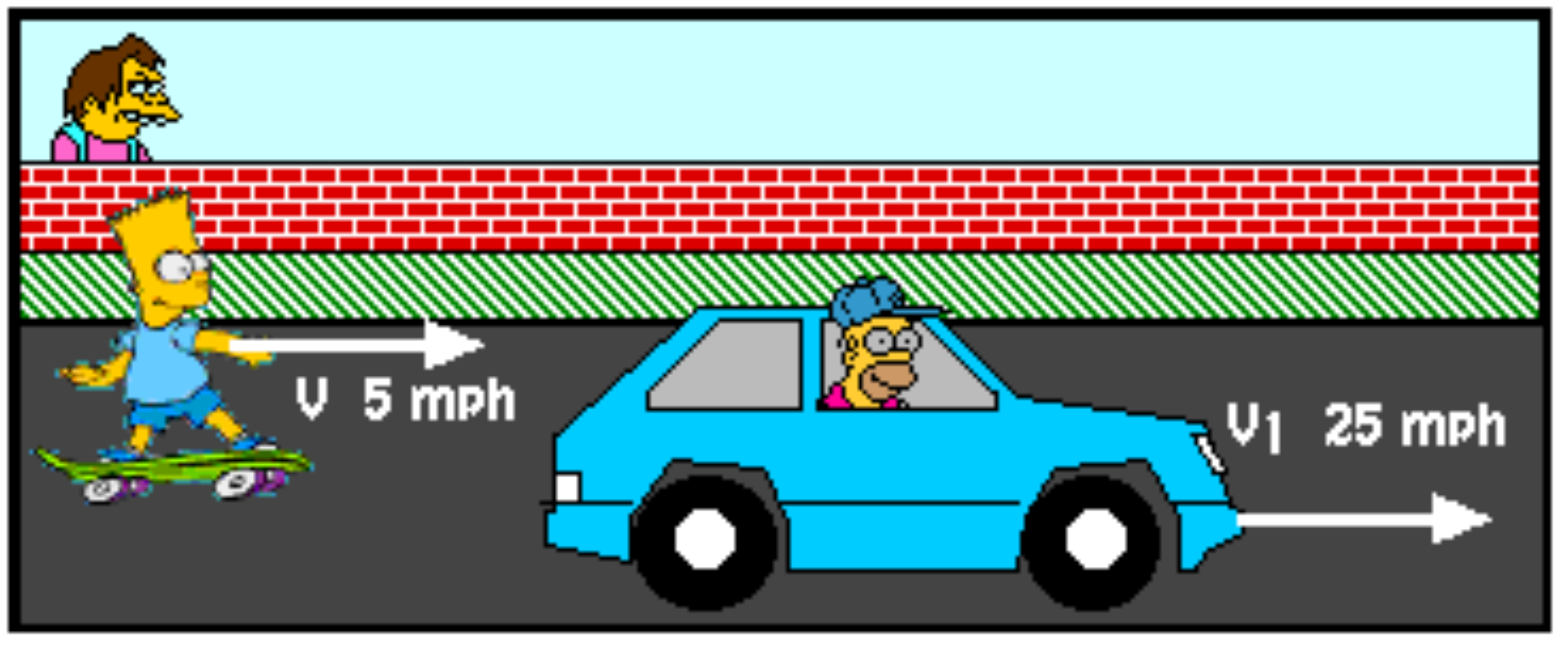
Announcements

- Midterm in-class Tuesday, 2/14
  - Content: everything through lecture 2/7, homework due 2/9 (Chapters 1,2,3,4)
  - You will get a formula sheet and all numbers you need
  - Closed book and notes.
  - Bring a pencil and a non-web-enabled calculator
  - Best practice is to review the homework problems and reading assignments
- Midterm review sessions:
  - Friday, Feb. 10 4-5 pm NatSci2 Annex 101 (Plato)
  - Monday, Feb. 13 4-5pm NatSci2 Annex 101 (Marie)
  - Neither is required, you can go to either or both

Before we get to Special Relativity, let's talk about the regular stuff:

Galilean "Everyday" Relative Motion



Overalls-guy sees what we see: skateboarder moving right at 5 mph
car moving right at 25 mph

Skateboarder sees: overalls-guy moving 5 mph to the left
car moving to the right at 25 - 5 = 20 mph

Driver sees: overalls-guy moving left at 25 mph
skateboarder moving left at 20 mph

Wave Intuition: Water Waves

Water waves move through water at the same speed (relative to the water) regardless of the speed or direction of the boat that makes them. Wave speed relative to water just depends on wavelength.

Wave speed we observe depends on our speed relative to the water

The boat is traveling at 10 miles per hour.
The speed of the waves in the water is 3 miles per hour.
Assume there is no current; the water is still other than the wake of the boat.

If you are sitting on the rock, what speed do you measure for the waves?

A 10 miles/hour
B 3 miles/hour
C 13 miles/hour
D 7 miles/hour

Water waves move through water at the same speed regardless of the speed or direction of the boat that makes them. The wave speed we observe depends on our speed relative to the water

