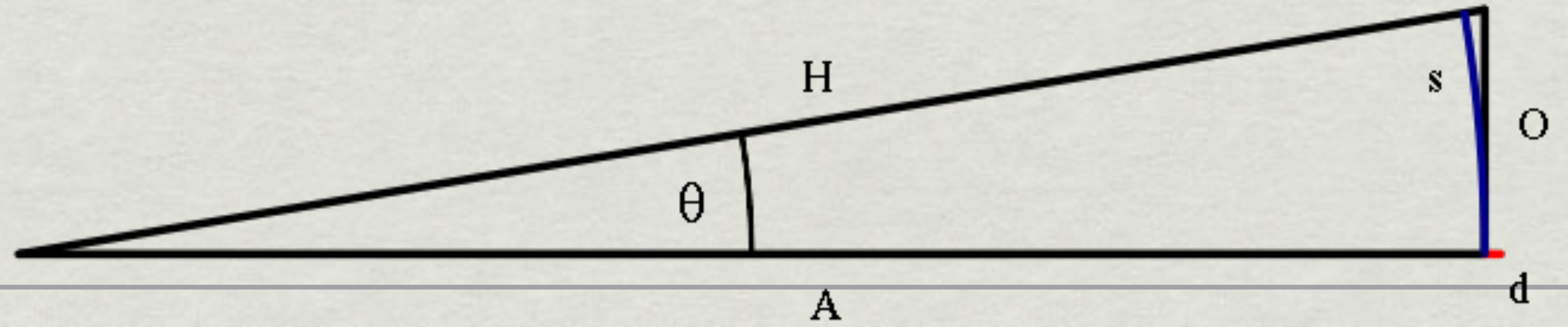
- We can even use it to weigh stuff we can't see, and prove that it is there
- A neat example: finding planets around other stars

Finding Planets Around Other Stars

Recap from last time

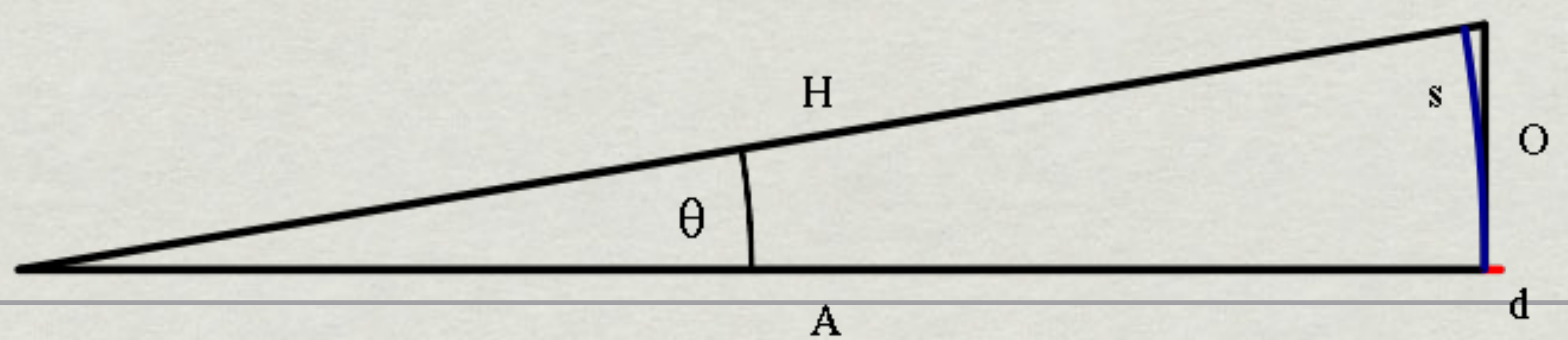
Why do we need gravity to do this?

- Even the most nearby stars are very far away
- Planets are ***much*** closer to their stars than the nearest stars are to us
- Angular-size distance relation: the angular separations we measure between the stars and their planets are tiny. Even something as big as a solar system is only a tiny angular size.
- Like being in San Francisco and trying to see the head of a pin 15 meters away from a grapefruit in Washington, DC



Finding Planets Around Other Stars
Recap from last time

- So this is not easy:
  - planets around other stars are much fainter than their stars
  - they are also very, very close to their stars, so very difficult to identify the faint planet light

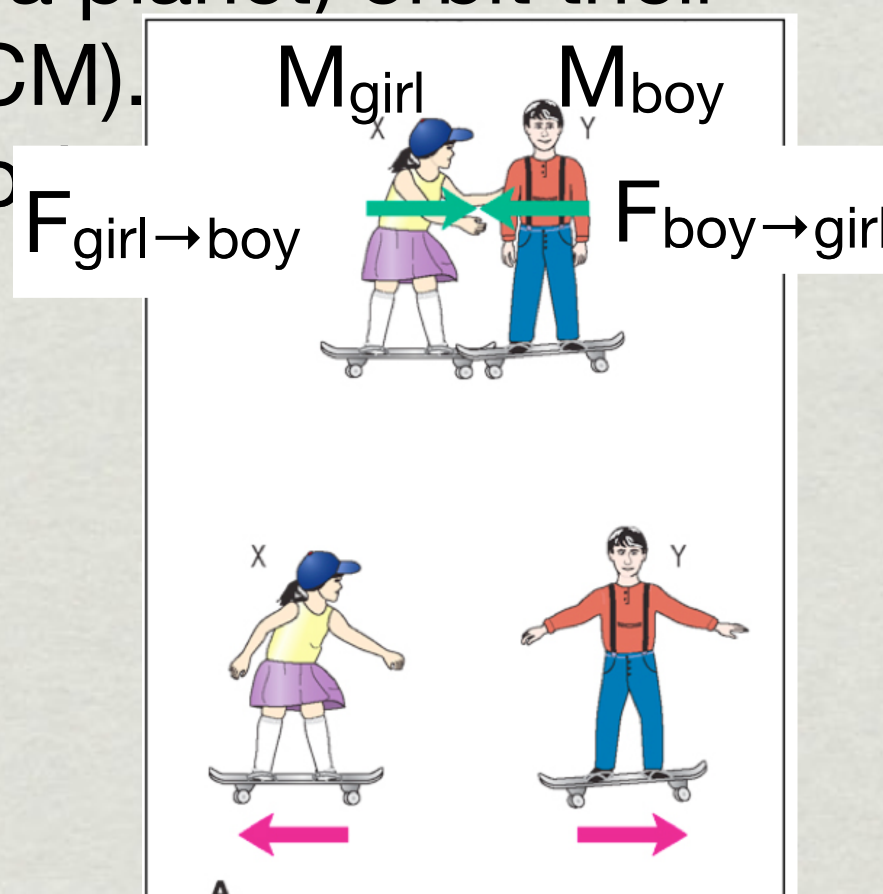


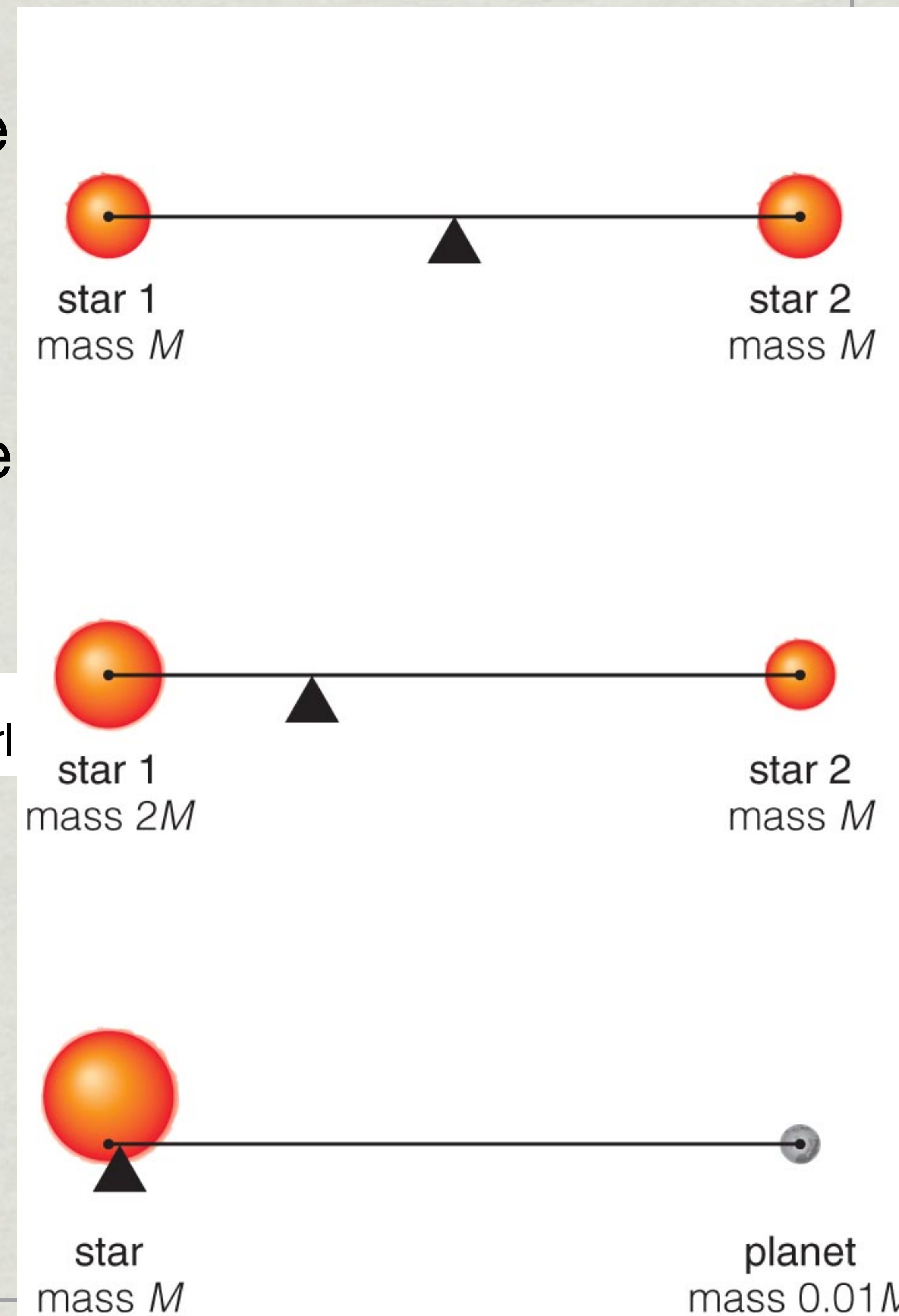
Center of Mass and Orbits

Recap from last time

Newton's 3rd Law: if something (like a moon) is in orbit around something more massive (like a planet), the massive object feels a force, too.

Two objects (two stars, the earth and the moon, a star and a planet) orbit their Center of Mass (CM). same orbital perio



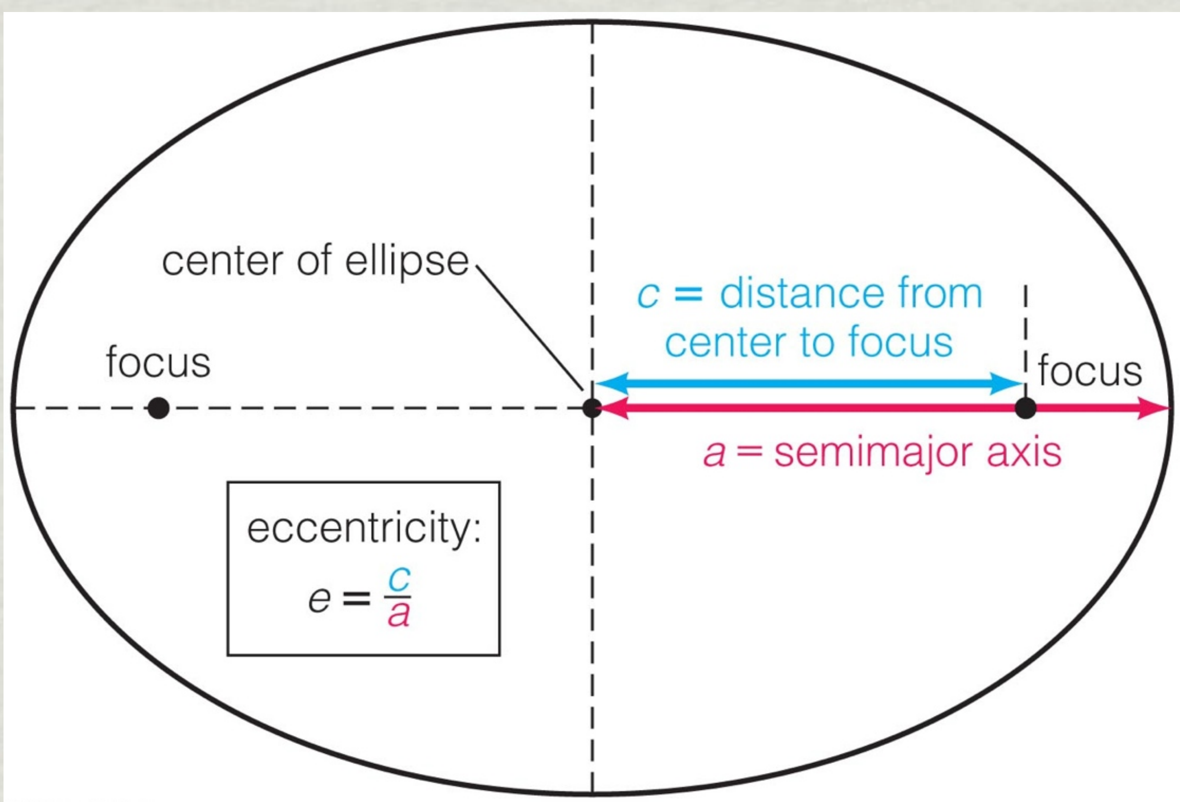


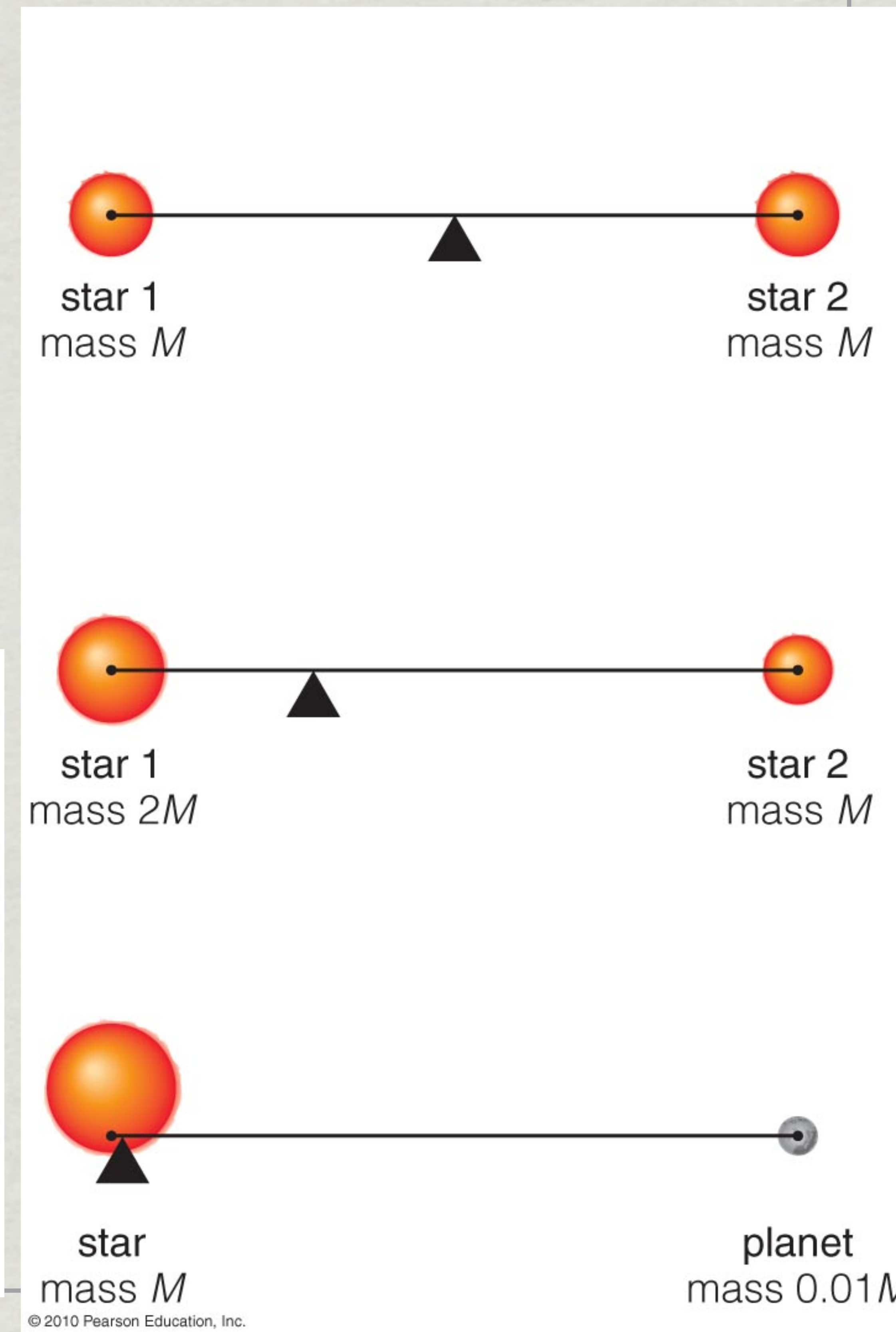
Center of Mass and Orbits
Recap from last time

Kepler's 1st Law: planets orbit in ellipses with the sun at one focus.

- The Center of Mass is what is really at the focus.

For one object much more massive than the other, the CM is very close to the center of the massive object, sometimes ***inside*** it





Newton's Version of Kepler's 3rd Law

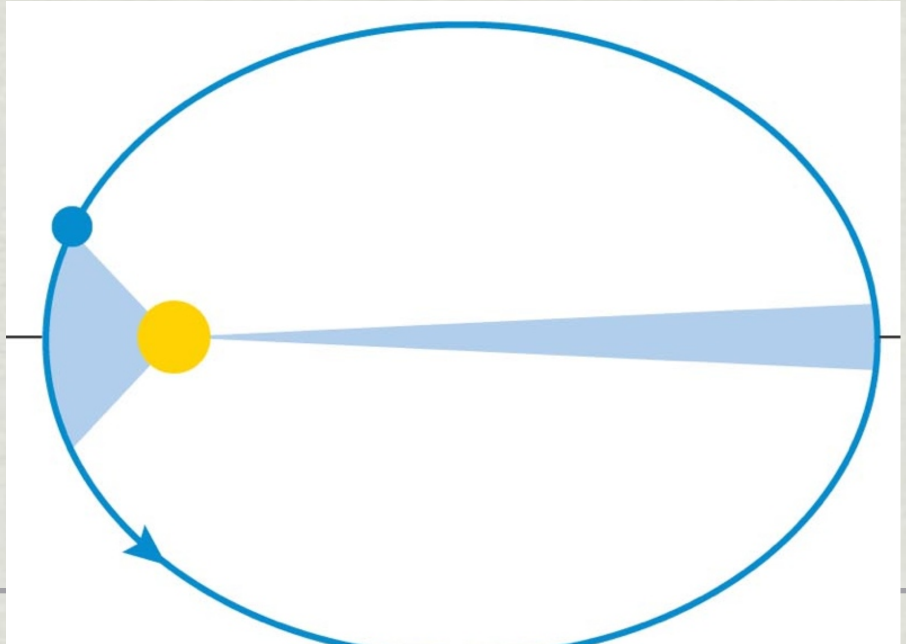
$$\frac{d^3}{P^2} = \frac{G\,(M_1+M_2)}{(2\pi)^2}$$

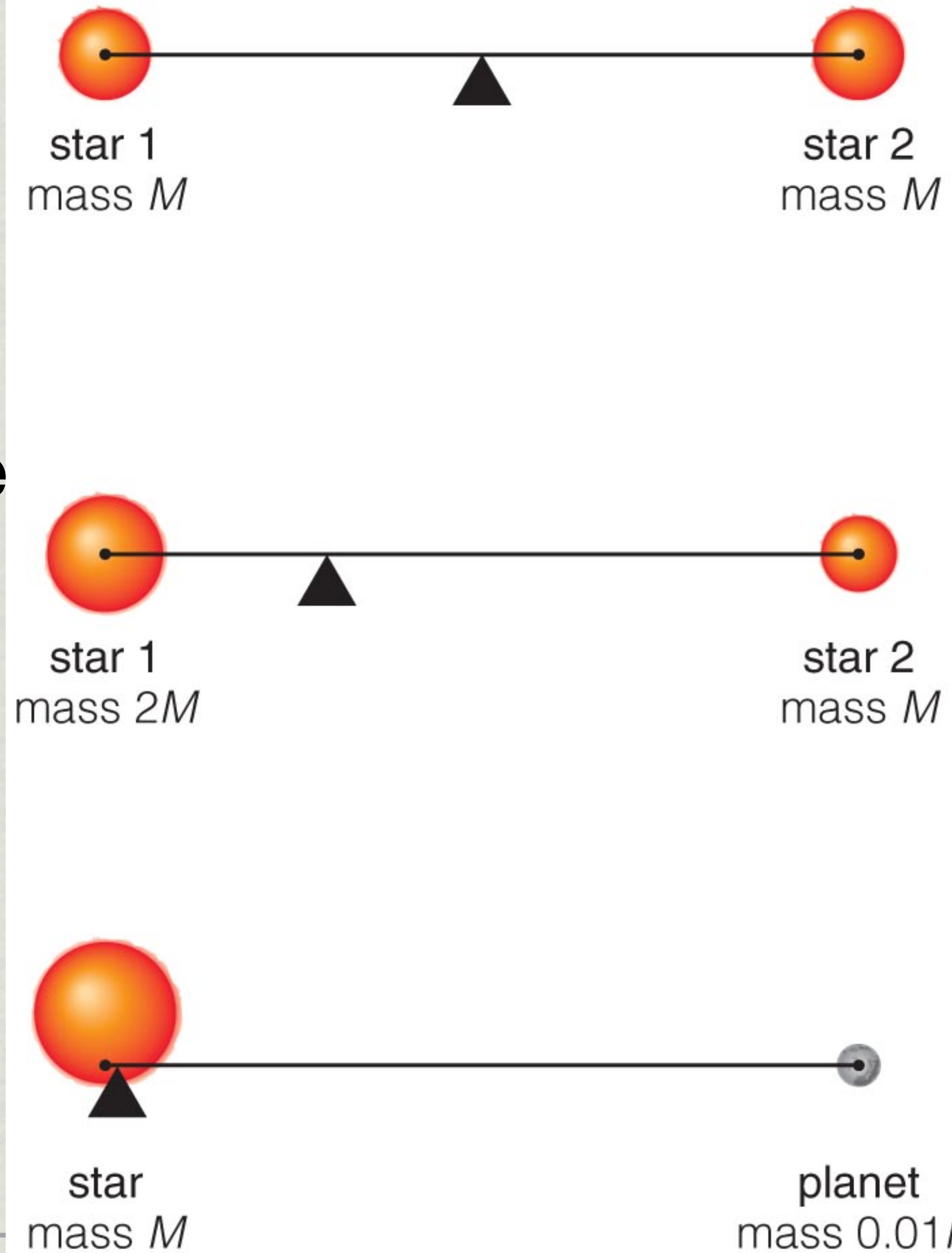
This is the more exact version.