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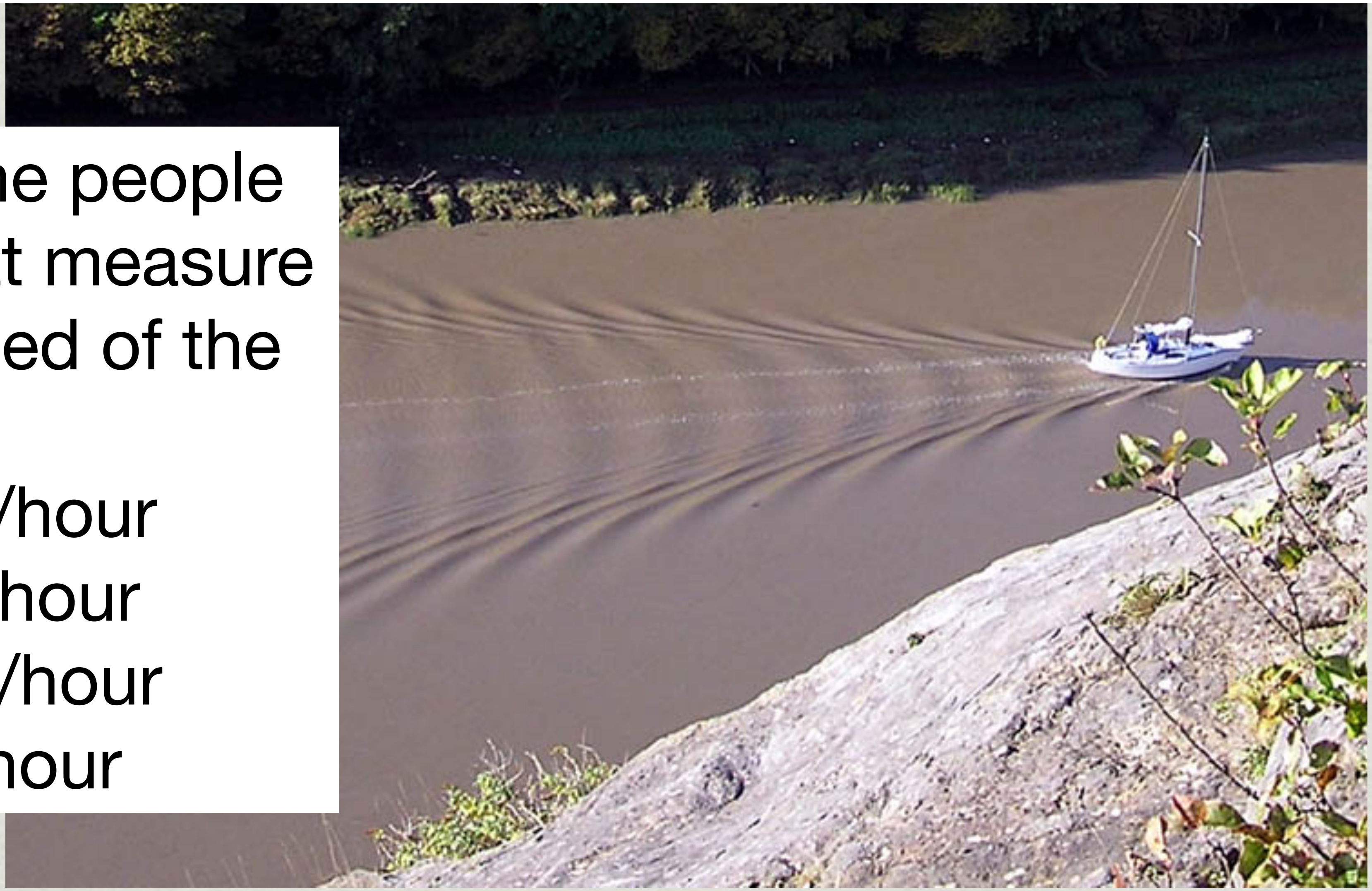

What do the people on the boat measure for the speed of the waves:

A 10 miles/hour
B 3 miles/hour
C 13 miles/hour
D 7 miles/hour

Water waves move through water at the same speed regardless of the speed or direction of the boat that makes them. The wave speed we observe depends on our speed relative to the water

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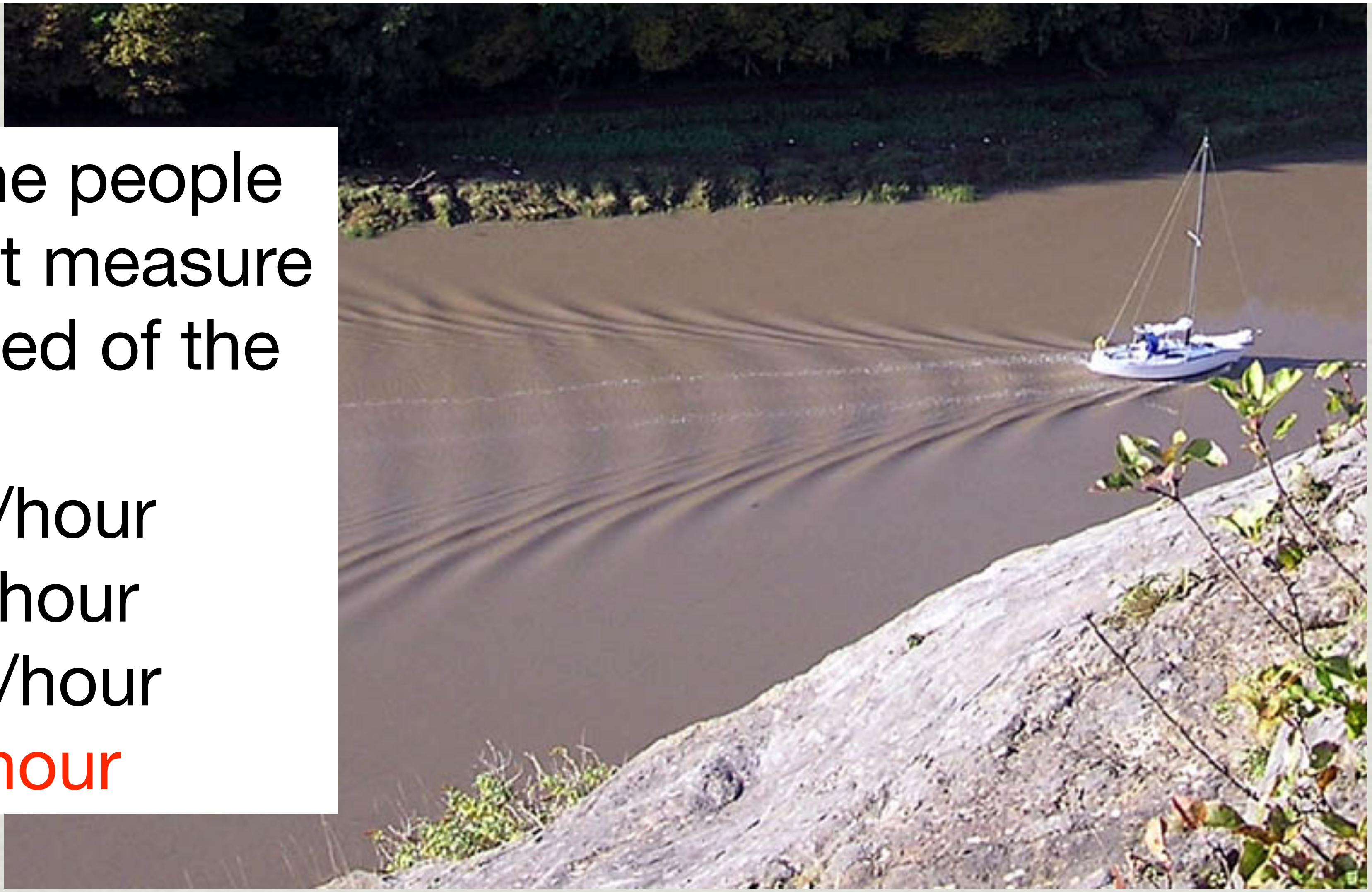

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A 10 miles/hour
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C 13 miles/hour
D 7 miles/hour

Water waves move through water at the same speed regardless of the speed or direction of the boat that makes them. The wave speed we observe depends on our speed relative to the water

What about Light?

We’ve said before that the speed of light is “constant”. Known for a long time thanks to some nice observations going back to Galileo.

double star animation

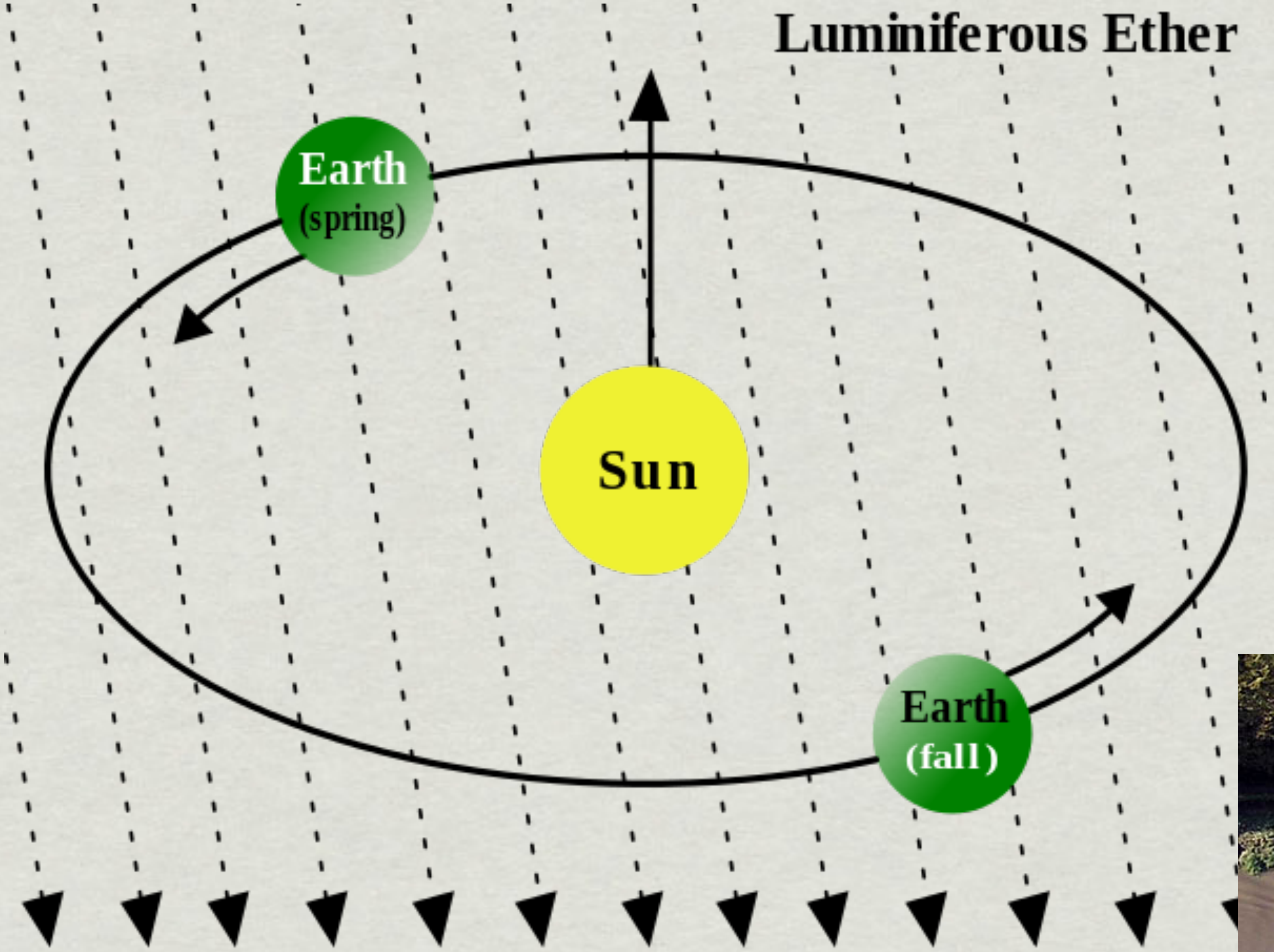
What about Light?

We've said before that the speed of light is "constant". Known for a long time thanks to some nice observations going back to Galileo.

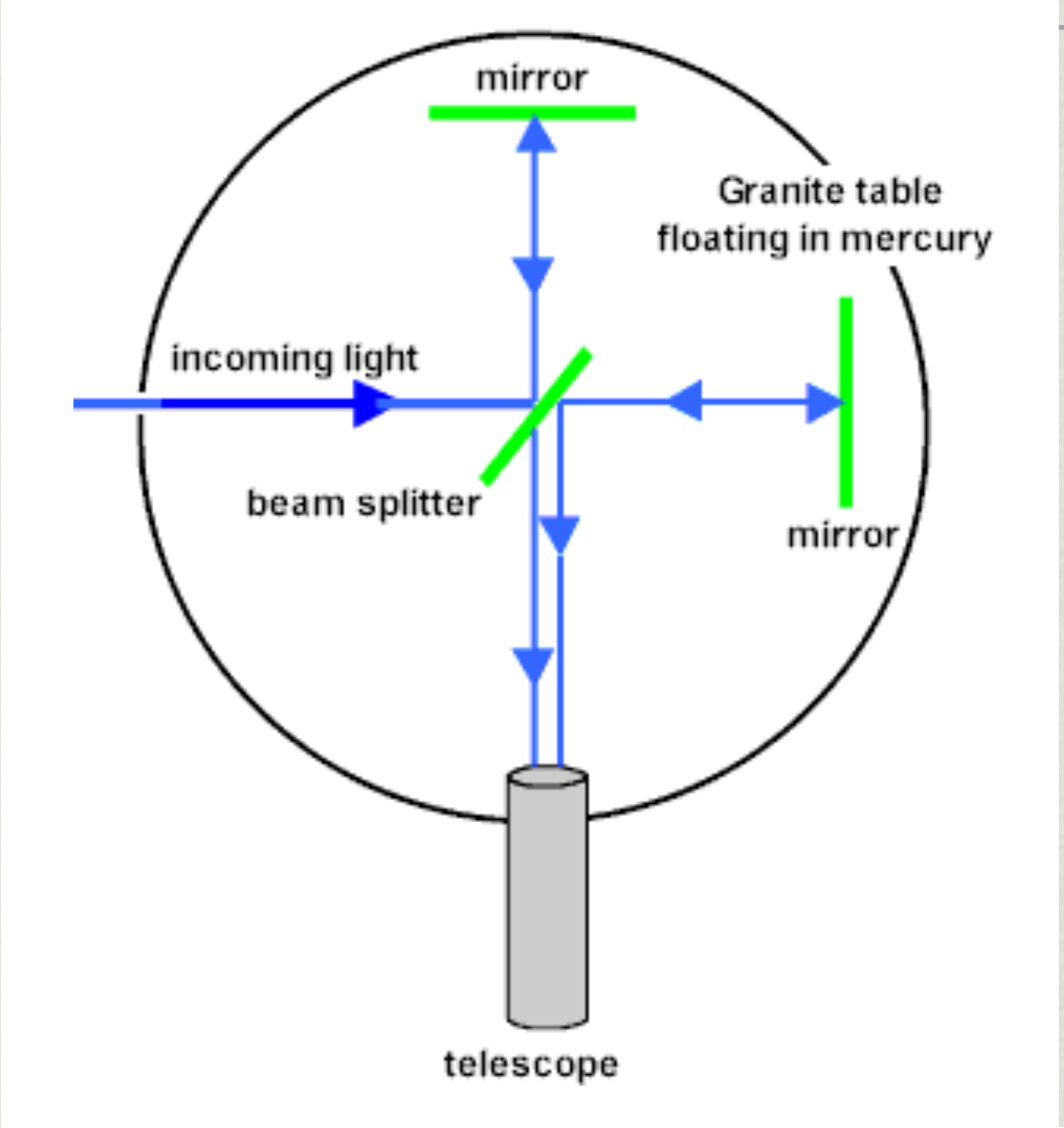
What does that mean? Constant in some goop that light propagates in?
    Like water waves in water?
    If so, we should measure different speeds for light if we move relative to the goop.