If $M_1 >> M_2$ then this can be simplified to the equation we derived earlier in class:

$$\frac{d^3}{P^2} = \frac{G\,M}{(2\pi)^2}$$

d = distance to the Center of Mass of the orbit. If $M_1 >> M_2$ then d is just the distance between M_1 and M_2.





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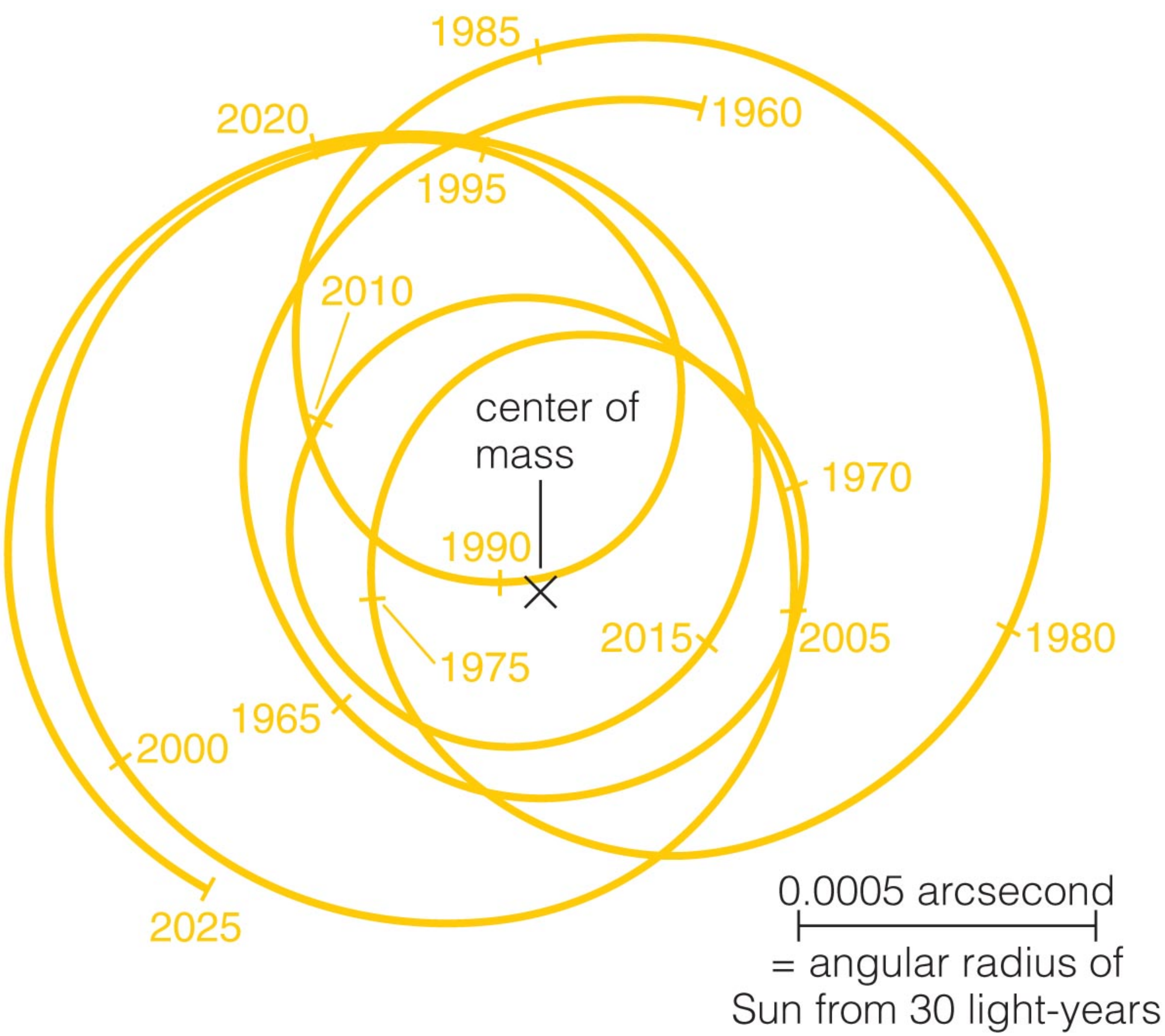
d = distance to the Center of Mass of the orbit. If $M_1 >> M_2$ then d is the distance between M_1 and M_2.

This is an important tool for how we learn about other solar systems:

1. Observe the motion of the star around the CM of the orbit
2. If we see the star move, even a little bit, we've found a planet!
3. The star's orbital period we measure is the same as the orbital period of the planet around the CM of the orbit, the planet's "year"

Gravitational Tugs



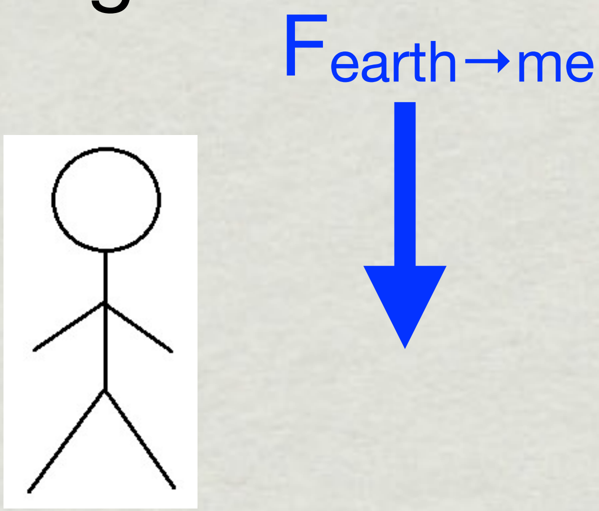
- The Sun's motion around the solar system's center of mass depends on tugs from all the planets.
- Astronomers around other stars that measured this motion could determine the masses and orbits of all the planets.

The motions are tiny: 0.001 arcseconds, just too tiny to measure from the ground. Gaia satellite is making this measurement now!

Surface Gravity and Weight

- The earth's surface gravity ***g***: acceleration you feel at the surface of the earth, pulls you toward the ground (actually, to the center of the earth, but the ground is in the way)
- **g** is an acceleration, but it is an important one so it gets its own letter

Gravity: $F_{earth \rightarrow me} = \dfrac{G\, M_{me}\, M_{earth}}{R^2_{earth}}$



The force of the earth's gravitational pull on me

$F_{earth \rightarrow me} = Ma = M_{me}\, \mathbf{g}$

= my mass x acceleration from Earth's gravitational pull

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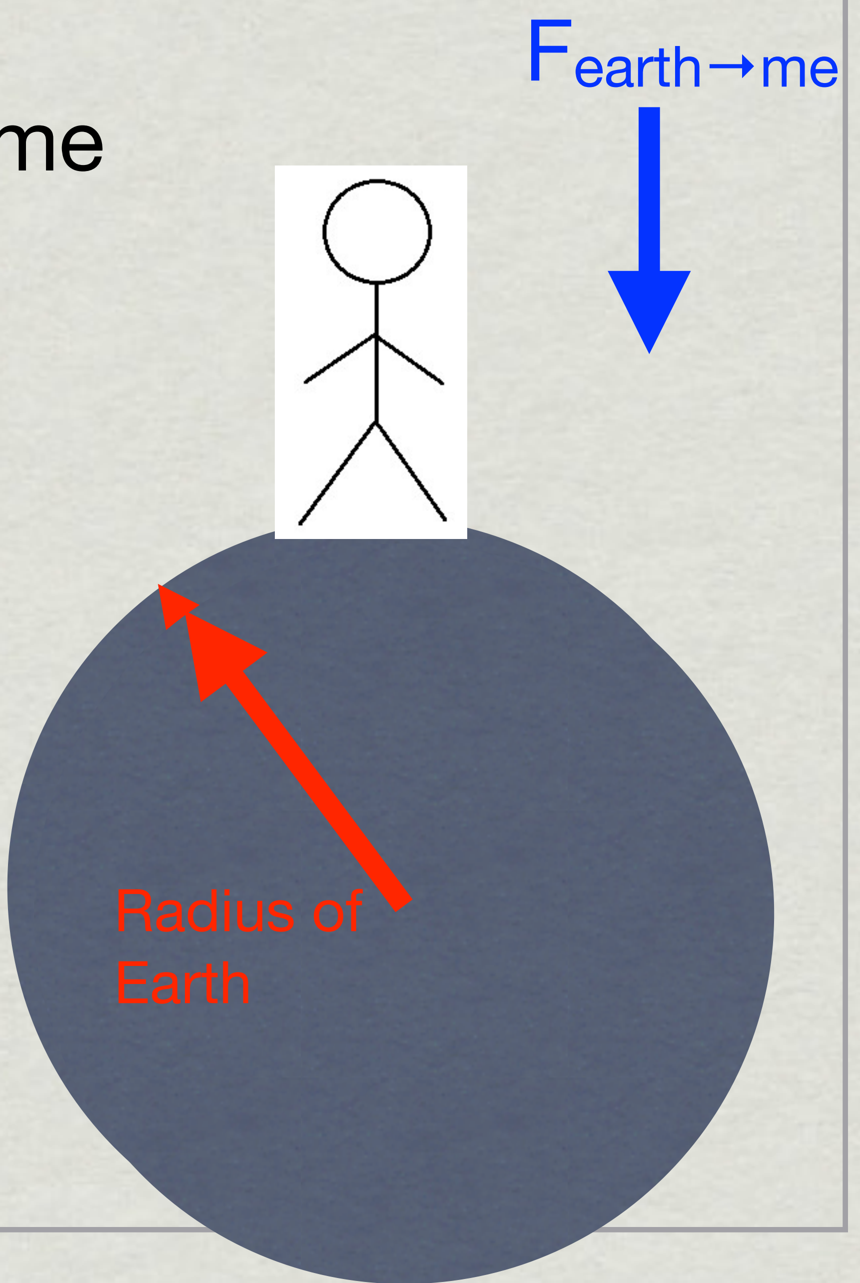
$$\cancel{M}_{me}\, \mathbf{g} = \frac{G\, \cancel{M}_{me}\, M_{earth}}{R^2_{earth}}$$

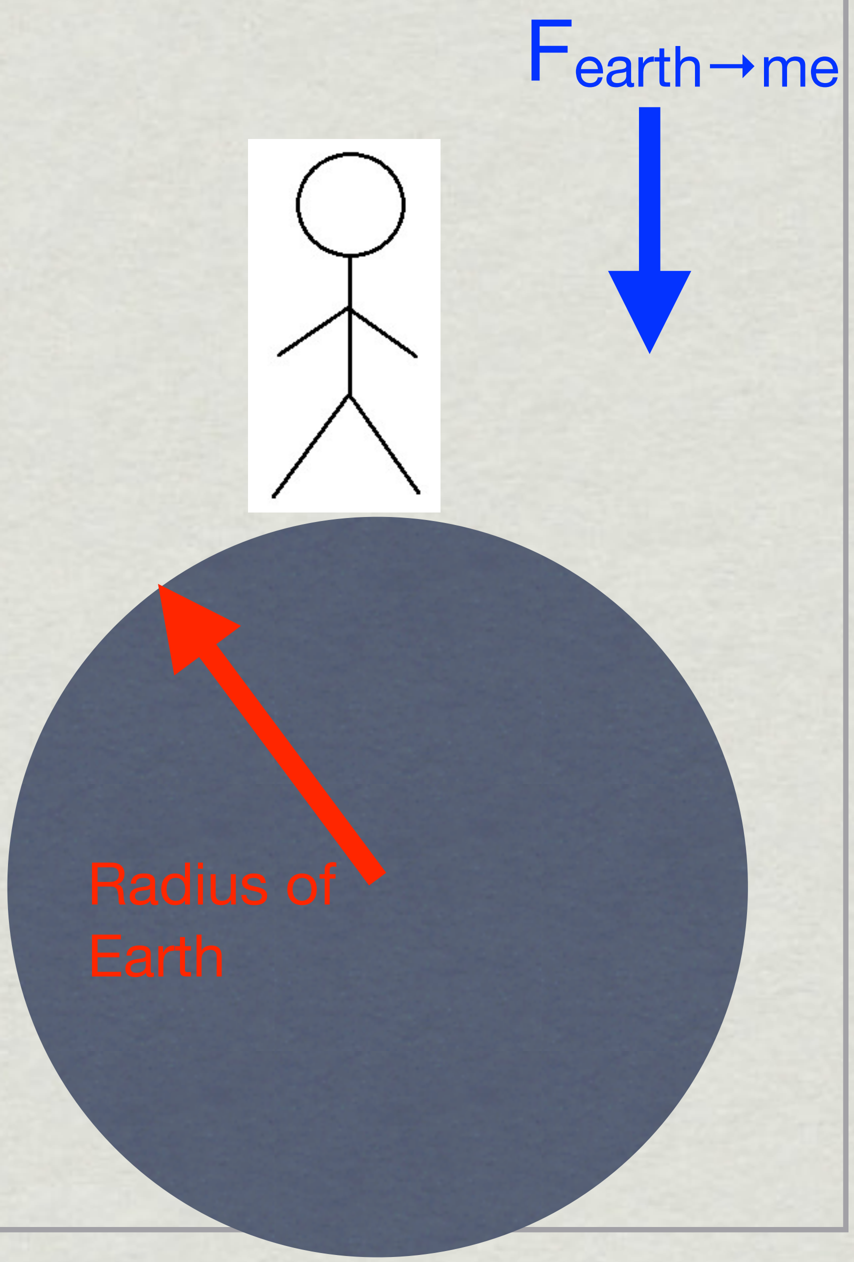
$$\mathbf{g} = \frac{G\, M_{earth}}{R^2_{earth}}$$

Doesn't depend on my mass



Force, Mass and Weight

- Your mass: amount of stuff in you
  - same anywhere in the universe
- Your weight: Force pushing up on your mass due to Earth's surface gravity
- My mass: 68 kg
- Acceleration of earth's gravity: 9.8 m/s^2
- Force: 668 kg m/s^2, my weight



Force, Mass and Weight

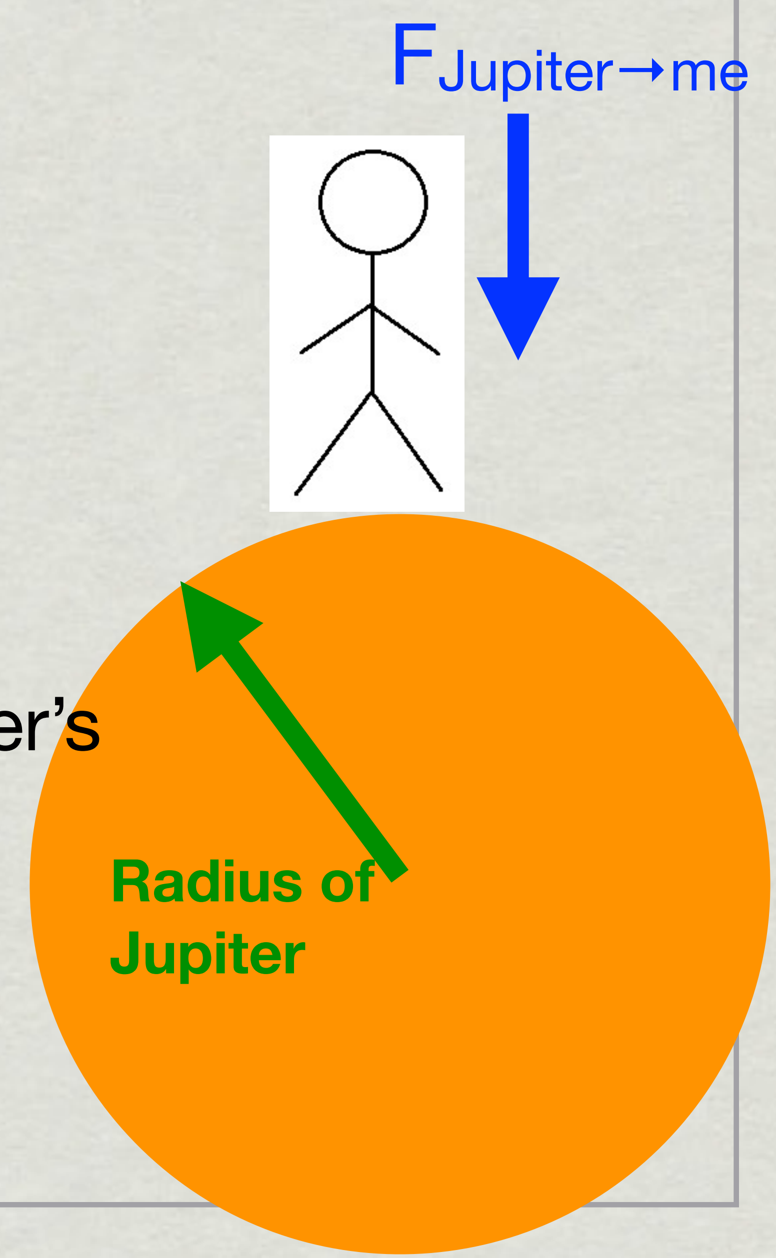
Jupiter has a different mass and radius than earth

$$\mathbf{a}_{\text{Gravity, Jupiter}} = \frac{G\,M_{Jupiter}}{R^2_{Jupiter}}$$

$M_{Jupiter} = 2 \times 10^{27}$ kg

$R^2_{Jupiter} = 7 \times 10^7$ m

$\mathbf{a}_{\text{Gravity, Jupiter}} = 27\ \text{m/s}^2$



Compare g, acceleration due to Earths gravity: 9.8 m/s^2

You would experience a greater acceleration from Jupiter's gravity

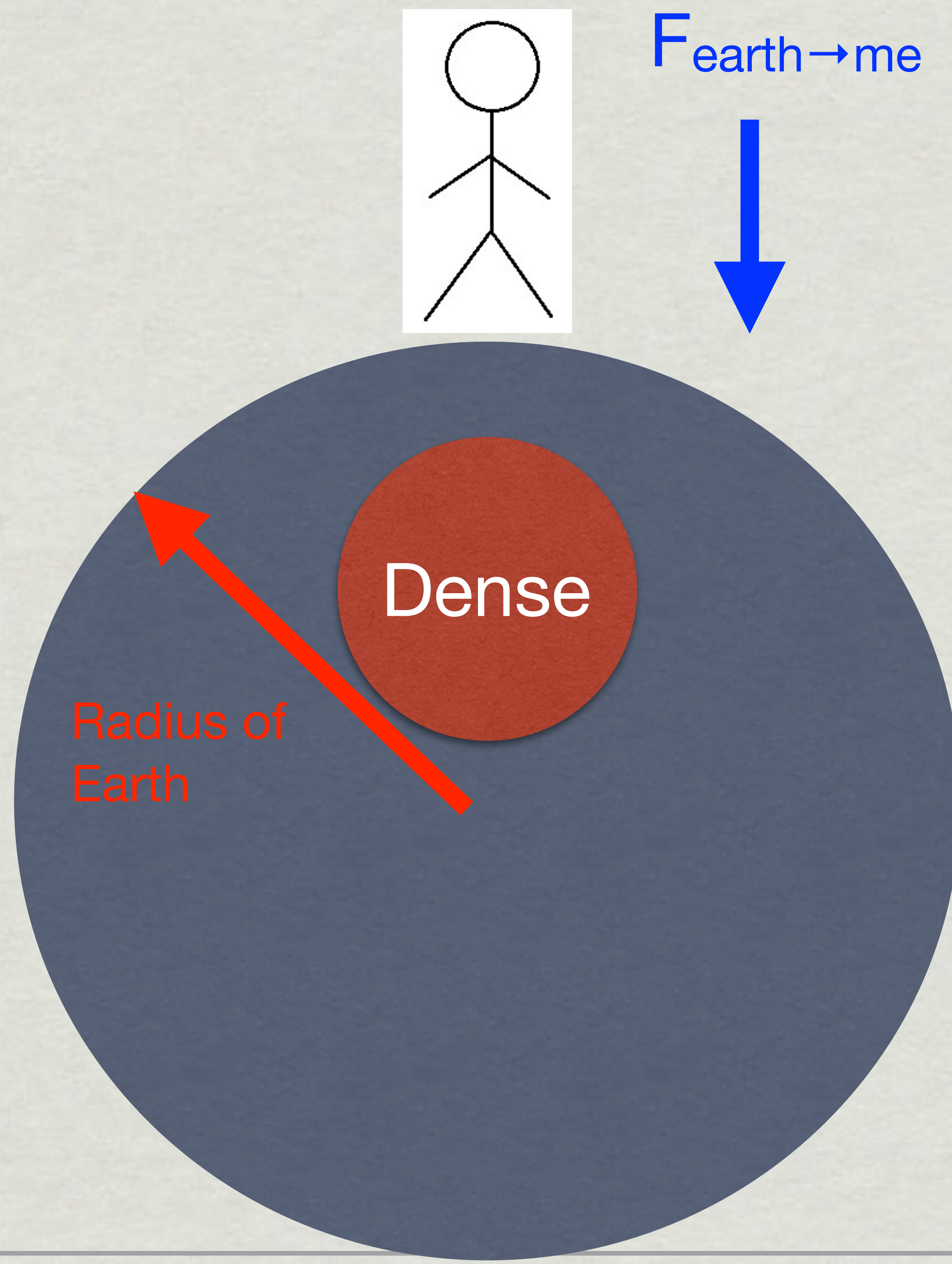
You would weigh more on Jupiter

Moon has weaker gravity: you weigh less on the moon

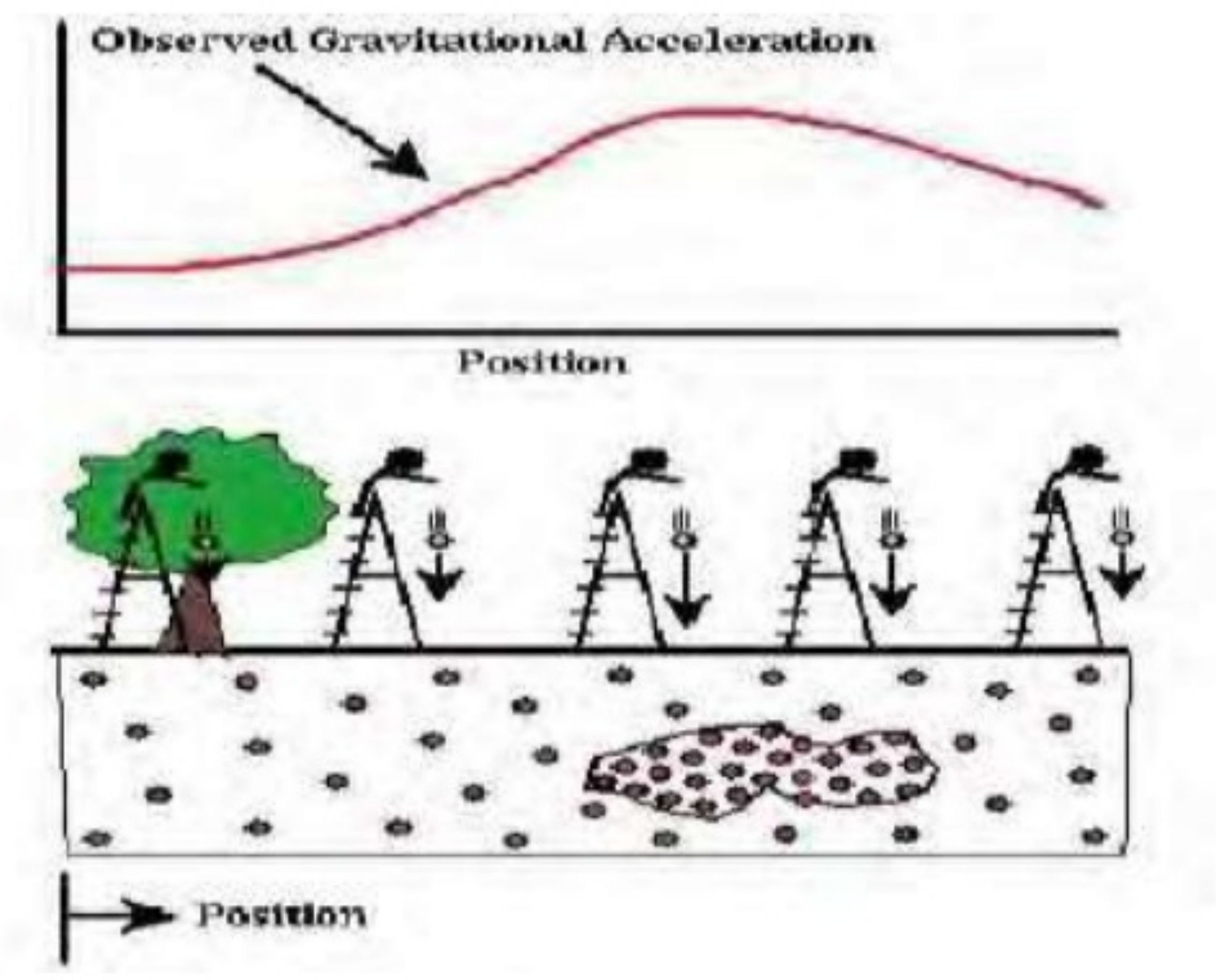
Force, Mass and Weight

- Blob of dense stuff: more local mass so gravity acceleration is larger locally

Gravity: $$F_{earth \rightarrow me} = \frac{G\, M_{me}\, M_{earth}}{R^2_{earth}}$$

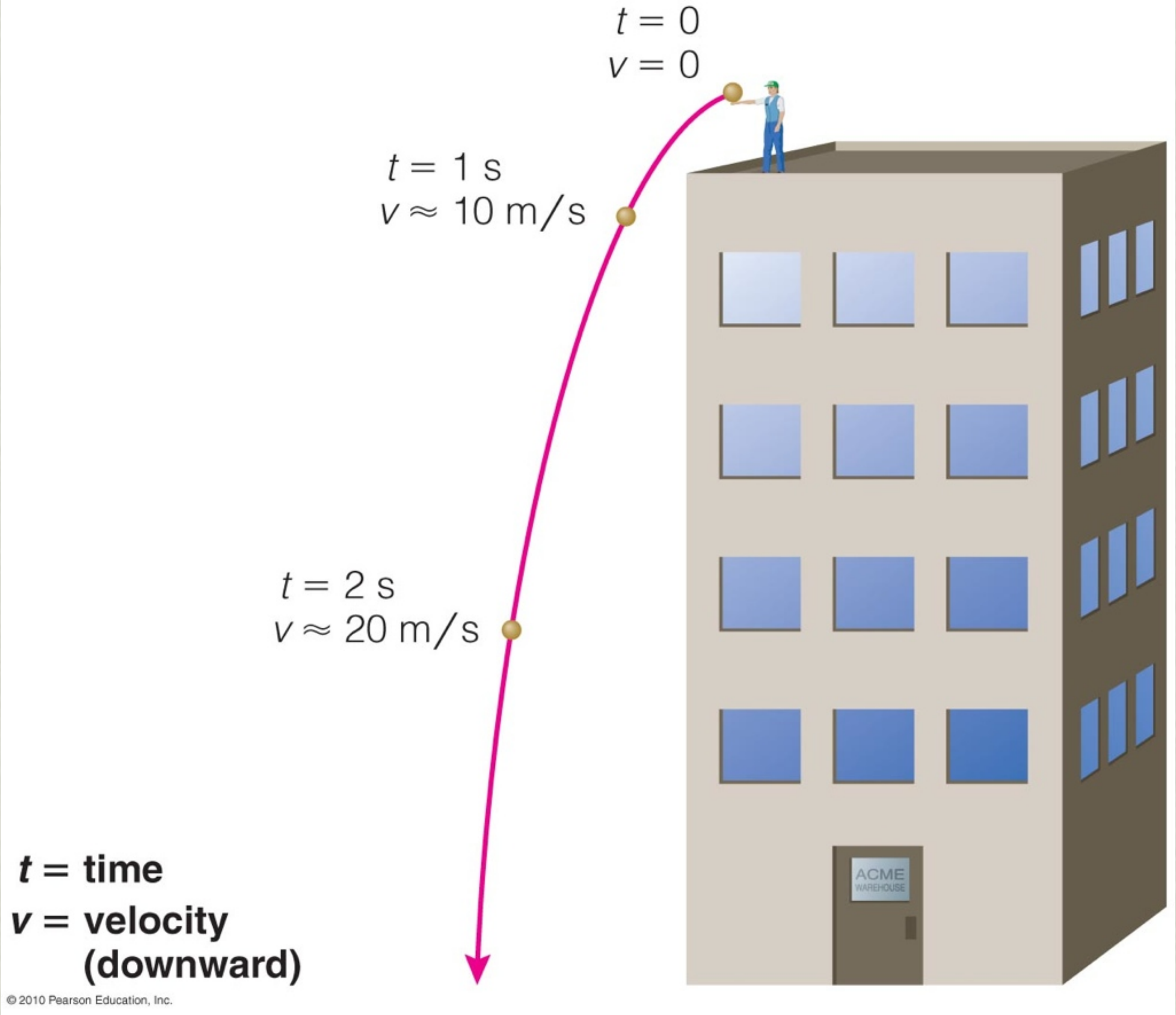


Force, Mass and Weight



Force, Mass and Weight
Recap from last week

- Weightlessness: if you jump off the building (wearing your parachute, of course):
  - you still have mass x acceleration = "weight"
- better description: free-fall
  - accelerating freely due to force of gravity



Mass, Weight, Force

"Felix Baumgartner, Daredevil"
Red Bull "Stratos"
A balloon to 24 miles above the Earth, then jump

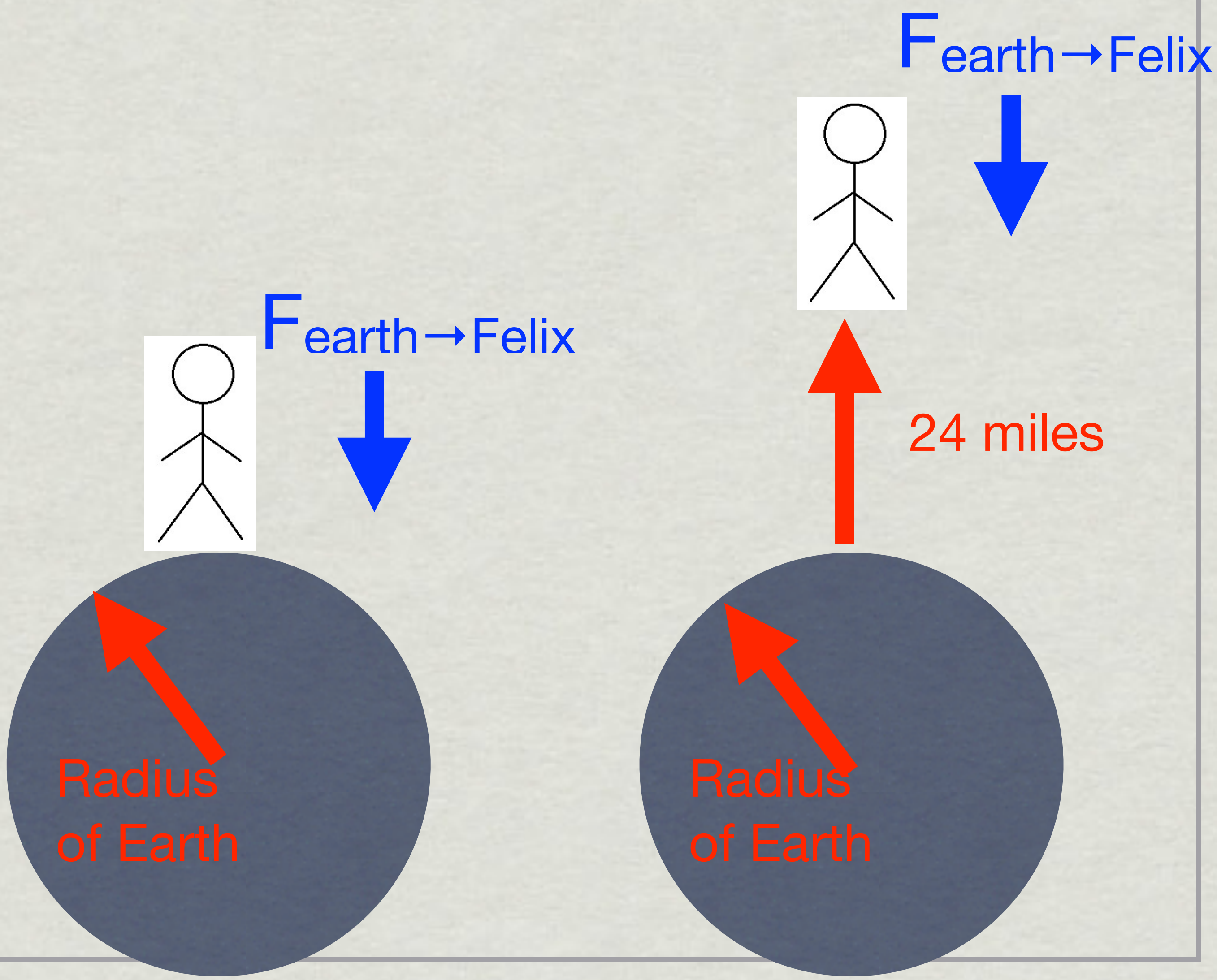
Mass, Weight, Force

What happens to Felix's velocity as he drops?

Mass, Weight, Force

What happens to Felix's velocity as he drops?
Felix accelerates

Does his acceleration change as he falls, starting at 24 miles above the surface of the earth?



Mass, Weight, Force

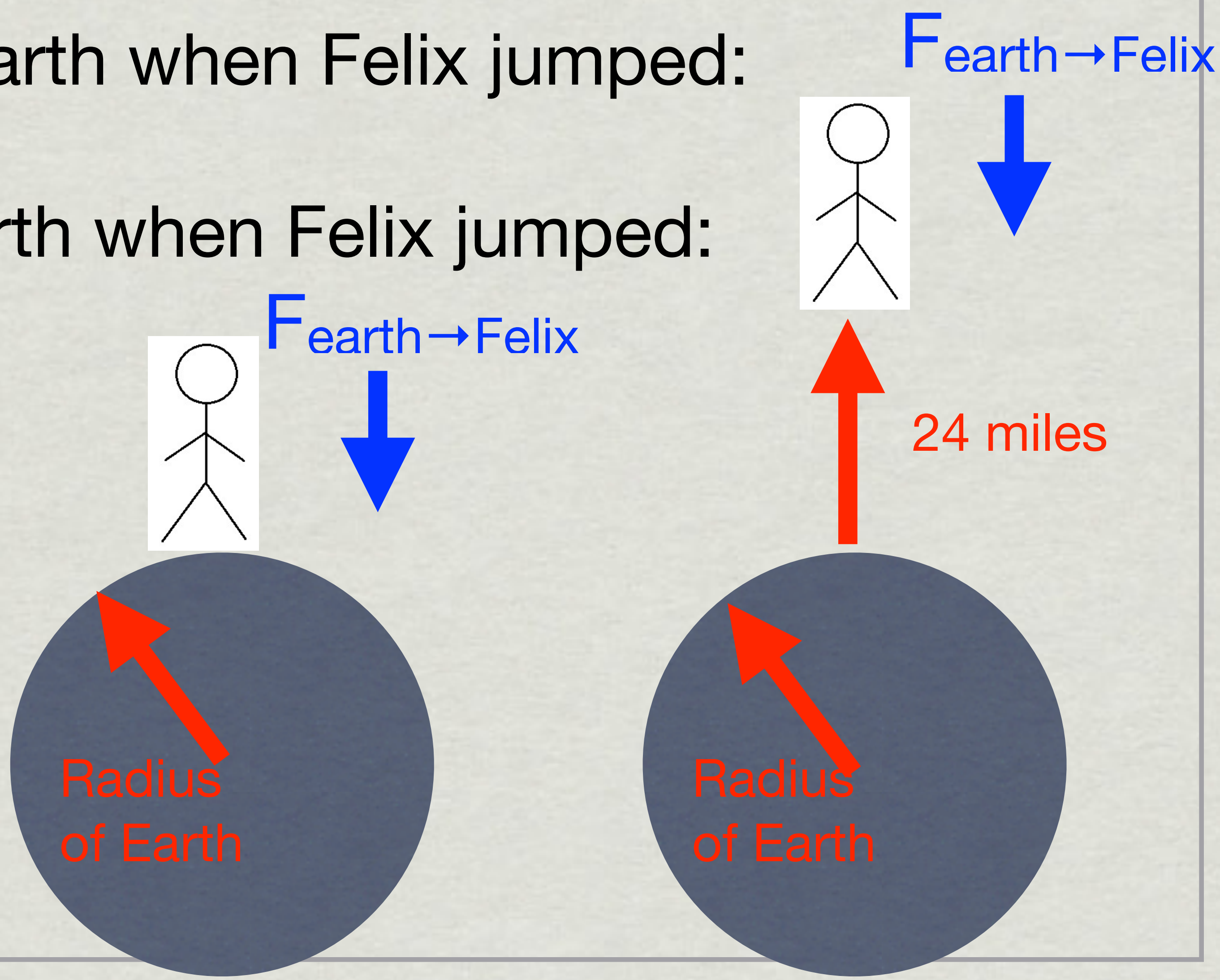
Does his acceleration change as he falls, starting at 24 miles above the surface of the earth?

$$\mathbf{g} = \frac{G\,M_{earth}}{d^2}$$

Radius of the earth: ~4000 miles
Distance from the surface of the earth when Felix jumped: 24 miles
Distance from the center of the earth when Felix jumped: 4024 miles

$4000^2 = 16$ million
$4024^2 = 16.2$ million

1% in the denominator of the acceleration equation. A tiny change.



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Polling question:

Felix stepped on a bathroom scale the morning of his jump, and it read 190 lbs.

If he had a scale in his capsule, would it read:

A 190 lbs
B a little bit less than 190 lbs
C a little bit more than 190 lbs
D you can't tell until he jumps

Mass, Weight, Force

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If he had a scale in his capsule, would it read:

A 190 lbs

B a little bit less than 190 lbs

C a little bit more than 190 lbs

D you can't tell until he jumps

x

Mass, Weight, Force

If Felix sat on a scale in his capsule, it would read about the same as on earth.

What would felix have to do to go into orbit after jumping out of his capsule?

From last time: Orbits and Circular Motion

- To get the velocity for an object in a circular orbit, combine:

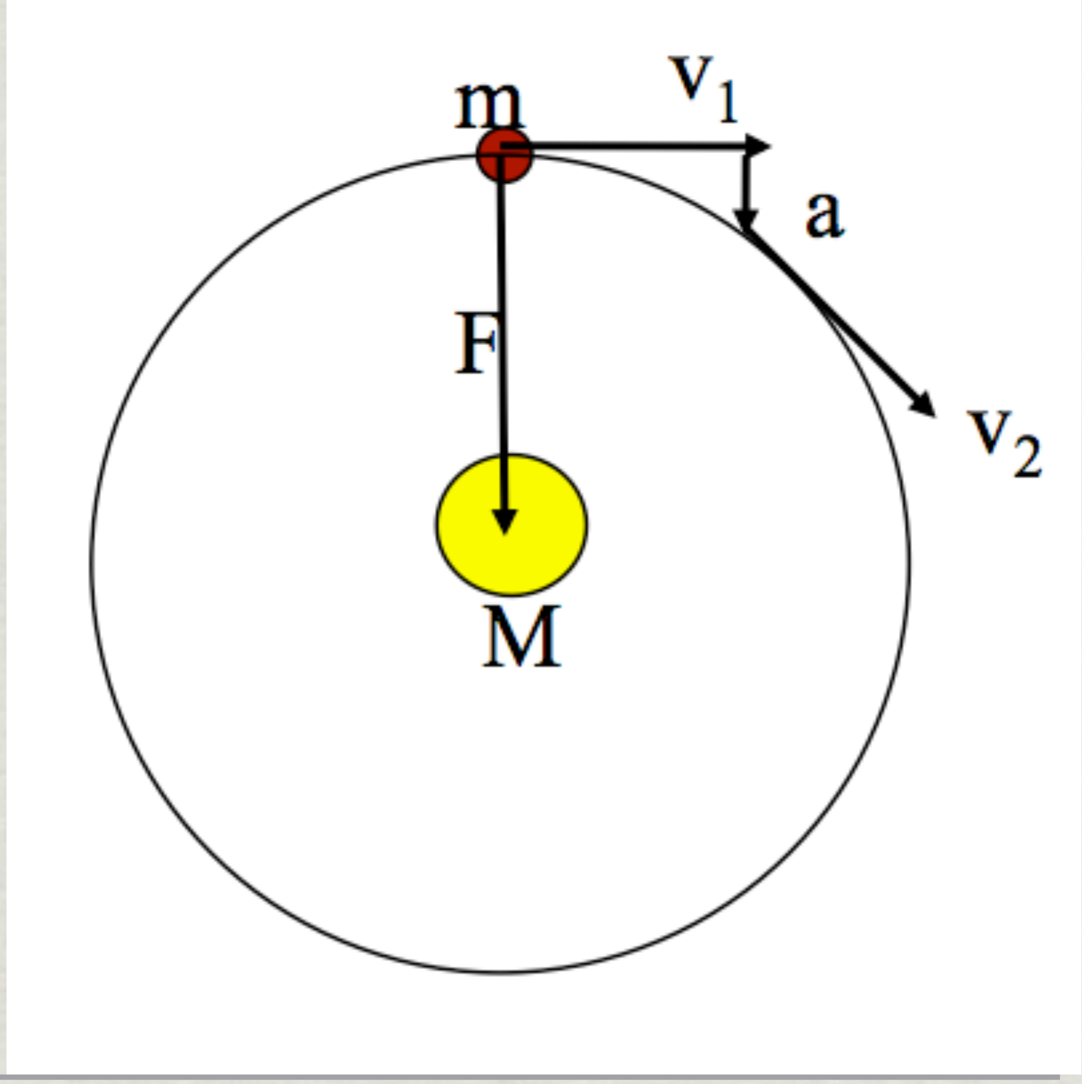
Acceleration required to keep an object in circular motion: $\frac{v^2}{d}$

with acceleration from Gravity $= \frac{G\,M}{d^2}$

$$\frac{v^2}{d} = \frac{G\,M}{d^2}$$

$$v = \sqrt{\frac{G\,M}{d}}$$

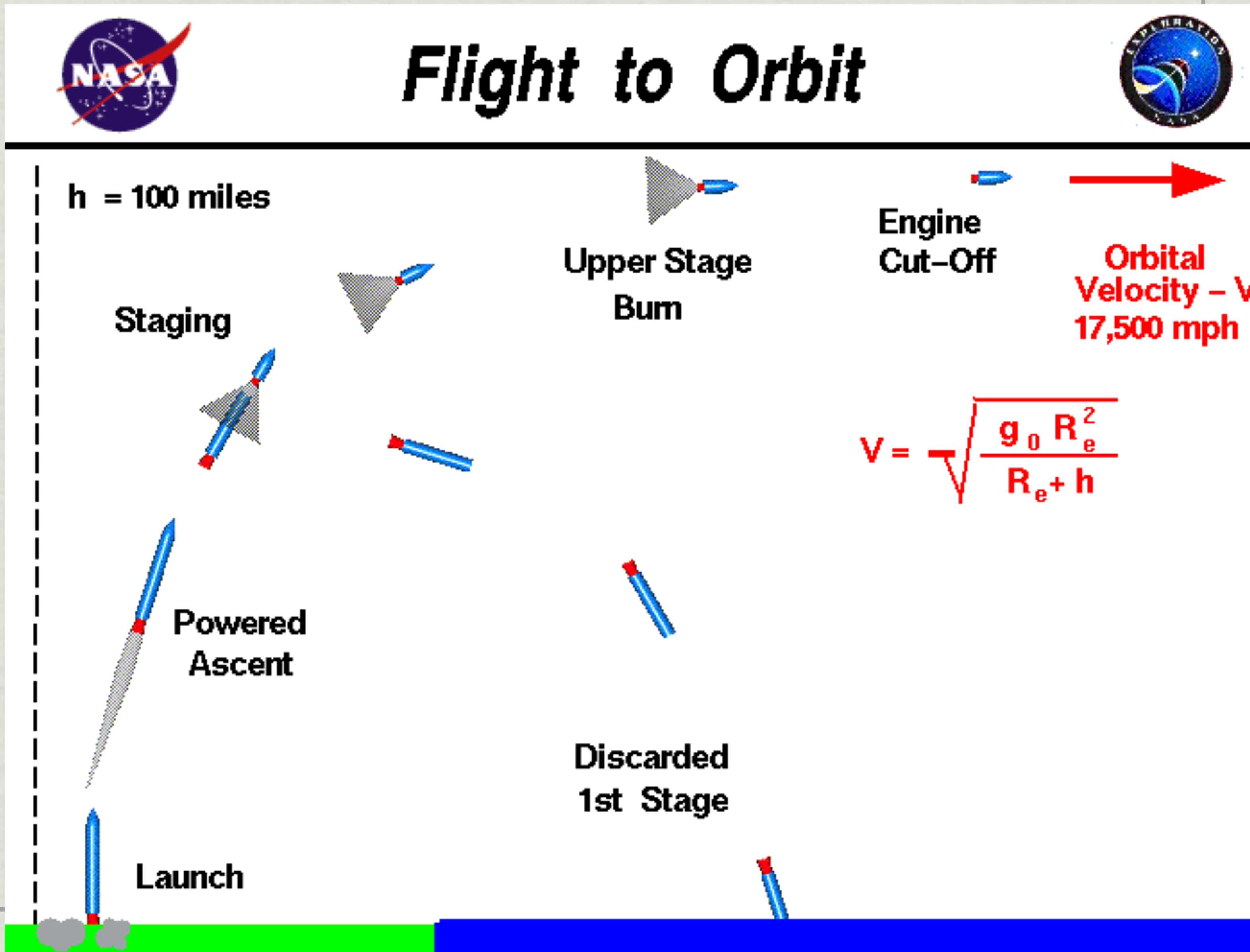
v = speed for an object in stable circular motion around mass M at distance d



Balloon vs. Rocket

Rocket starts out vertical, moving away from earth due to force (thrust) generated by the engine.