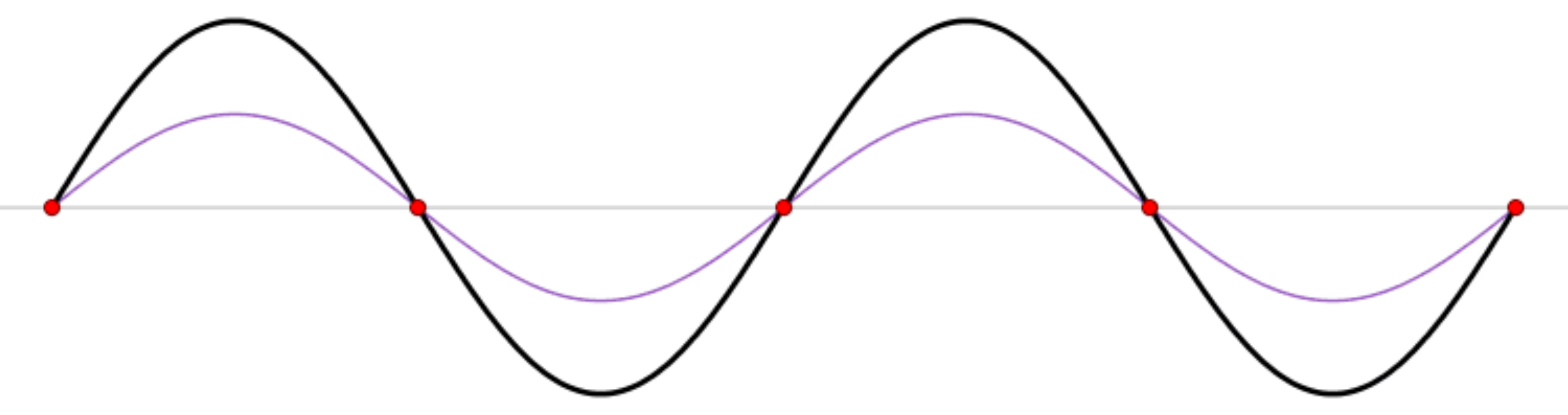
Goop was called "aether"
Experiments in the 1800's to measure the change in the speed of light as earth moved relative to the aether.

Trying to Measure the Aether
Michelson and Morley, 1887



Does light propagate in the aether like water waves?


mirror
Granite table
floating in mercury
incoming light
beam splitter
mirror
telescope



Trying to Measure the Aether
Michelson and Morley, 1887

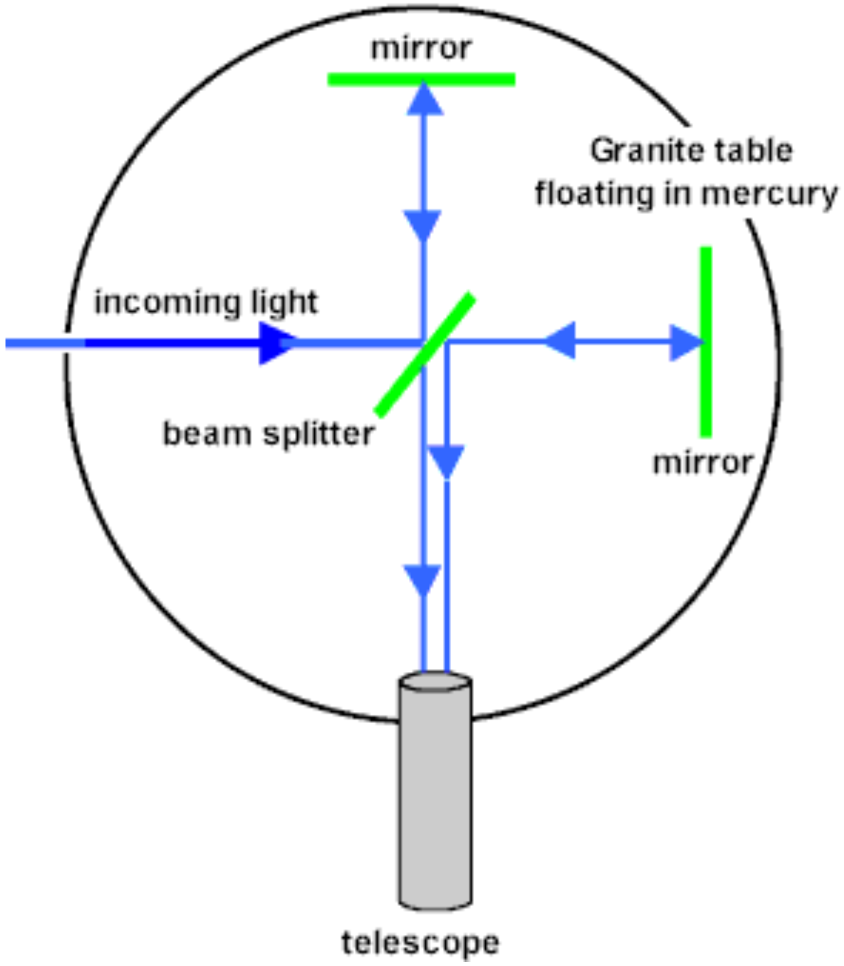
- Albert Michelson & Edward Morley:

"Light moves at 'c' no matter what."

(c = 3×10^8 m/s in a vacuum)

Albert Michelson at Chicago University

Edward Morley



Light takes the same time to travel along both arms, any day of the year, any time of day.

Special Relativity

Albert Einstein, 1910, wonders, “Really? What if I throw light off a moving train. It still moves at c ?”

Special relativity: What happens because light moves at c, no matter what, even if you are moving(*) relative to the motion of the light.