Later the rocket turns so it gets horizontal acceleration to reach high enough velocity for circular orbit.



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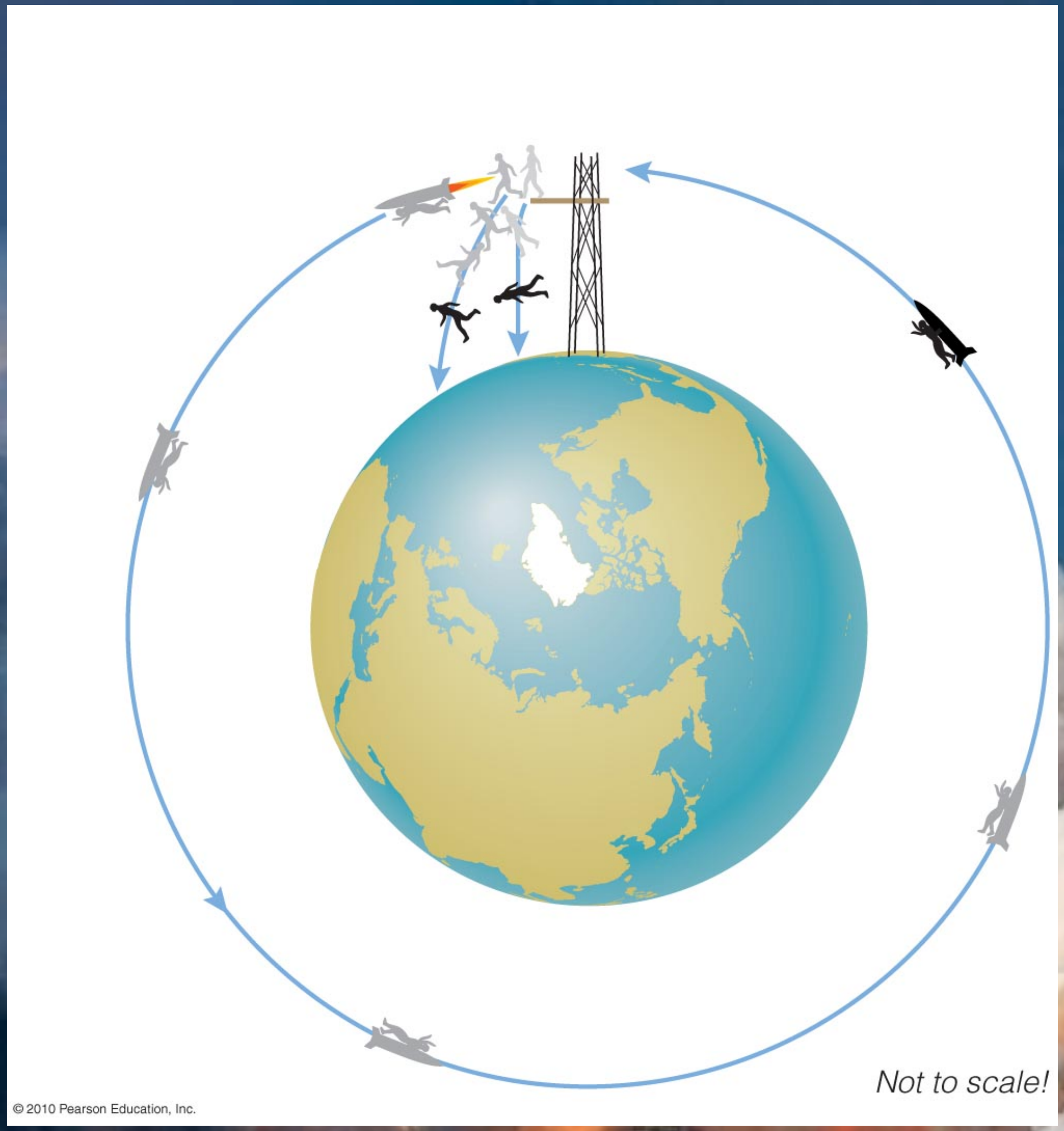

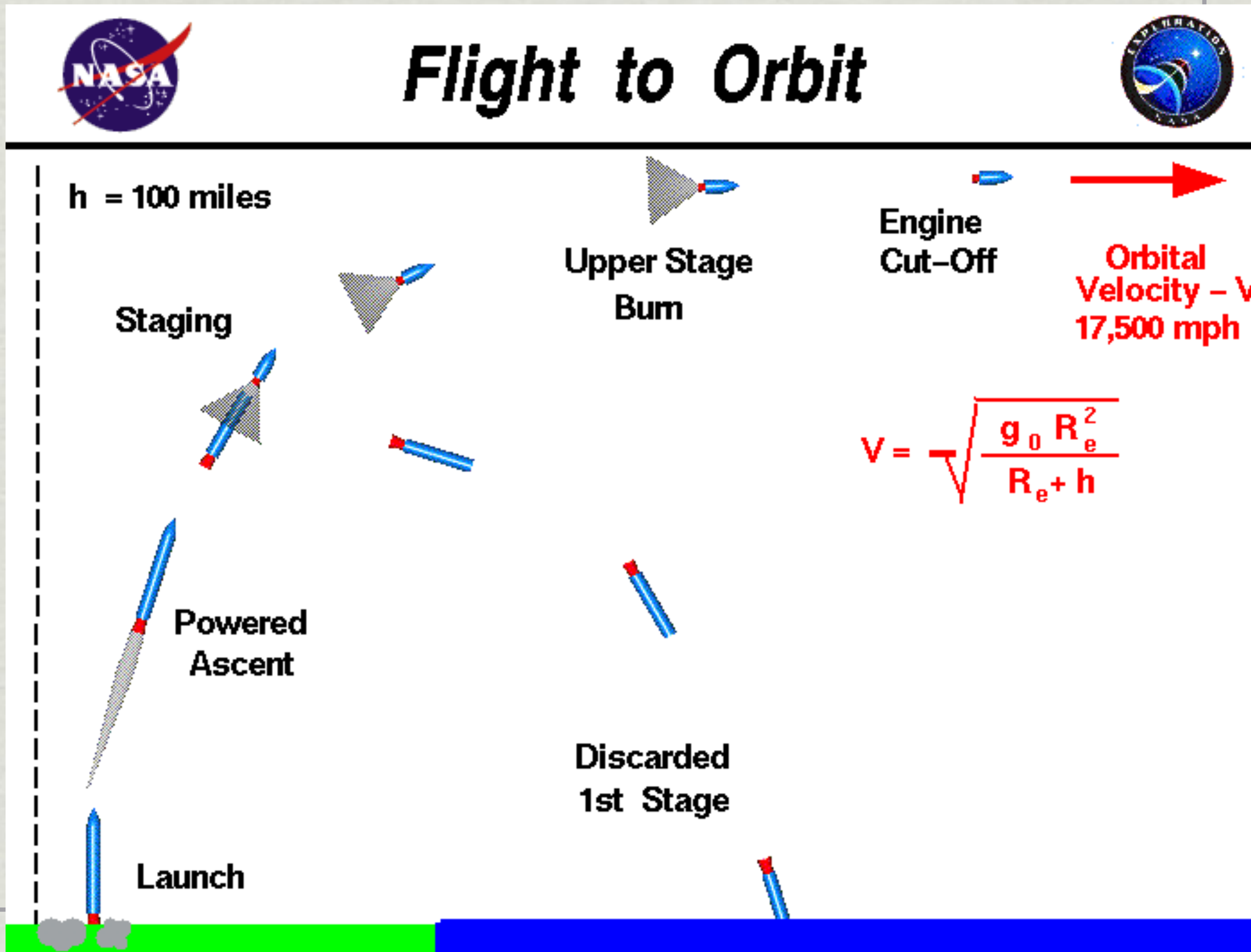

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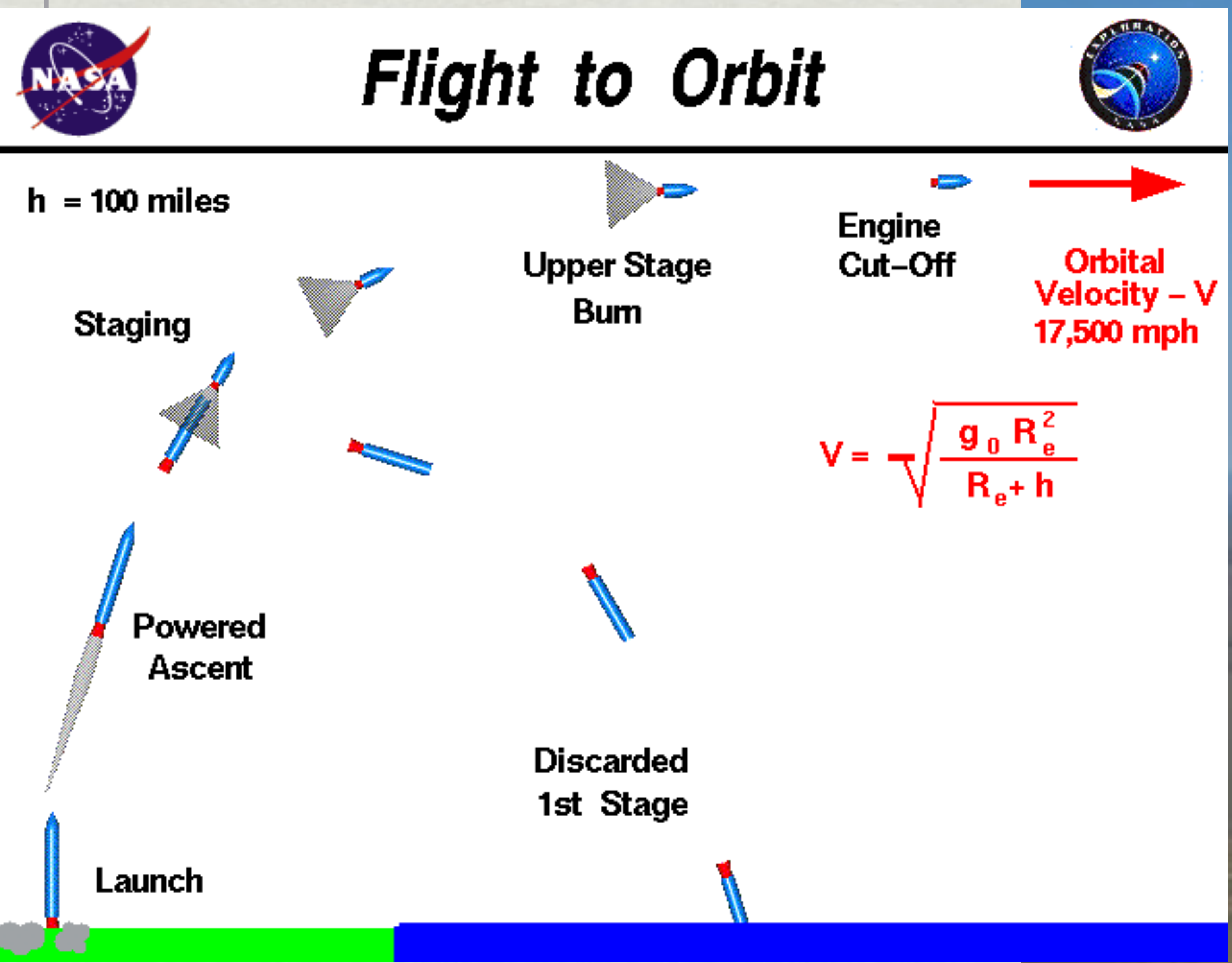
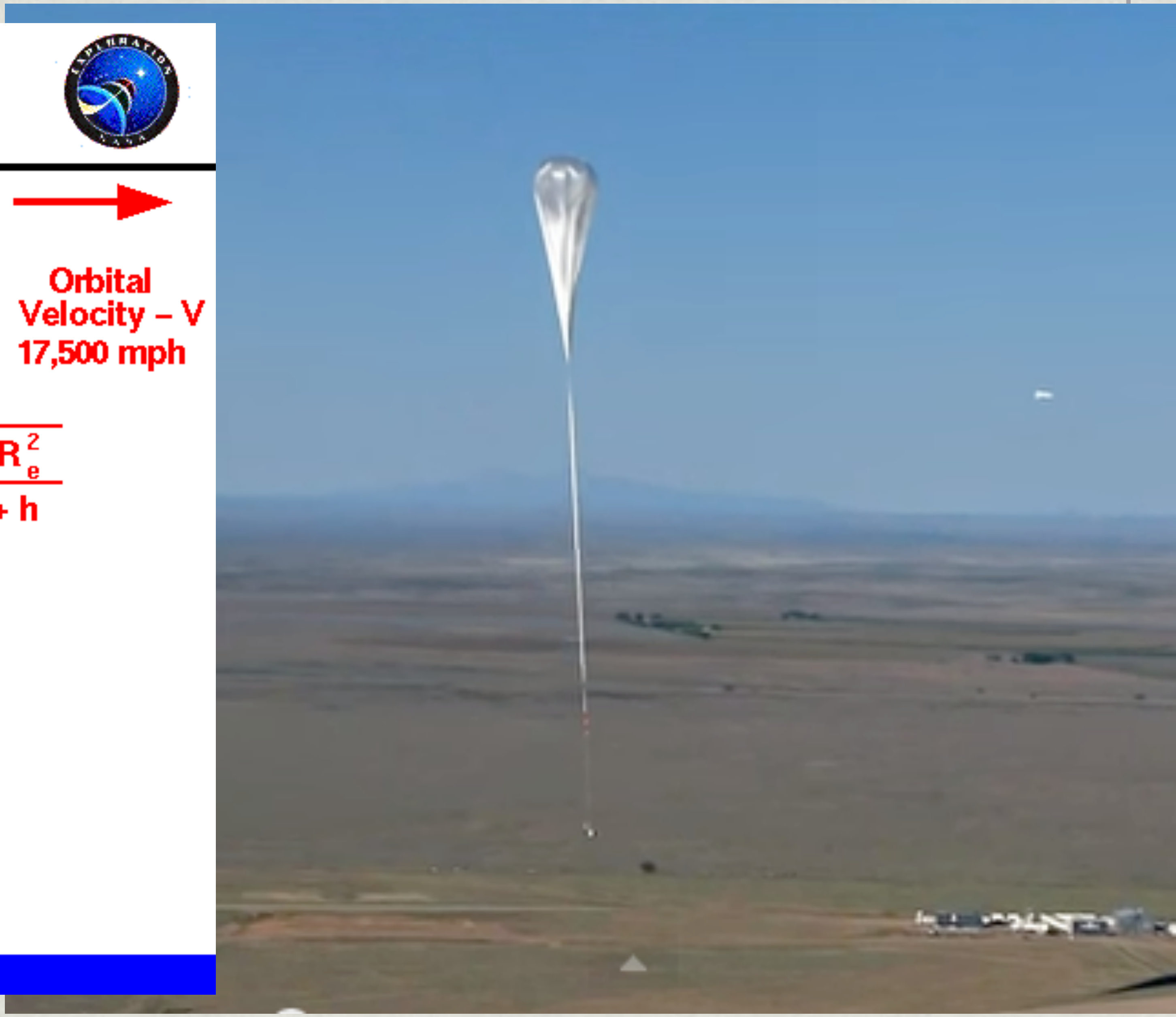




Energy: Kinds of Energy

Kinetic energy = energy of motion: $\frac{m v^2}{2}$

Potential energy = stored energy

Gravitational potential energy = $-\frac{GM_1M_2}{d}$

Energy stored by working against a gravitational force

Notice resemblance to the equation for gravitational force:

$$F = \frac{GM_1M_2}{d^2}$$

Kinds of Energy

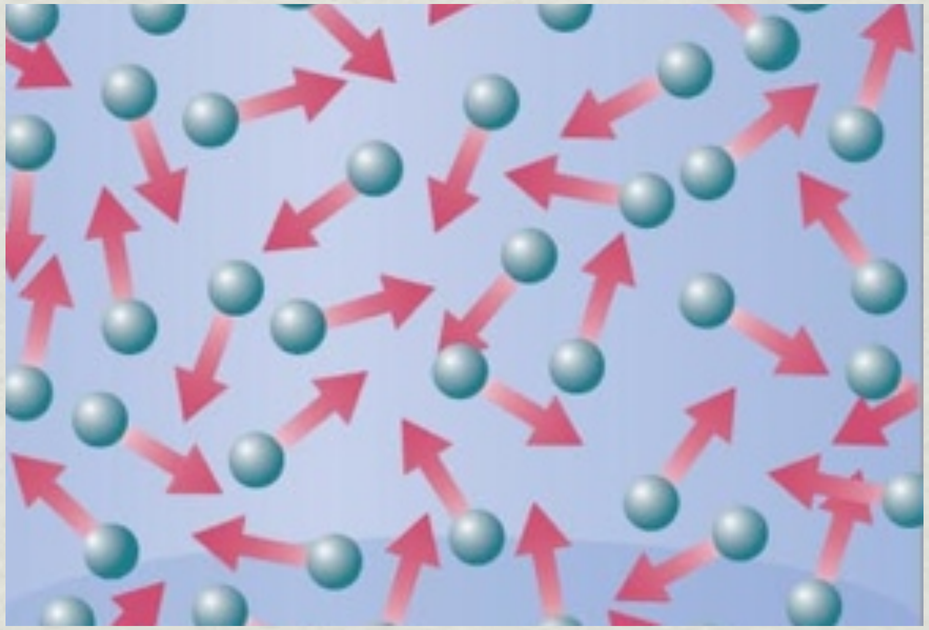
Kinetic energy = energy of motion: $\frac{m\,v^2}{2}$

Gravitational potential energy = $-\frac{GM_1M_2}{d}$

Energy units: kg m^2/s^2 (Joules)

Other kinds of energy:
Chemical potential: energy stored in chemical bonds

Thermal: kinetic energy due to random motions of particles



Radiative: light

Energy

In an isolated system: Energy is conserved

sum of all the energy is a constant value

Energy can change form:

gravitational potential → kinetic

radiative → thermal

or be exchanged between objects

Energy

In an isolated system: Energy is conserved

sum of all the energy is a constant value

Examples of closed, isolated systems (includes all interacting objects):

Earth and pendulum

A planet or planets orbiting a star

Conservation of Energy: Examples

A swinging pendulum: each swing reaches the same height

Total Energy $E_{total} = E_{kinetic} + E_{grav\ potential} = \text{Constant}$

At top of swing, v=0 and d is at largest value as the ball turns around:

$$E_{kinetic} = \frac{m\ v^2}{2} = 0$$

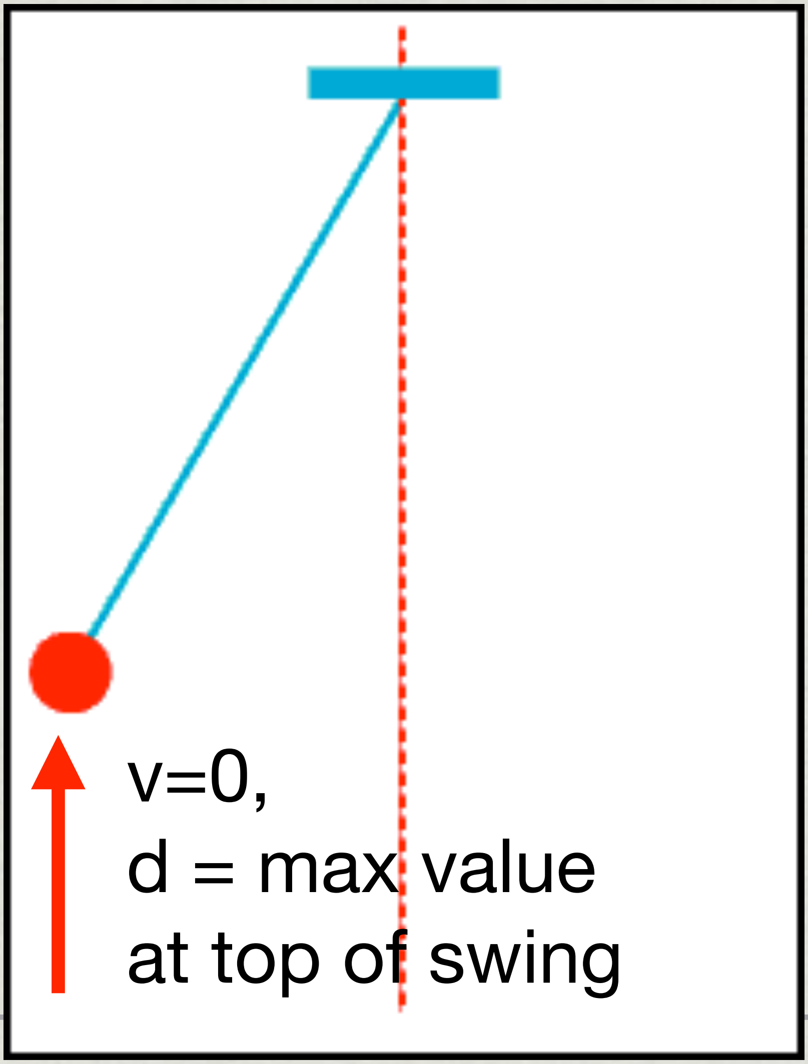
$$E_{grav\ potential} = \frac{-\ GM_1M_2}{d}$$