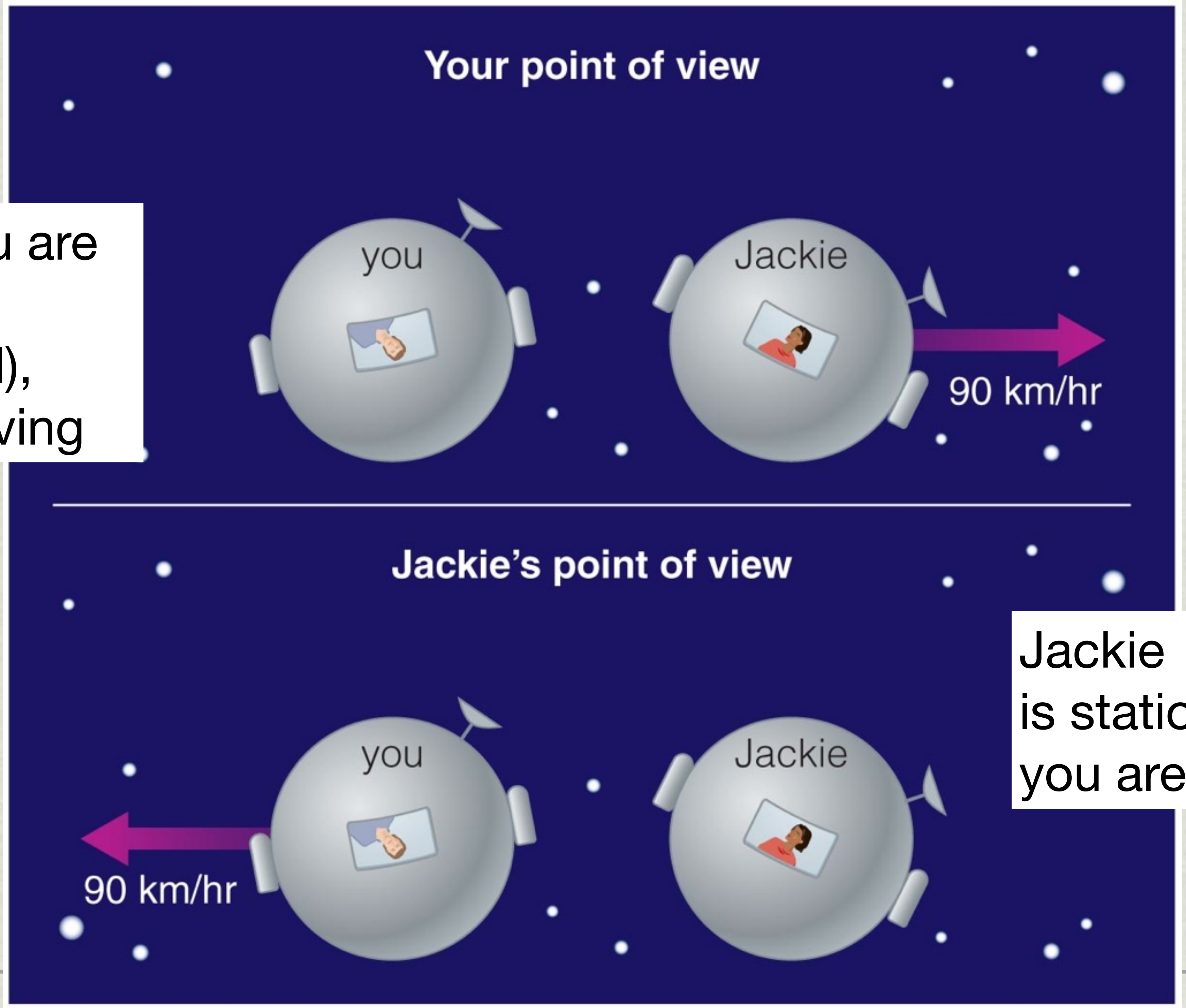
Also: The laws of nature (conservation of momentum, etc.) are the same for everyone.

(*) must be moving at constant velocity.

Not constant velocity? The you are ***accelerating***! Need General Relativity to understand what happens then.

Special Relativity

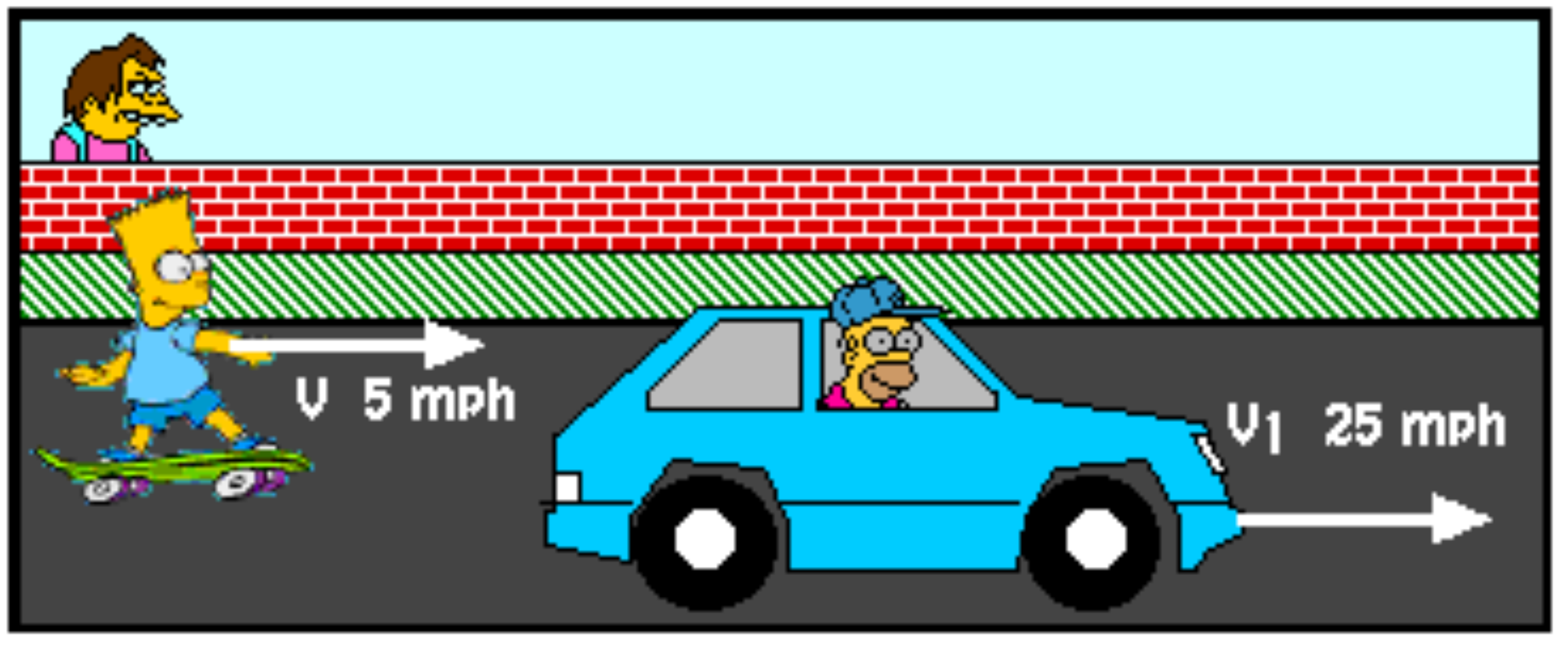
Fact #1:
If everyone is moving at constant velocity, then you can't tell who is moving. You each have a stationary "reference frame"



You think you are stationary (standing still), Jackie is moving

Jackie thinks she is stationary and you are moving

Galilean “Everyday” Relative Motion



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skateboarder moving left at 20 mph

Special Relativity

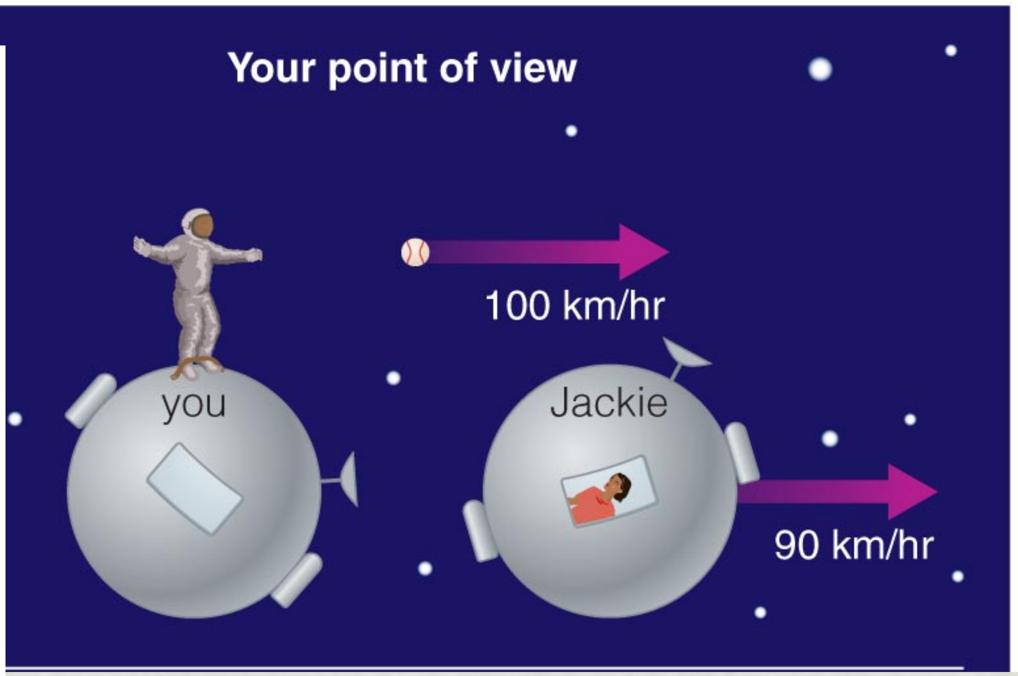
Fact #2:

If you throw something, like a ball, off a moving object, you measure a different velocity than Jackie does. You and Jackie have different reference frames.

You think you are stationary.

You see Jackie moving away from you at 90 km/hr.

You see the ball moving away from you at 100 km/hr



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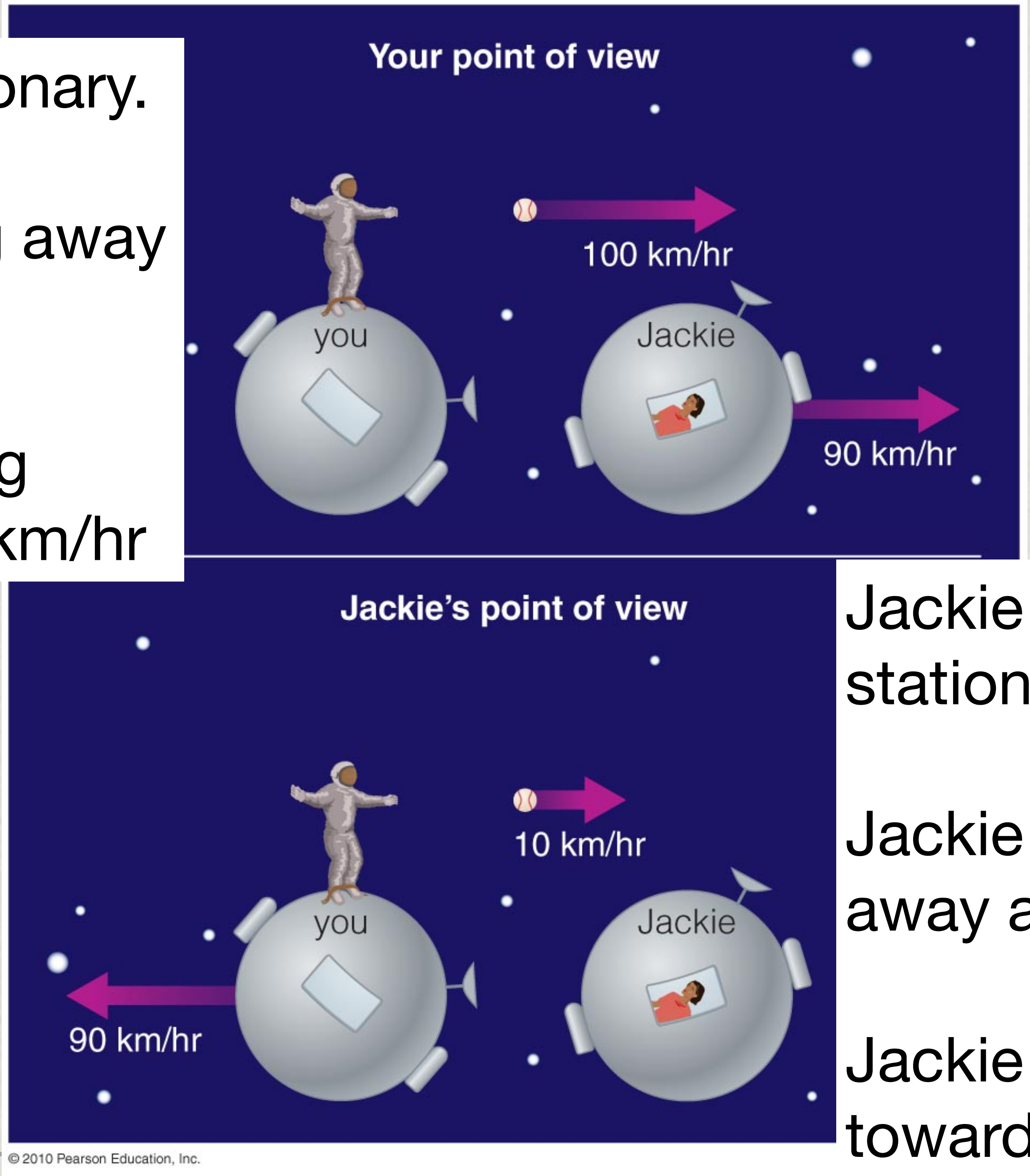

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Jackie sees herself stationary.