$E_{grav\ potential} = E_{total}$, largest possible value

At bottom of swing,
v = largest value, d = smallest value

$E_{kinetic} = \frac{m\ v^2}{2}$ largest value

$E_{grav\ potential}$ = minimum, ball is at its smallest distance from earth

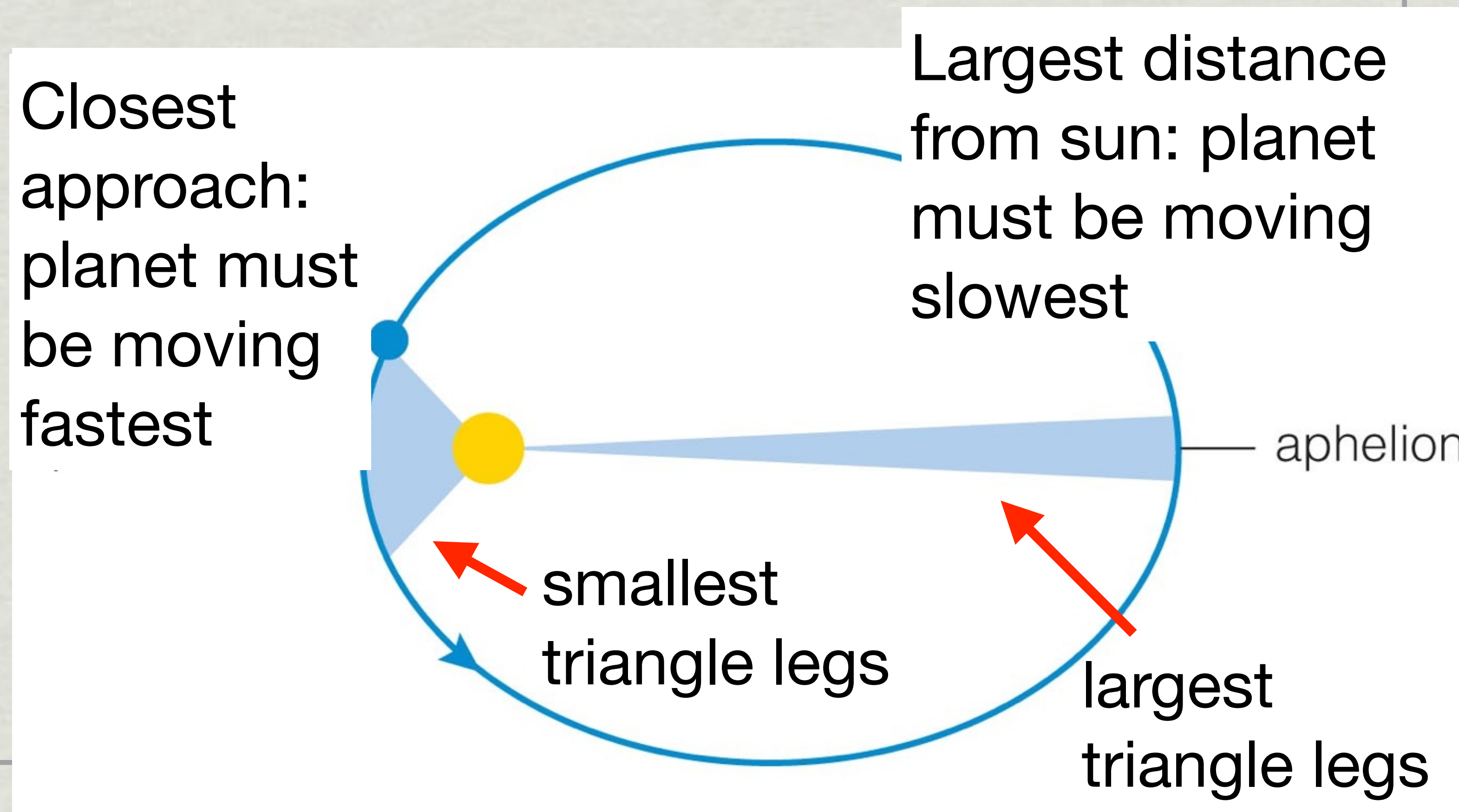


Conservation of Energy and Orbits

Total Energy = $E_{kinetic} + E_{grav\ potential}$ = Constant

Kepler's 2nd Law: Planets sweep out equal area in equal time

Remember: planet must be moving fastest when closest to the sun in order to sweep out equal area when the triangle legs are small.



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At perihelion: smallest distance

$$E_{grav\ potential} = \frac{-\ GM_1M_2}{d}$$ is smallest (a big negative number)

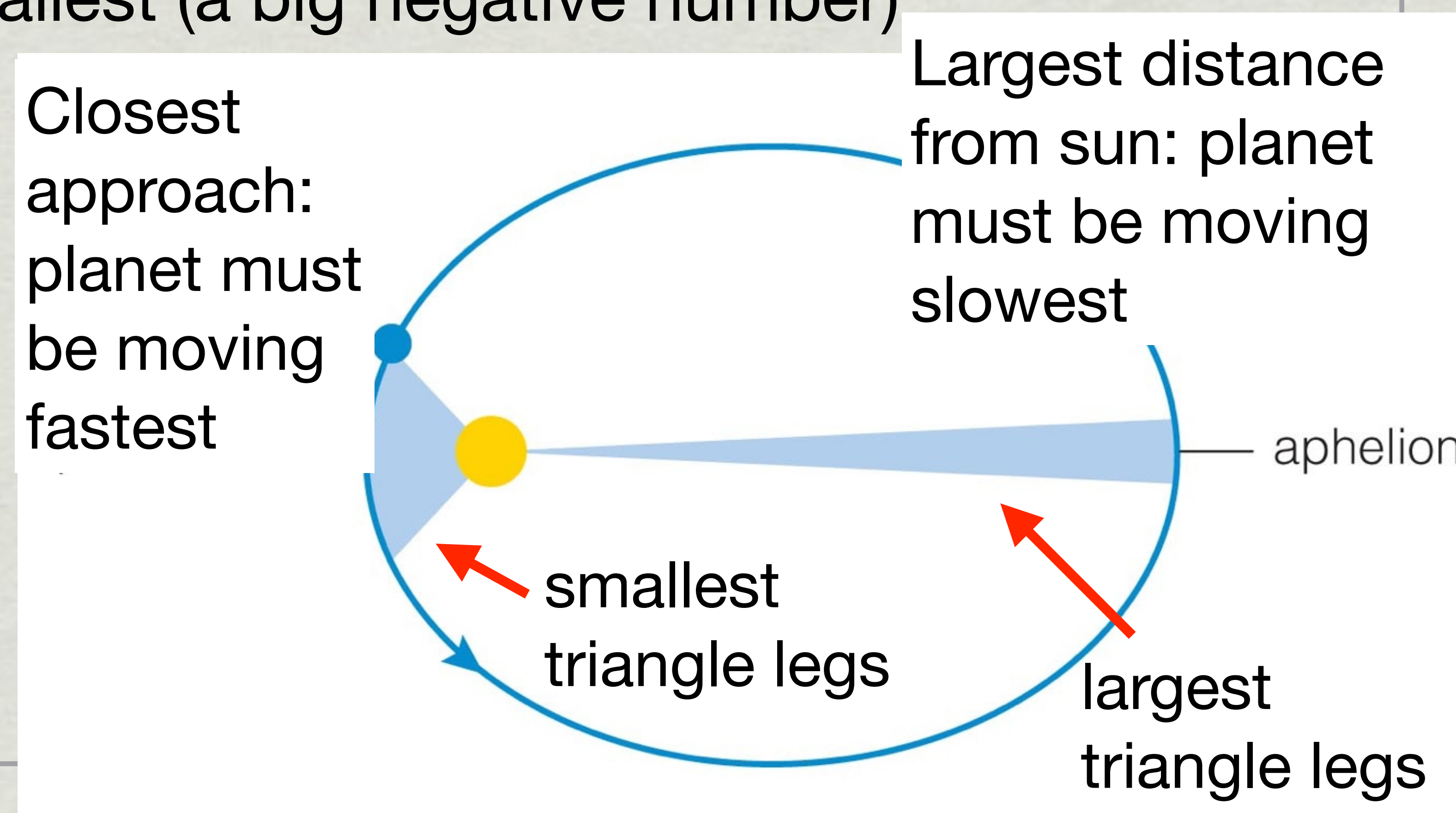
$$E_{kinetic} = \frac{m\ v^2}{2}$$ is largest

At aphelion: largest distance

$$E_{grav\ potential} = \frac{-\ GM_1M_2}{d}$$

is largest (least negative)

$$E_{kinetic} = \frac{m\ v^2}{2}$$ is smallest



Escape Speed

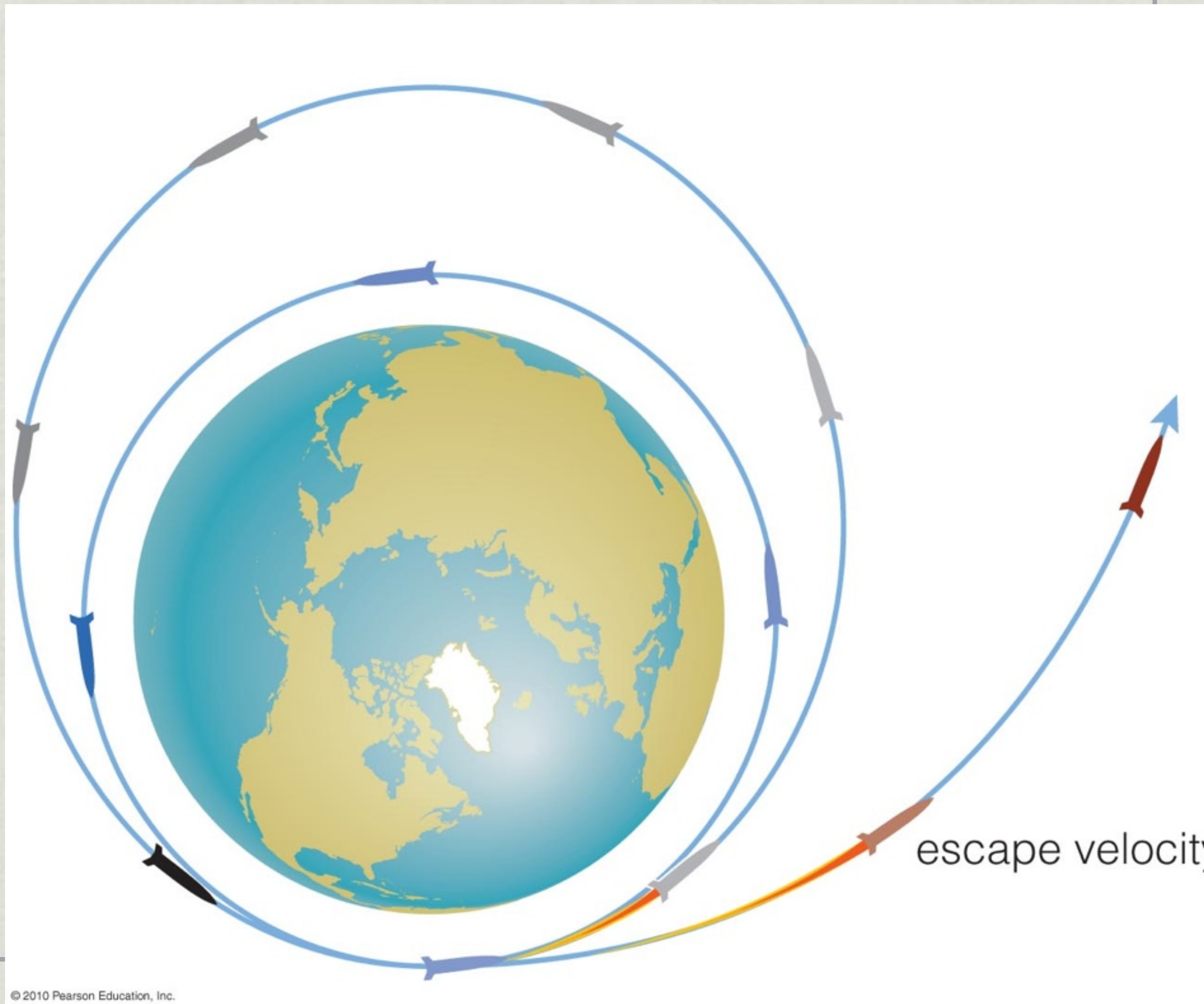
Kinetic Energy + Gravitational Potential Energy = 0 for an orbit that just barely has a large enough velocity to escape the gravitational pull of a planet (or sun, etc.)

KE PE

$$\left(\frac{m\,v^2}{2}\right) + \left(-\frac{GM\,m}{R}\right) = 0$$

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

Chapter 3.8 in the book tells you that formula, we can see where it comes from



Conservation of Energy and Orbits

Total Energy = $E_{kinetic} + E_{grav\ potential}$ = Constant

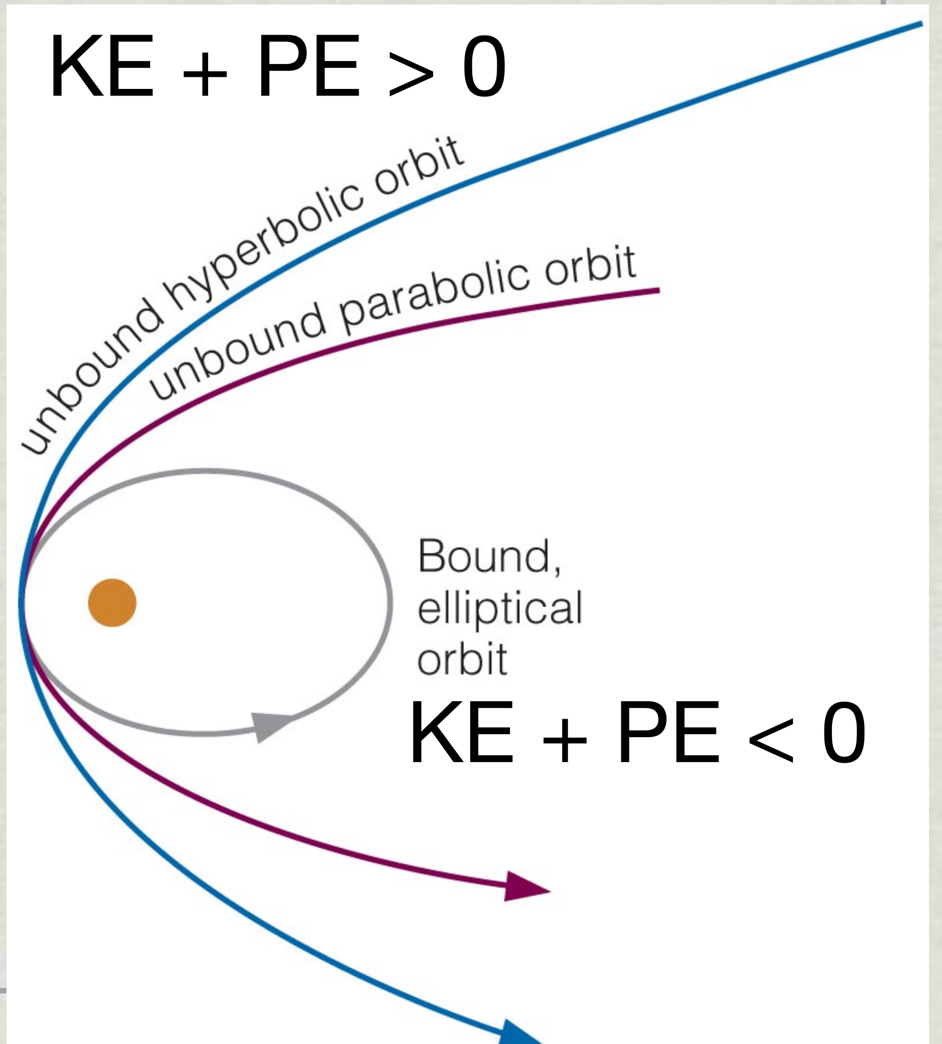
Orbits are defined by the total energy of the object in orbit.

Bound Orbit: closed ellipse, object stays with the thing it is orbiting

Kinetic Energy + Potential Energy < 0

Kinetic Energy $\left(\frac{m\ v^2}{2}\right)$

Gravitational Potential Energy $\left(-\frac{GM\ m}{R}\right)$



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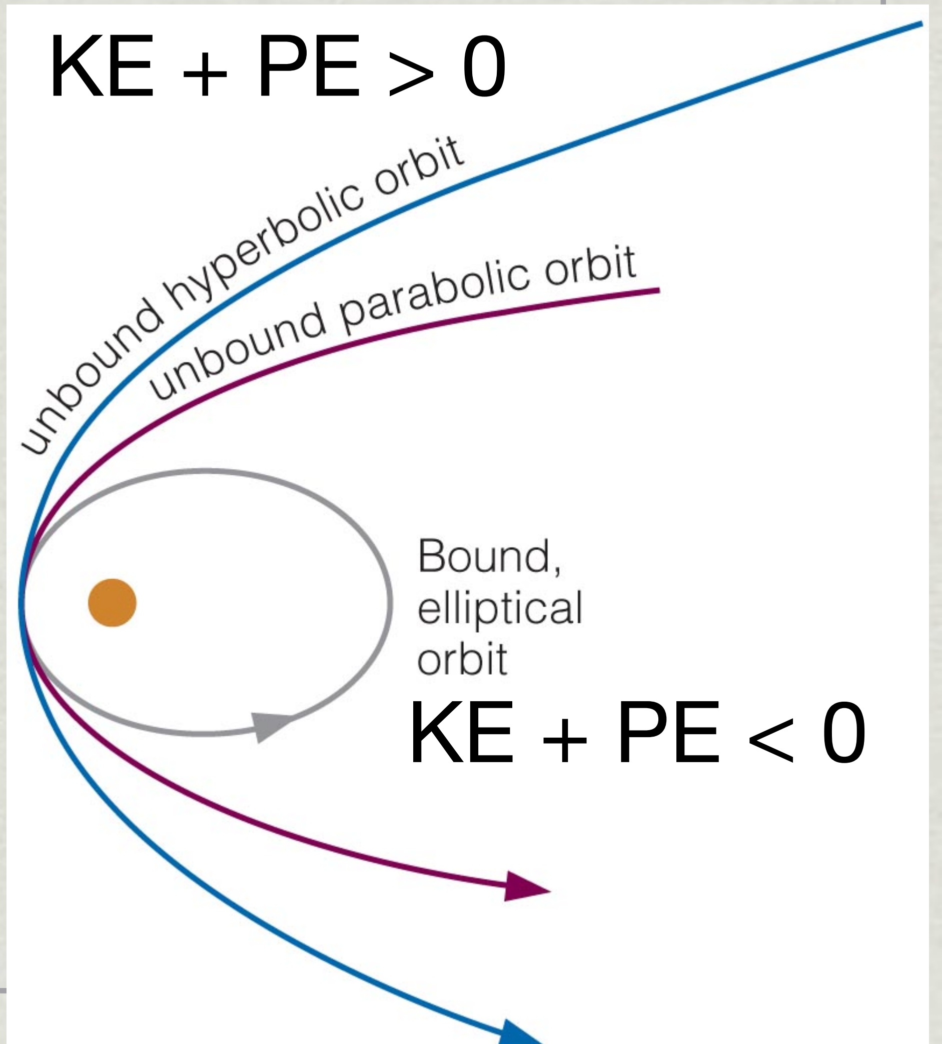
Potential Energy



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Potential Energy

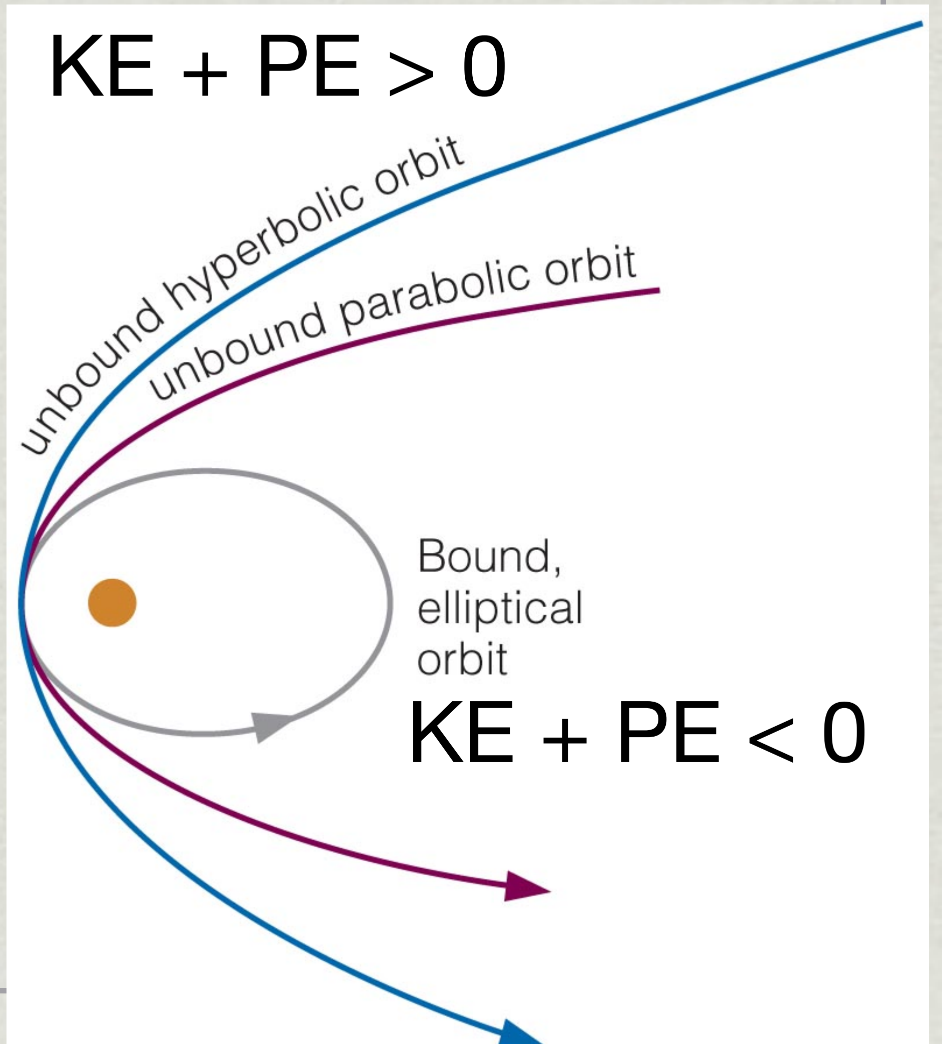
$$\left(\frac{m\,v^2}{2}\right) + \left(-\frac{GM\,m}{R}\right) < 0$$

$$\left(\frac{m\,v^2}{2}\right) < \left(\frac{GM\,m}{R}\right)$$

$$\frac{v^2}{2} < \frac{GM}{R}$$

$$v^2 < \frac{2GM}{R}$$

$$v < \sqrt{\frac{2GM}{R}}$$



Escape Speed

Kinetic Energy + Potential Energy = 0 for an orbit that is just barely unbound:

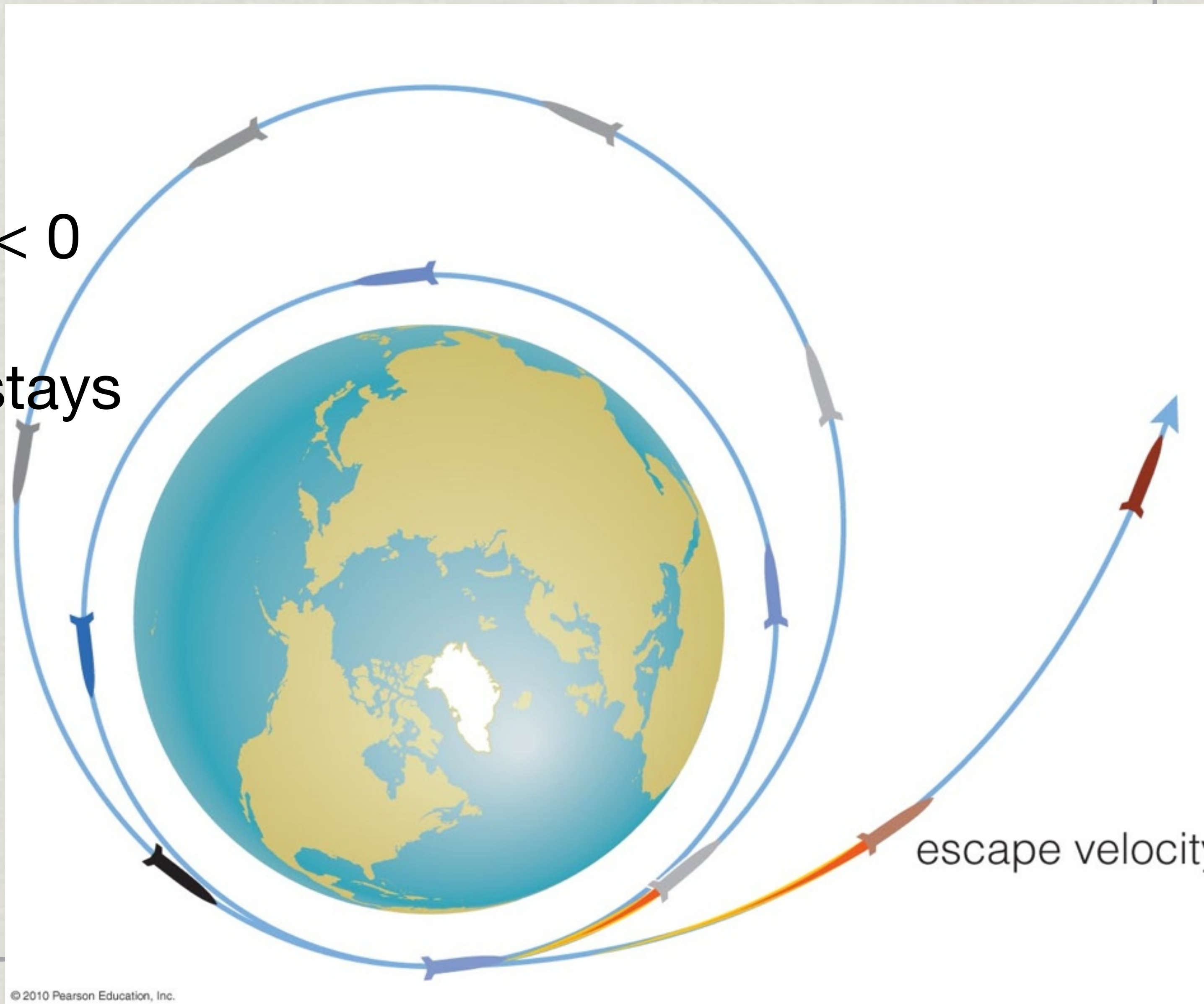
KE PE

$$\left(\frac{m\,v^2}{2}\right) + \left(-\frac{GM\,m}{R}\right) < 0$$

If Kinetic Energy + Potential Energy < 0

Bound Orbit: closed ellipse, object stays with the thing it is orbiting

$$v < \sqrt{\frac{2GM}{R}}$$



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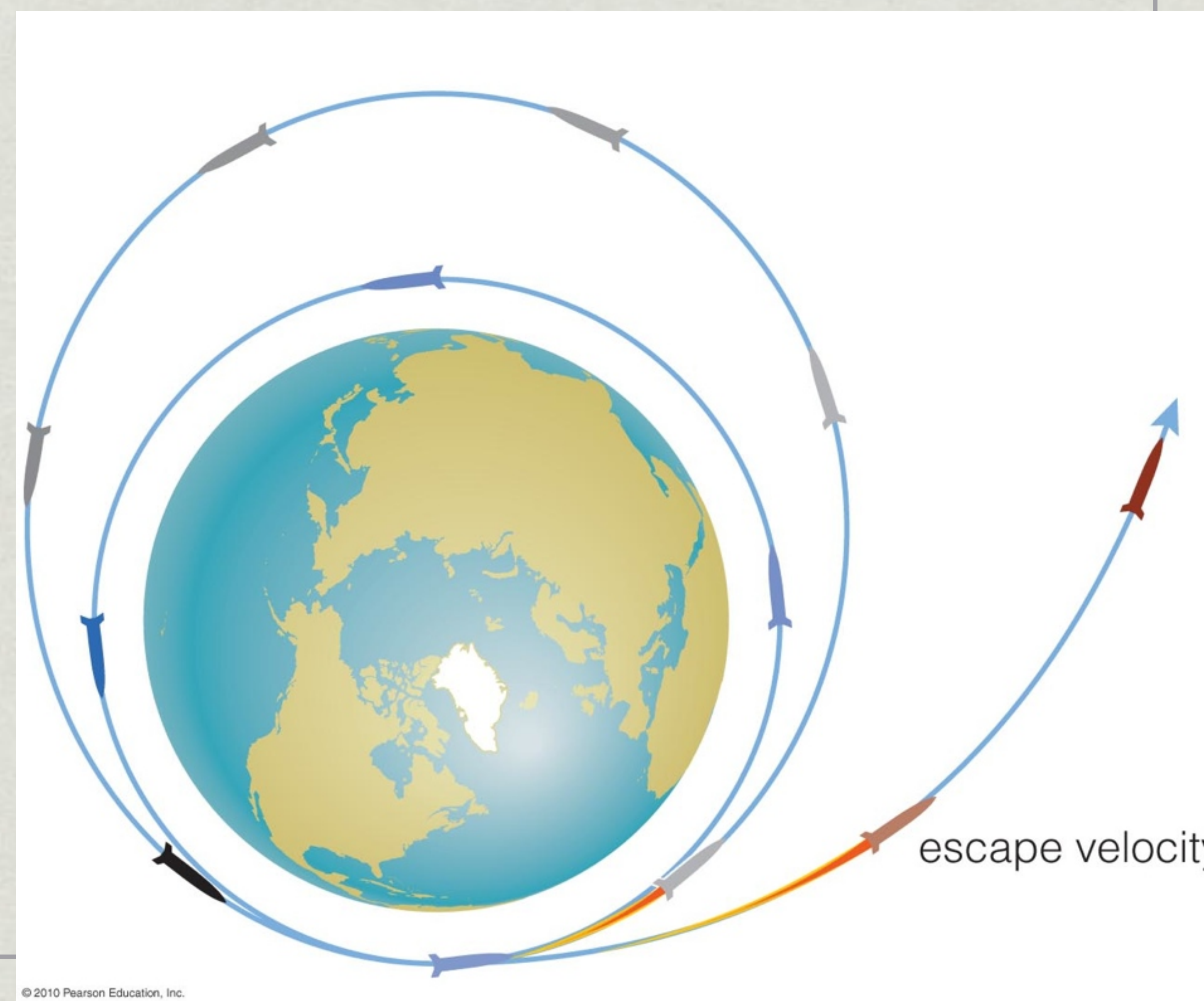

Energy and Orbits

$$v_{escape} = \sqrt{\frac{2\,G\,M}{d}}$$

escape speed for an object at distance d from mass M

Question:

You are building a rocket to send the next rover to Mars.
If the cost of the rocket depends on its escape speed from the Earth, where should you build the rocket?

A Earth
B the International Space Station

Energy and Orbits

$$v_{escape} = \sqrt{\frac{2\,G\,M}{d}}$$

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From last time: Orbits and Circular Motion

- Combine:

Acceleration required to keep an object in circular motion: $\frac{v^2}{d}$

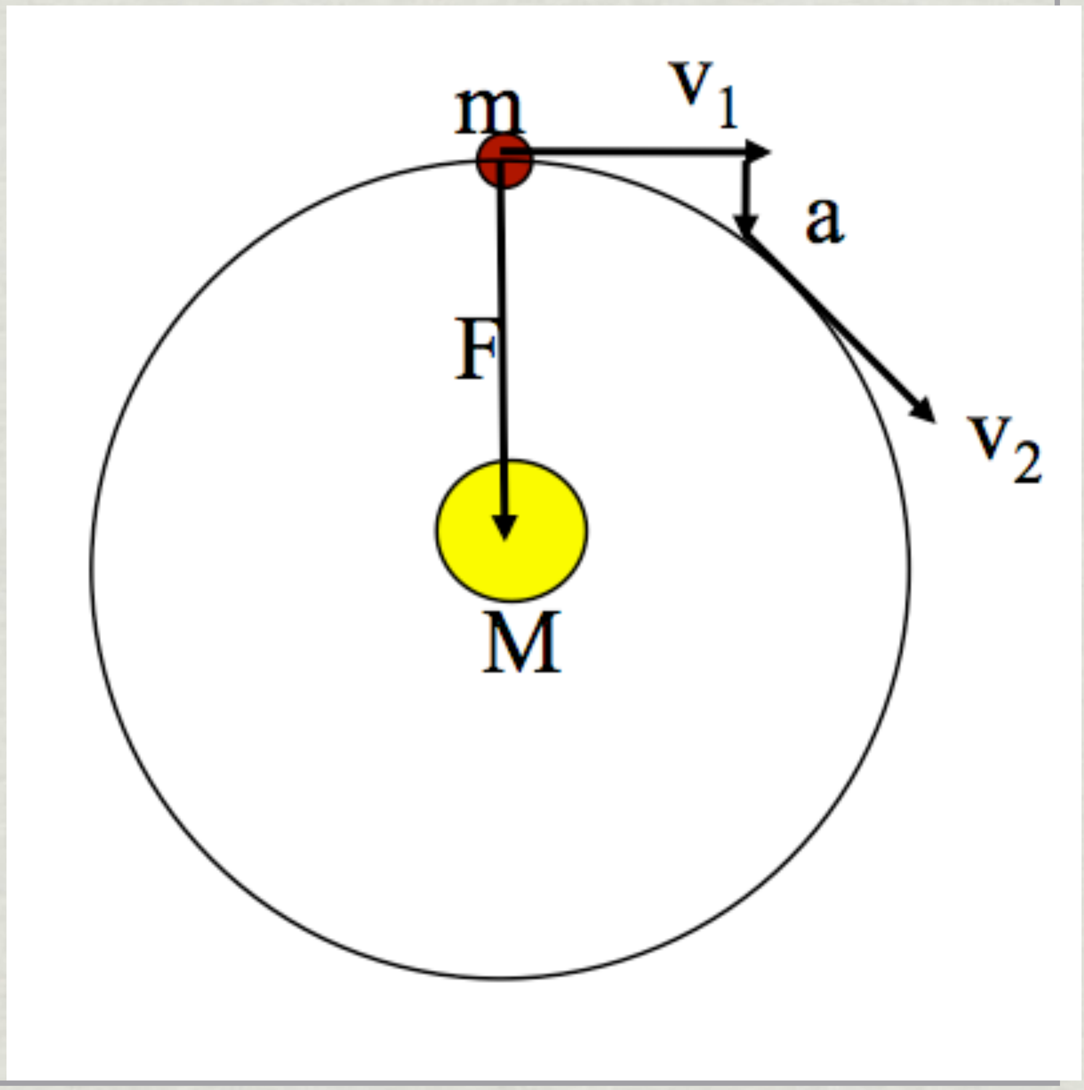
with acceleration from Gravity $= \frac{G\,M}{d^2}$

$$\frac{v^2}{d} = \frac{G\,M}{d^2}$$

$$v = \sqrt{\frac{G\,M}{d}}$$

v = speed for an object in stable circular motion around mass M at distance d

For an orbit, larger d = smaller v



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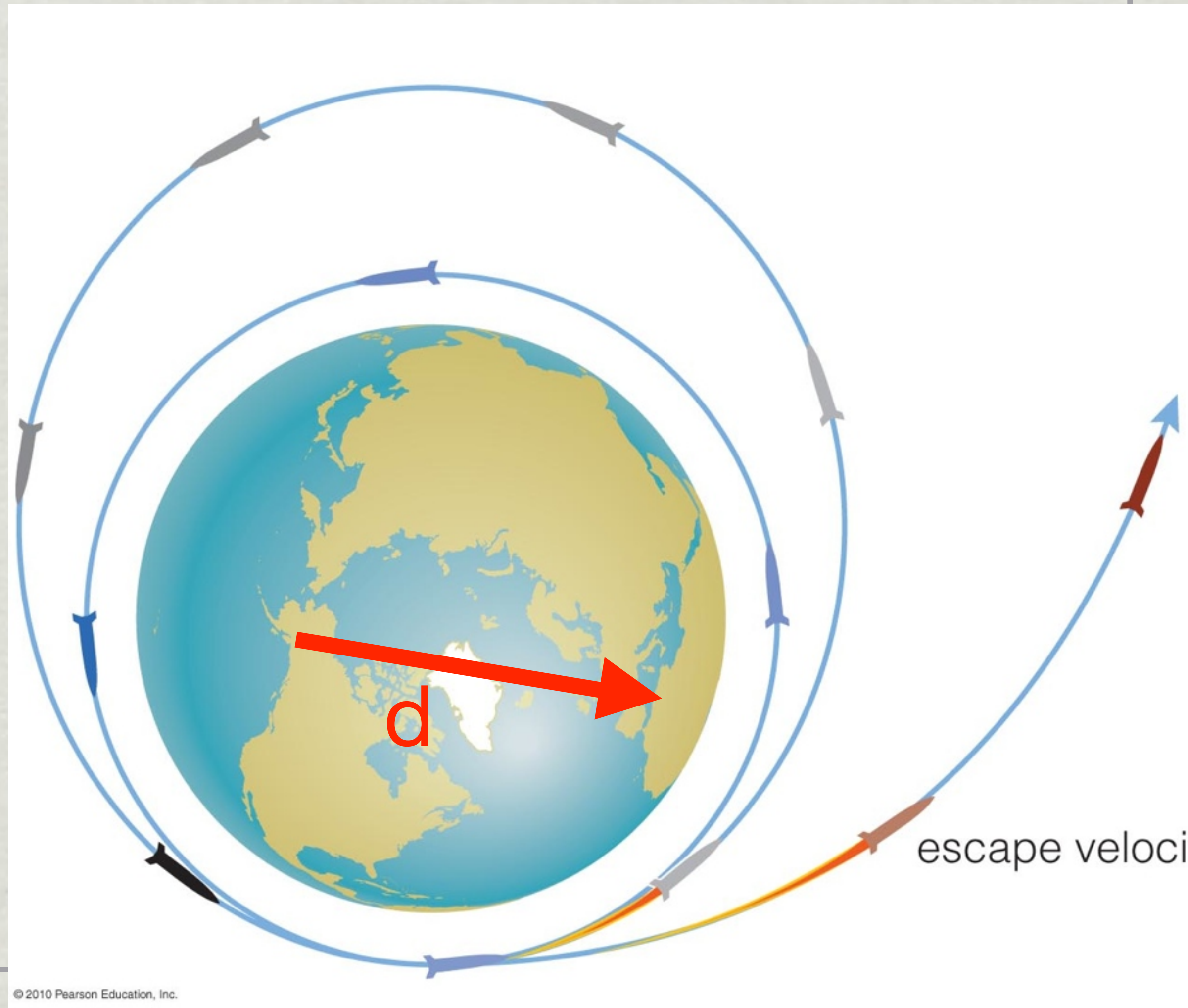
For an object in orbit at d, increase v by $\sqrt{2}$ and it will leave its orbit and escape.



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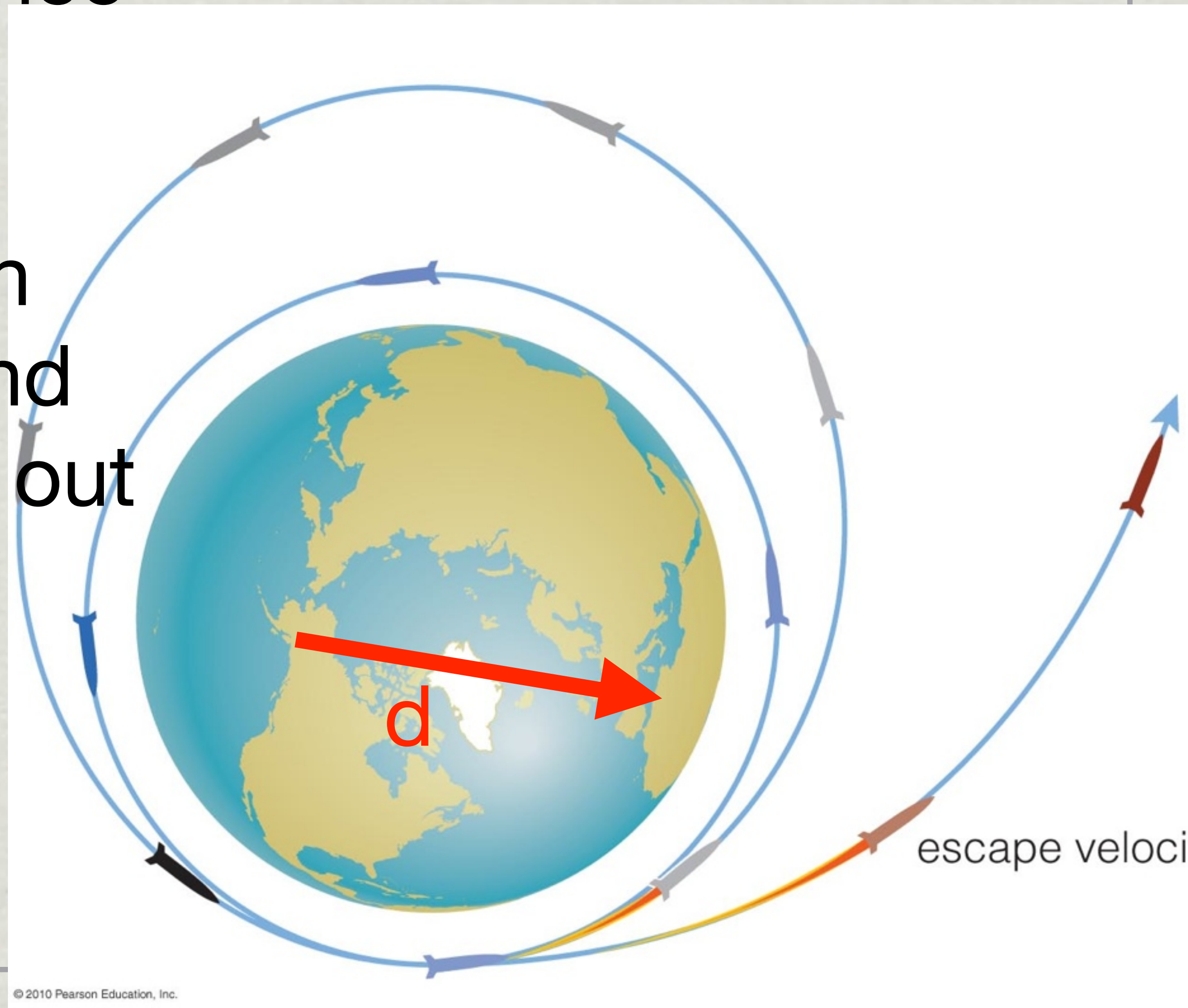
You are in orbit, with speed v at distance d from earth.

You turn on the rocket engine, point in the direction you are going in orbit and accelerate to increase v. But you run out of fuel before you increase v by √2.



Do you escape from earth?