Jackie sees you moving away at 90 km/hr.

Jackie sees the ball moving toward her at 10 km/hr

Nothing weird so far.

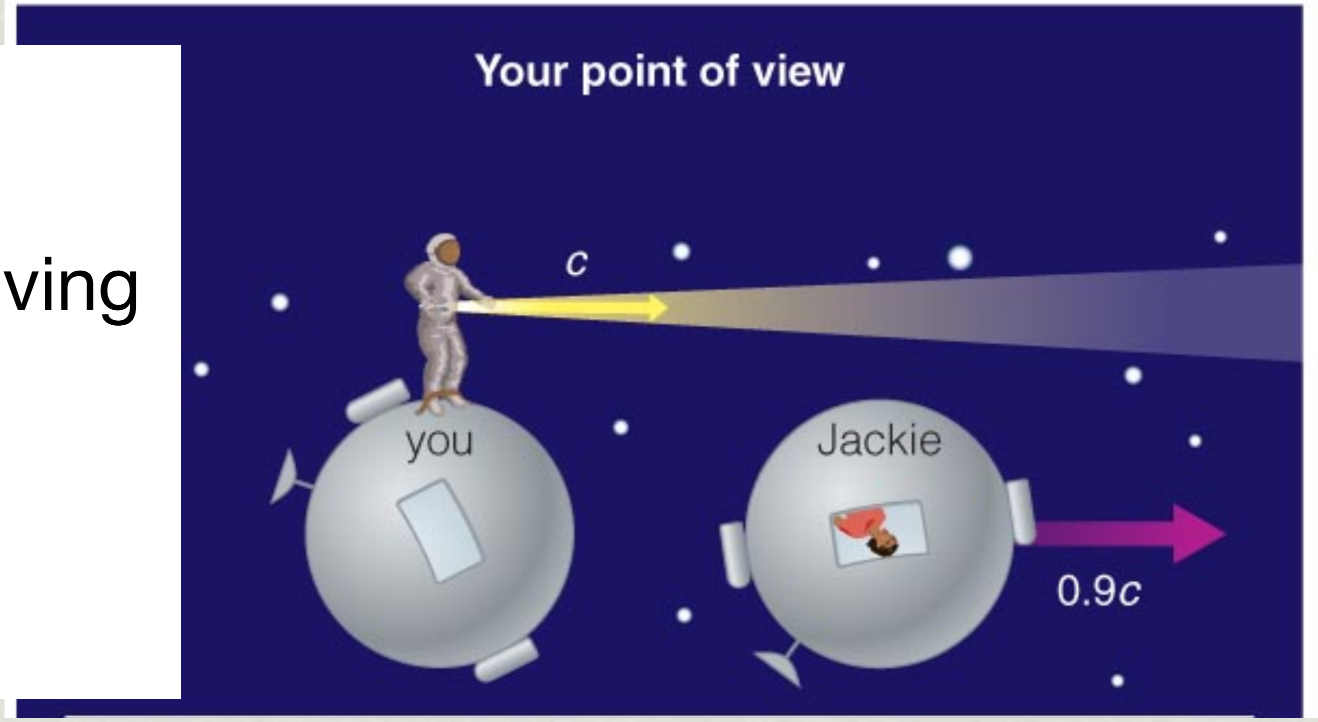
Special Relativity

Fact #3:
Reference frame does not matter for light. You and Jackie both see light moving at c, even from different reference frames.

You are stationary.

You see Jackie moving away at 0.9c

You see the light moving at c



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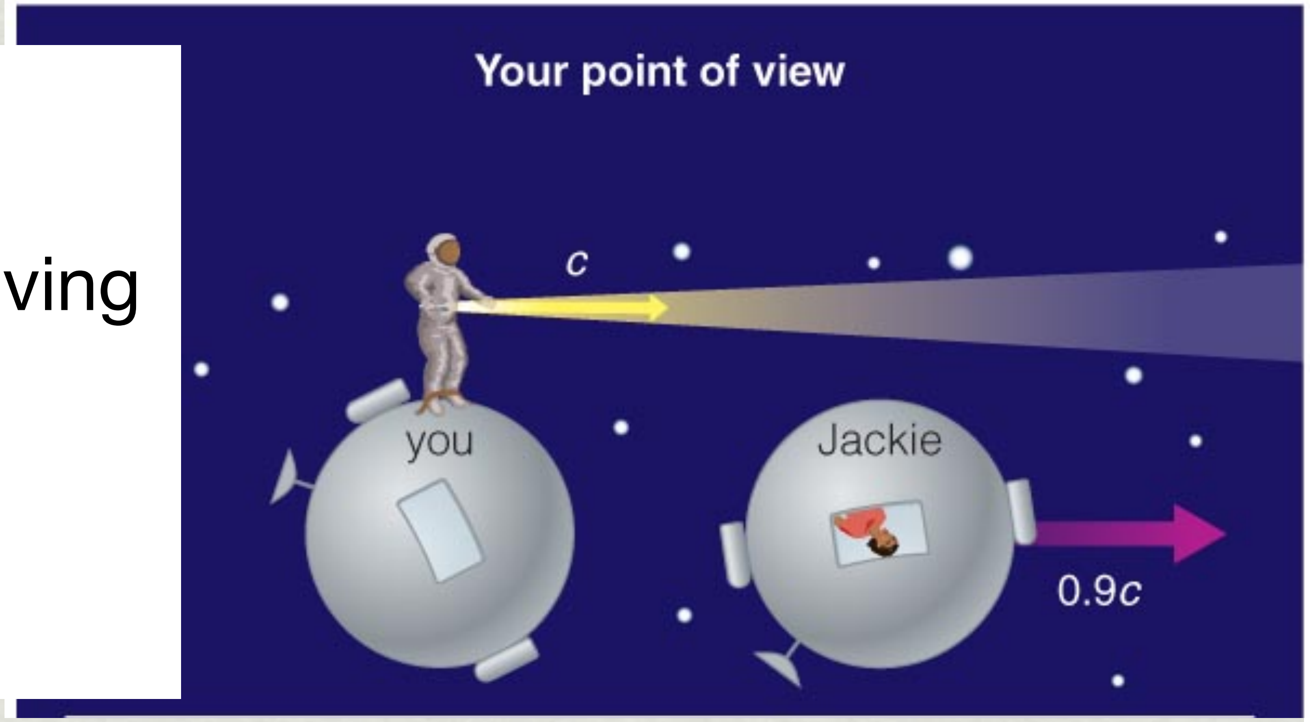