A yes

B no

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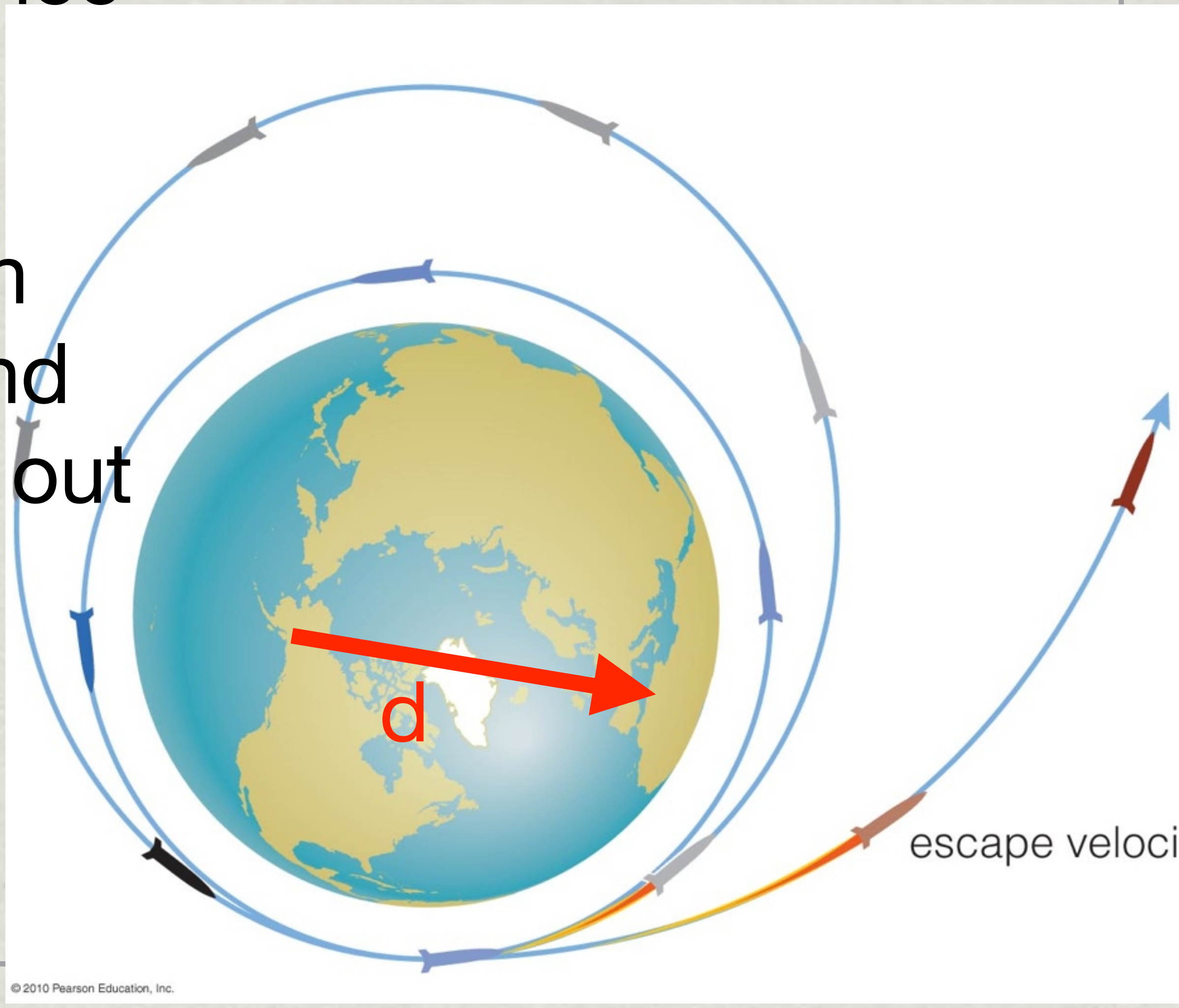

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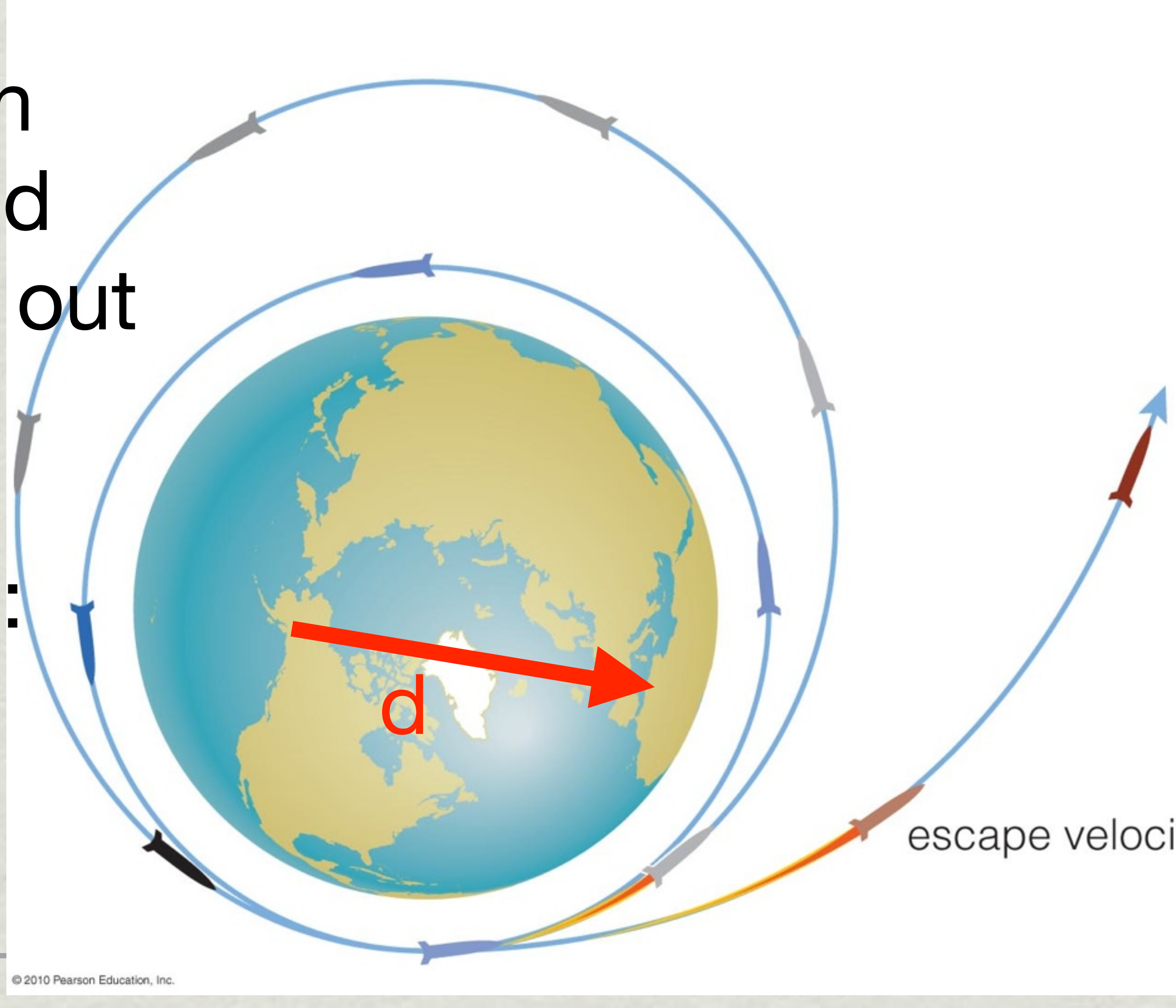
Your new distance d from the earth is:

A bigger

B smaller

C stays the same



Escape Speed

Kinetic Energy + Potential Energy = 0 for an orbit that is just barely unbound:

KE PE

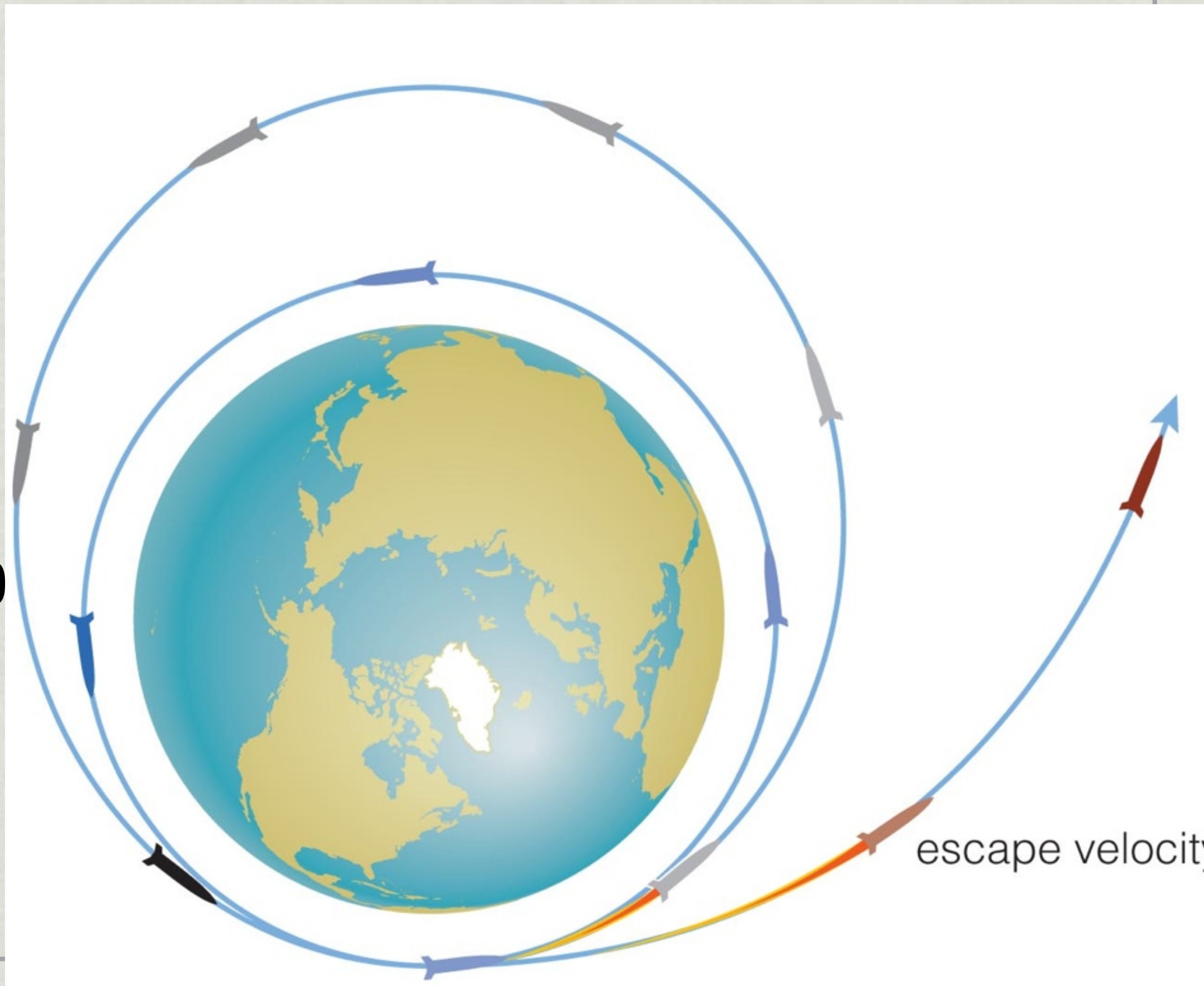
$$\left(\frac{m\,v^2}{2}\right) + \left(-\frac{GM\,m}{R}\right) = 0$$

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

Bound Orbit: closed ellipse, object stays with the thing it is orbiting

Kinetic Energy + Potential Energy < 0

$$v < \sqrt{\frac{2GM}{R}}$$



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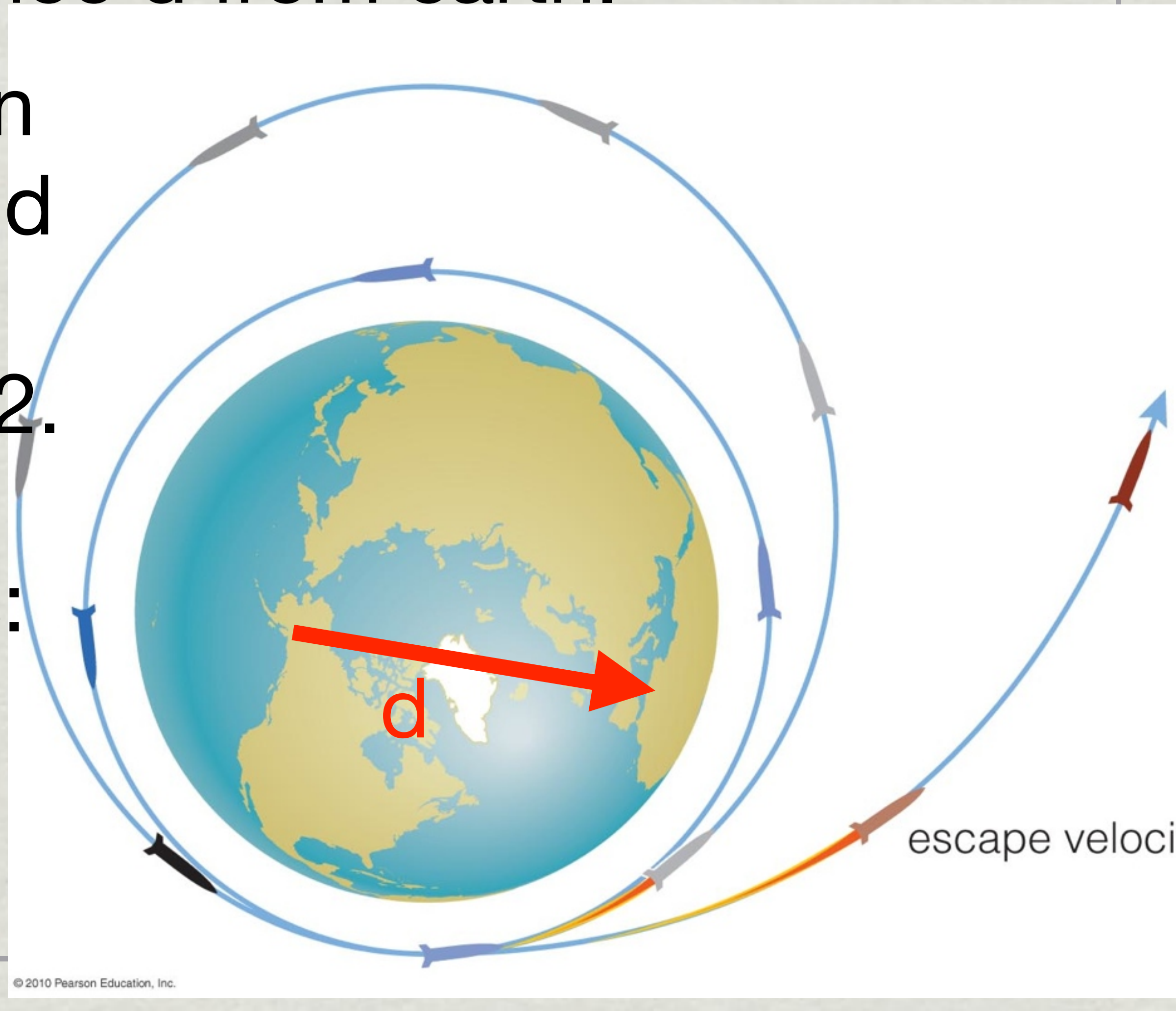

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After the rocket stops accelerating, your new velocity v is:

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B smaller

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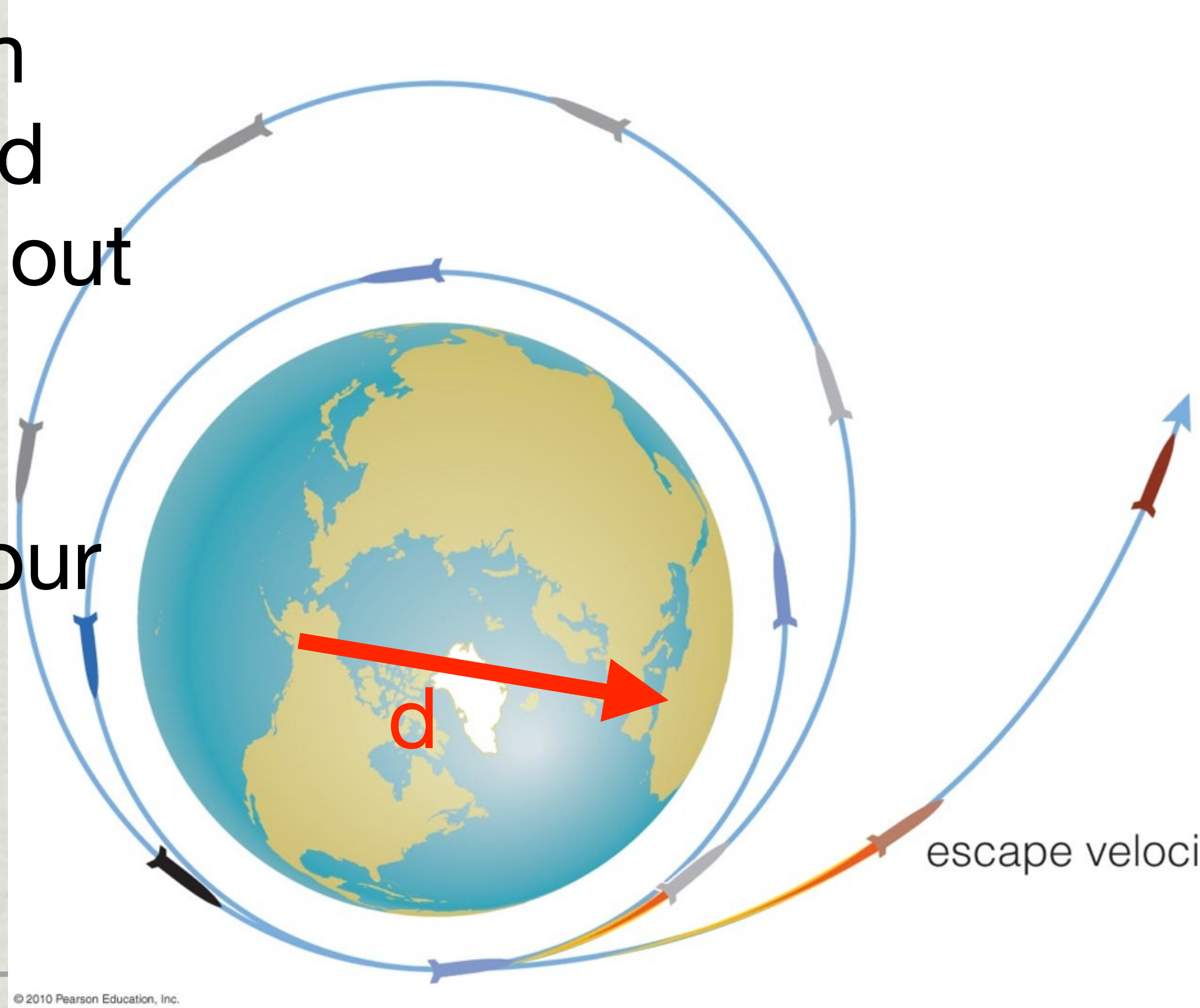

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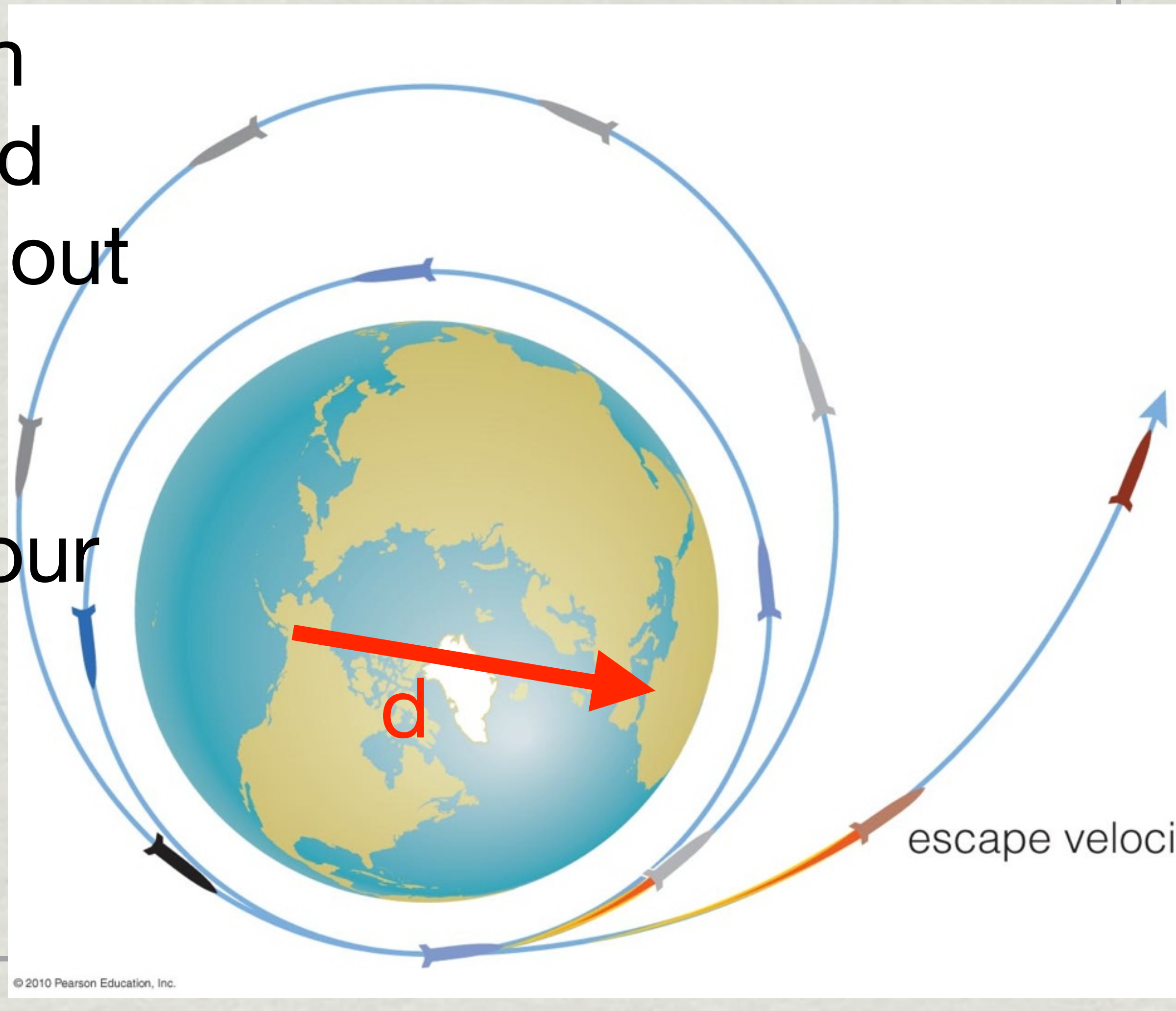

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Angular Momentum

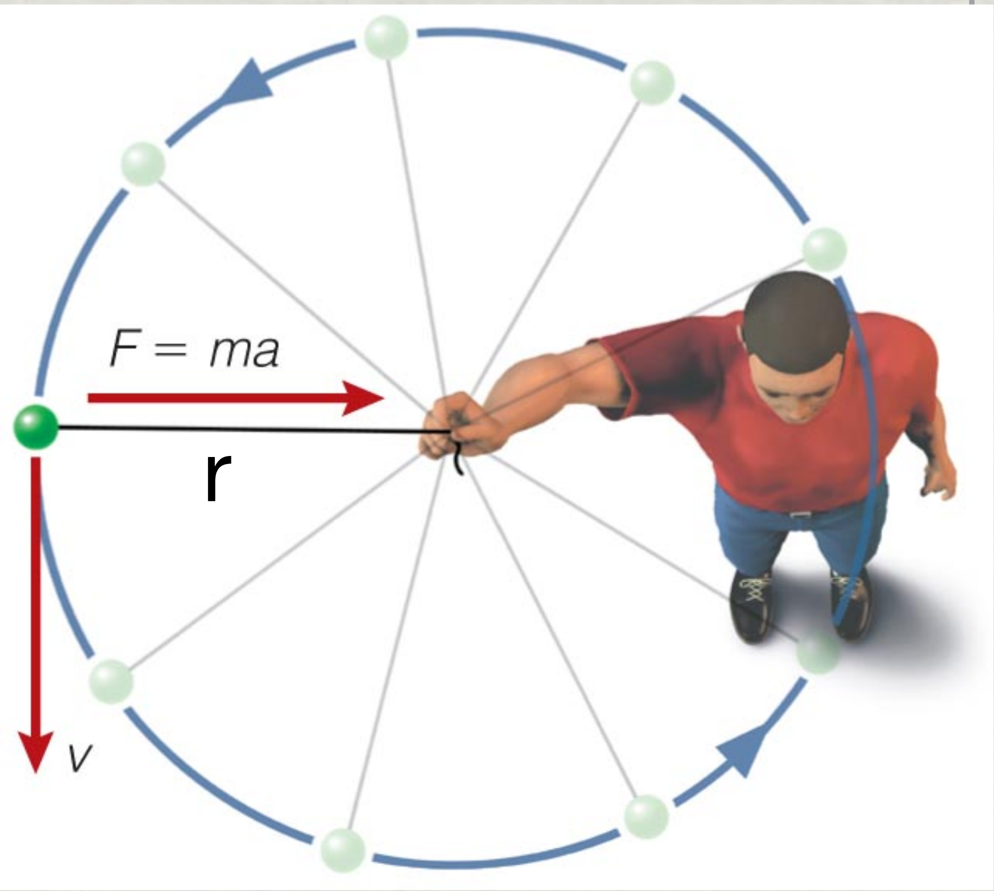
Momentum: mass x velocity

change in momentum requires force

Angular momentum: mass x velocity x radius

The angular momentum of an isolated system is conserved.

So is linear momentum, m x v



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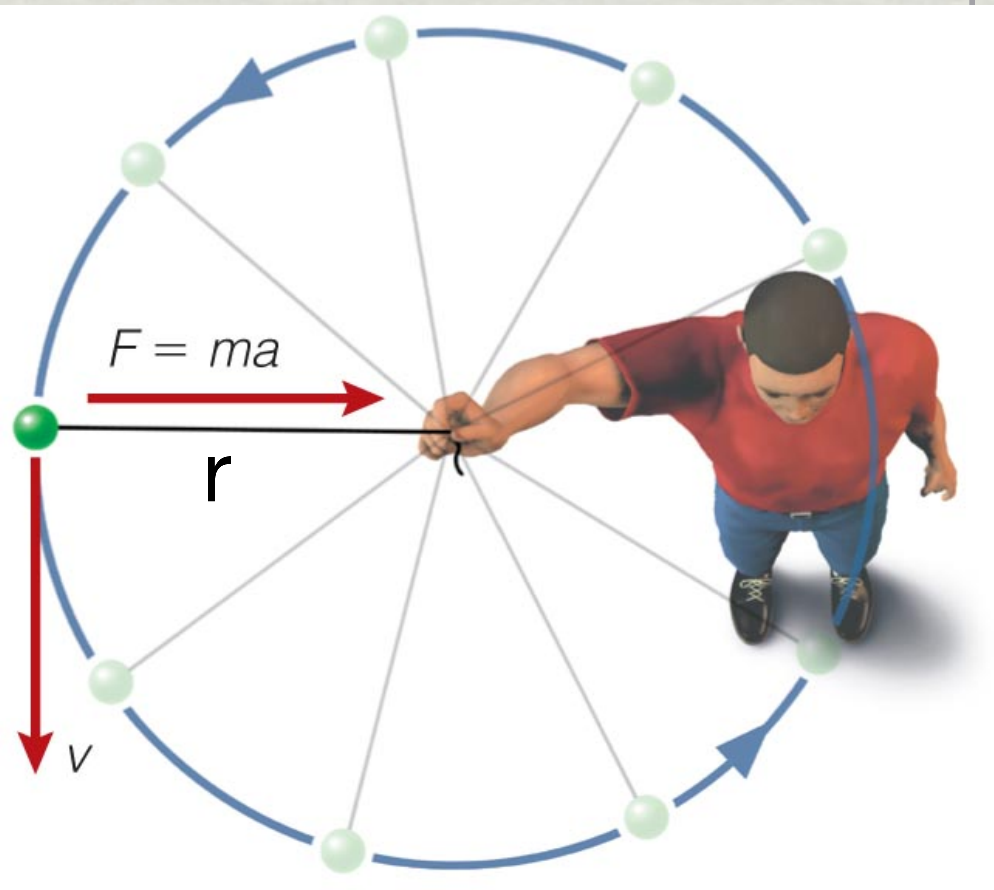
iClicker question:

If this is a closed system, what happens to the speed v as I make the string length r shorter?

A Stays constant, velocity is conserved

B Slower

C Faster



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Just like linear momentum.

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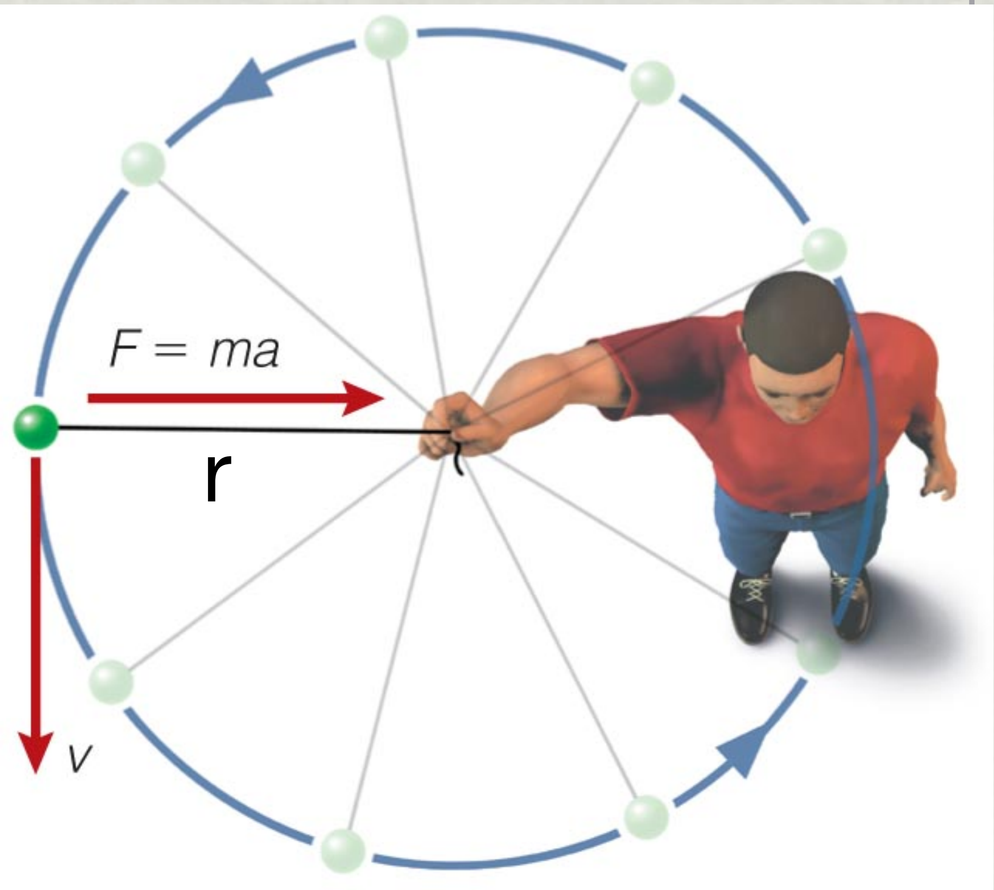

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Angular Momentum and Orbits

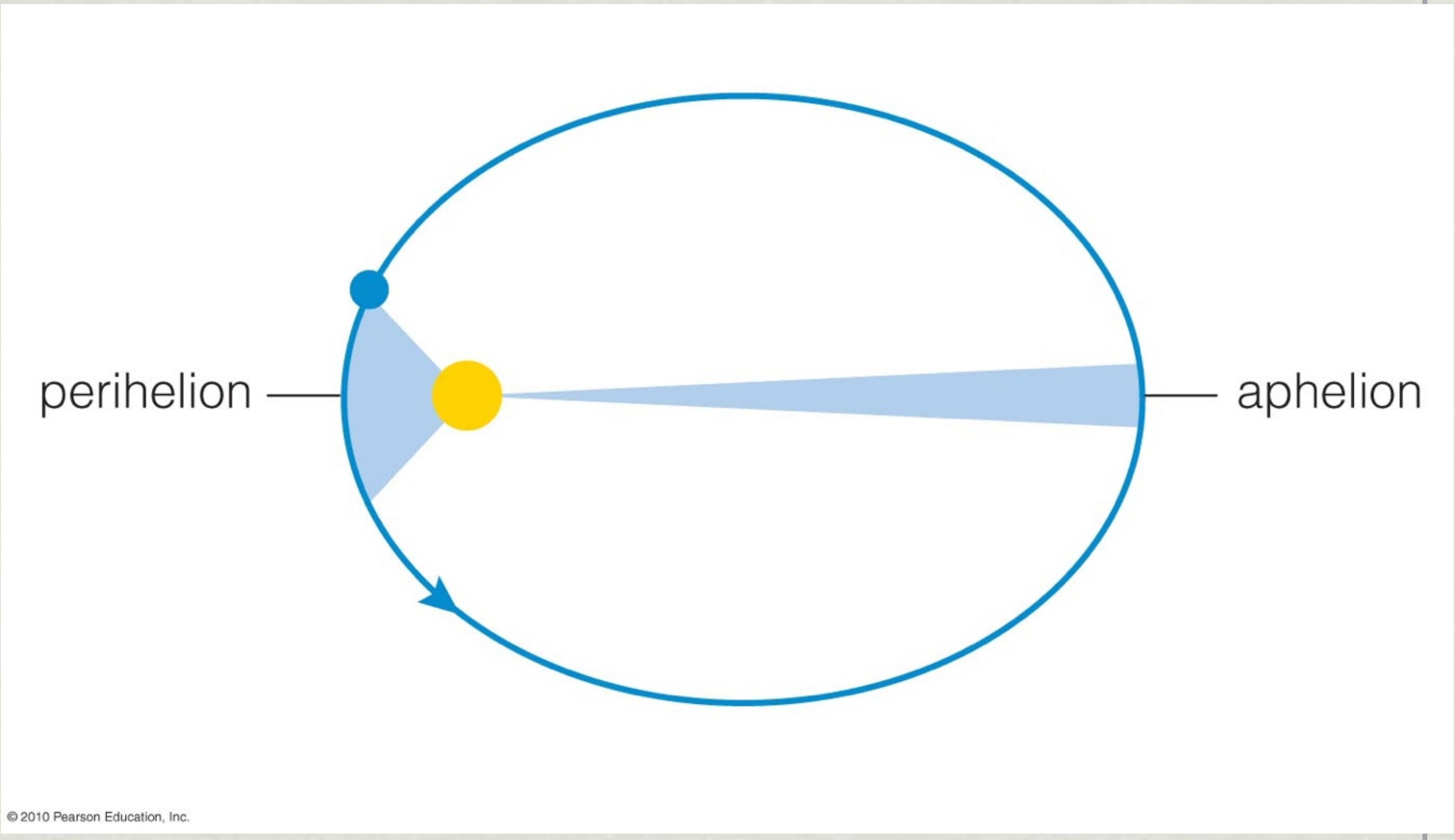
Kepler's 2nd Law: Planets sweep out equal area in equal time

Angular momentum: mass x velocity x radius

Assume the planet mass doesn't change:

At perihelion: smallest distance (radius)
 v must be largest

At aphelion: largest distance
 v must be smallest



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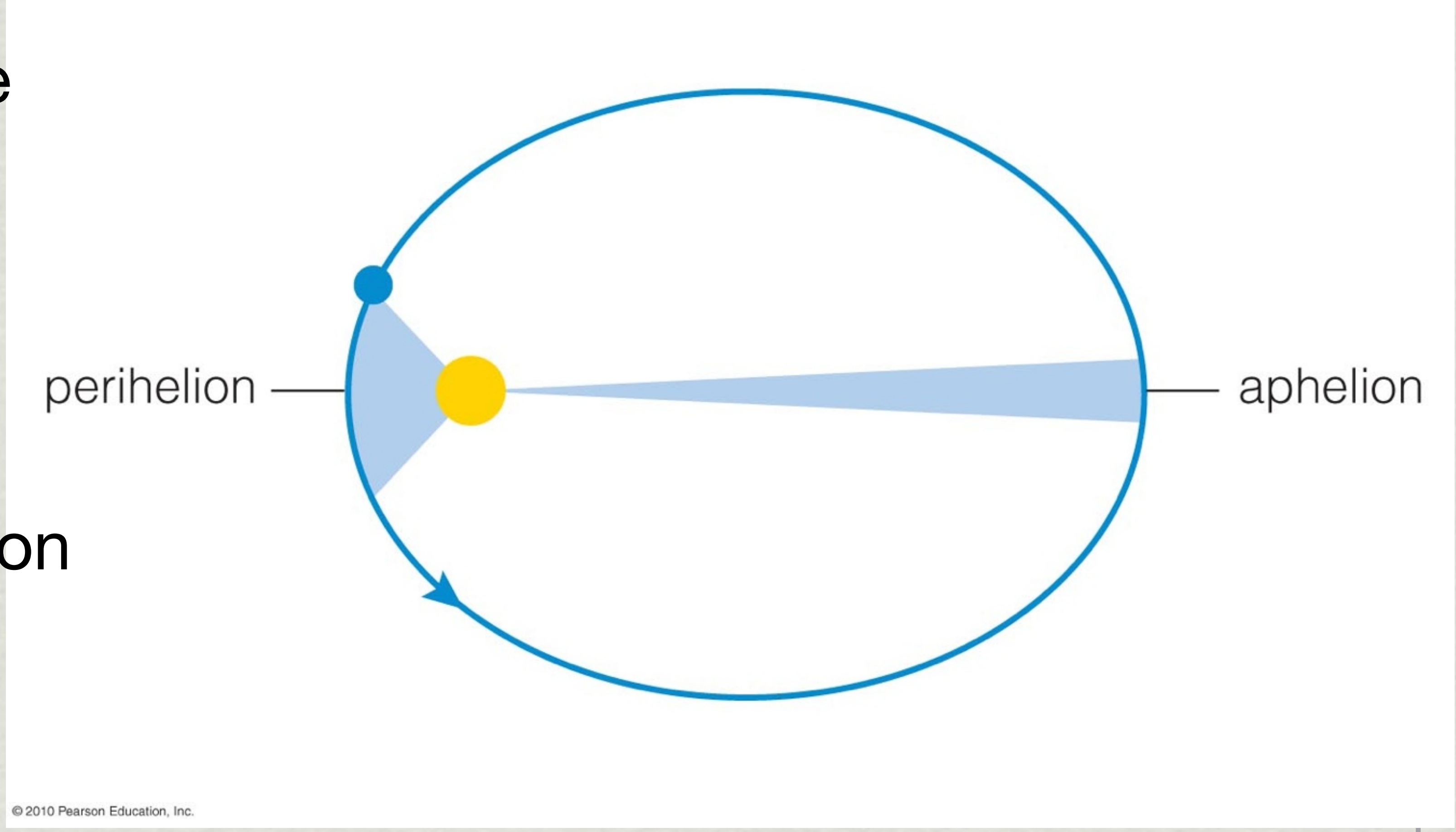


At aphelion: largest distance
 v must be smallest

We came to the same conclusion thinking about energy:

$$E_{grav\ potential} = -\frac{GM_1M_2}{d}$$

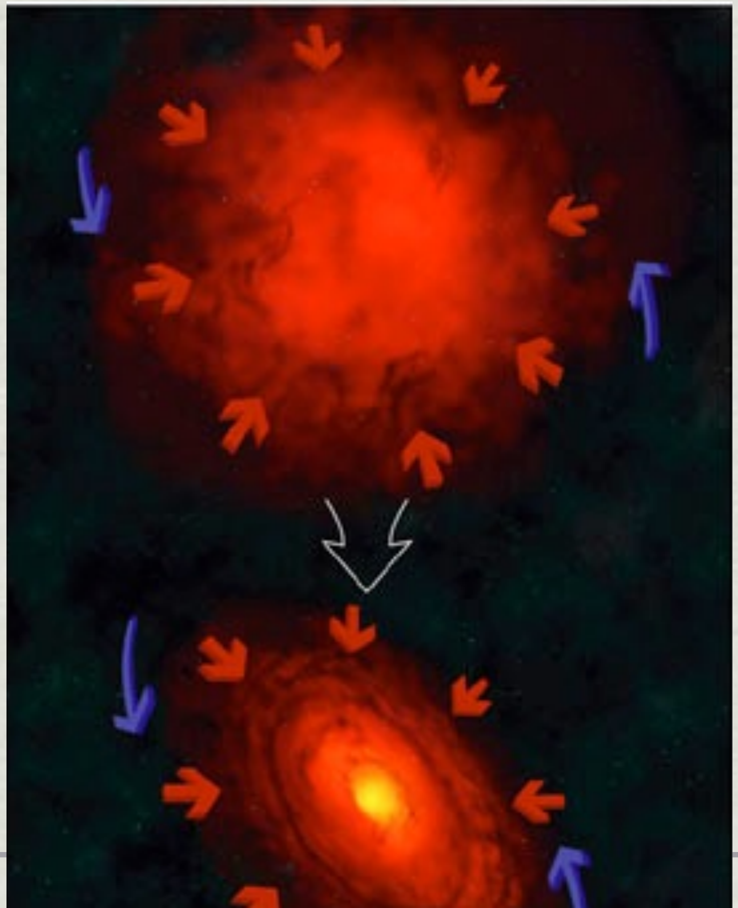
$$E_{kinetic} = \frac{m\,v^2}{2}$$

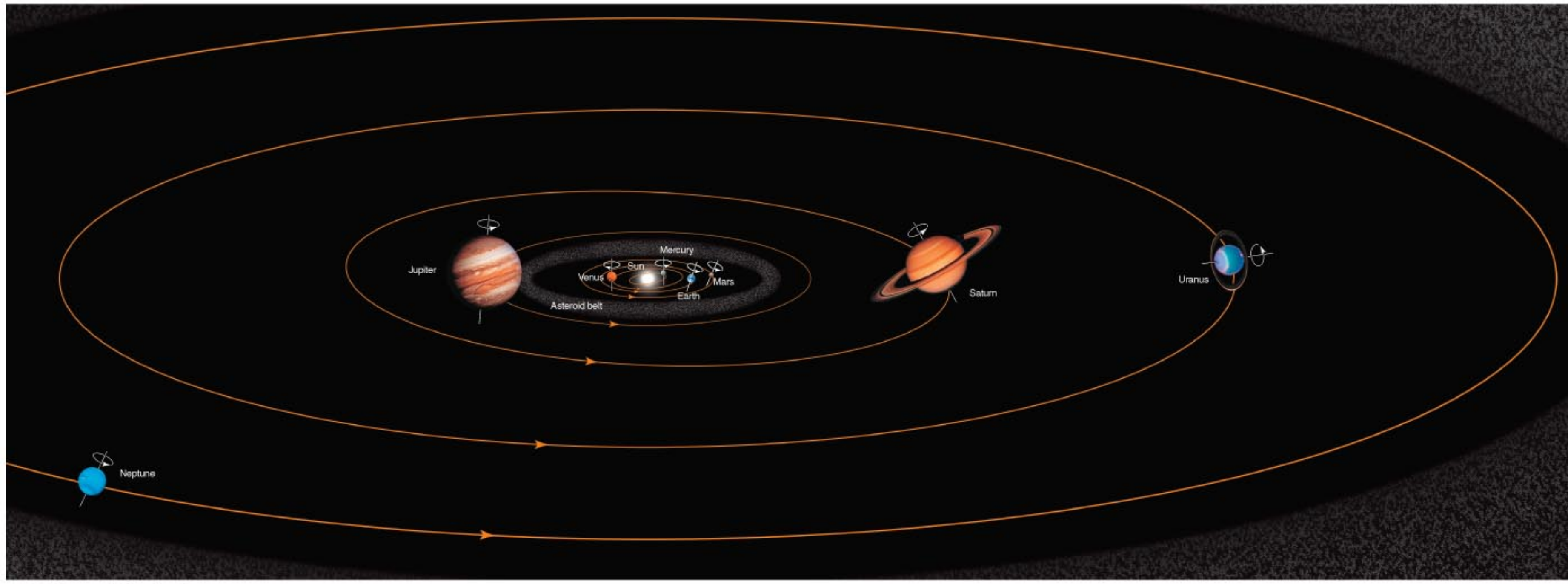
At perihelion, d is smallest so potential energy is smallest, kinetic energy must be largest -> velocity is largest

Angular Momentum

Galaxies and the solar system form when clouds of stuff, mostly hydrogen, collapse due to their own gravitational force

The clouds are very large. As they collapse, they become smaller. Conserve angular momentum → radius goes down, so v must go up. Gas orbits faster, "spins up", becomes flat, disk-like.





Angular Momentum and Galaxies