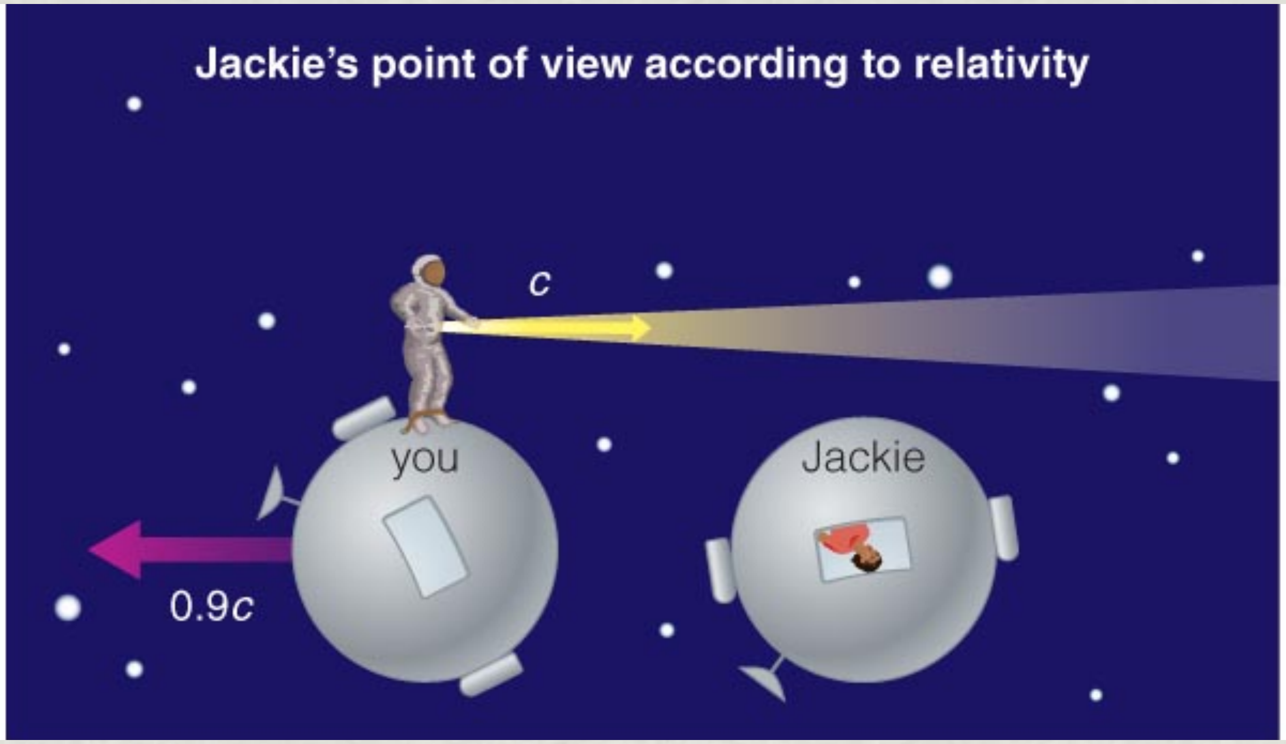

Jackie thinks she is stationary.

Jackie sees you moving away at 90 km/hr.

Jackie sees the light moving at speed:

A 0.1 c
B c
C 1.1 c

Special Relativity

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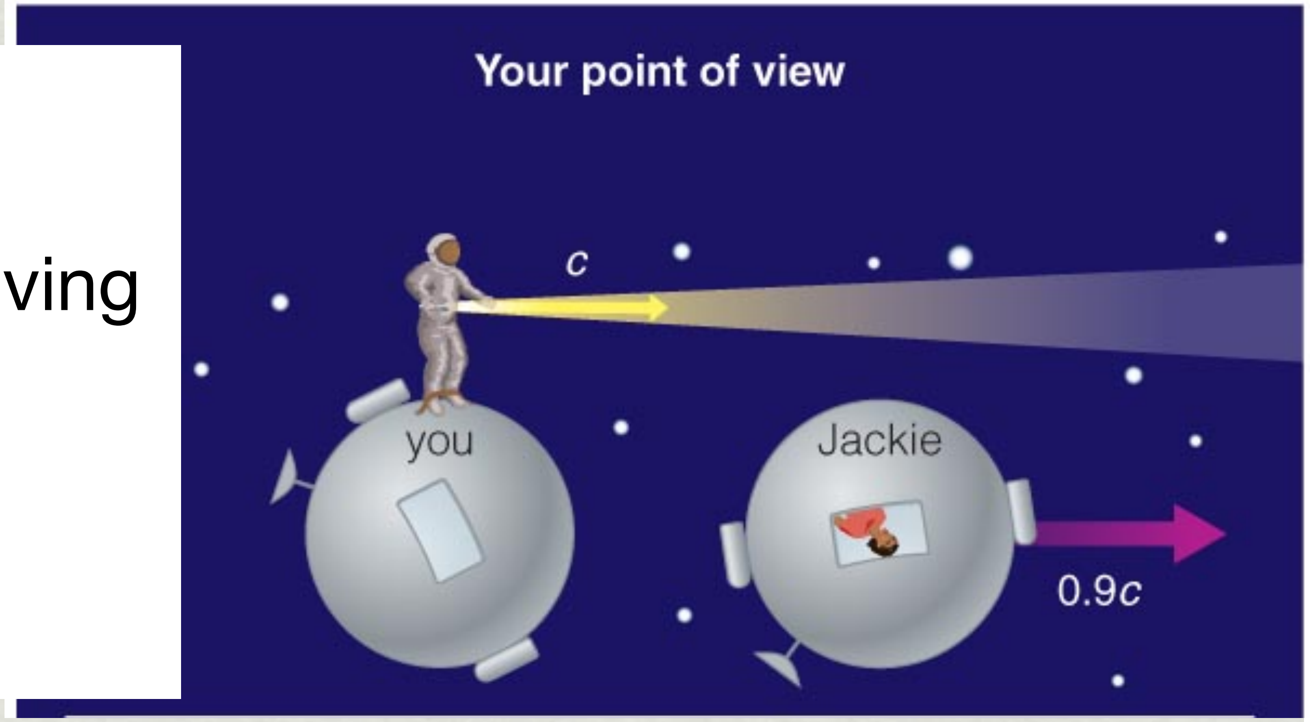
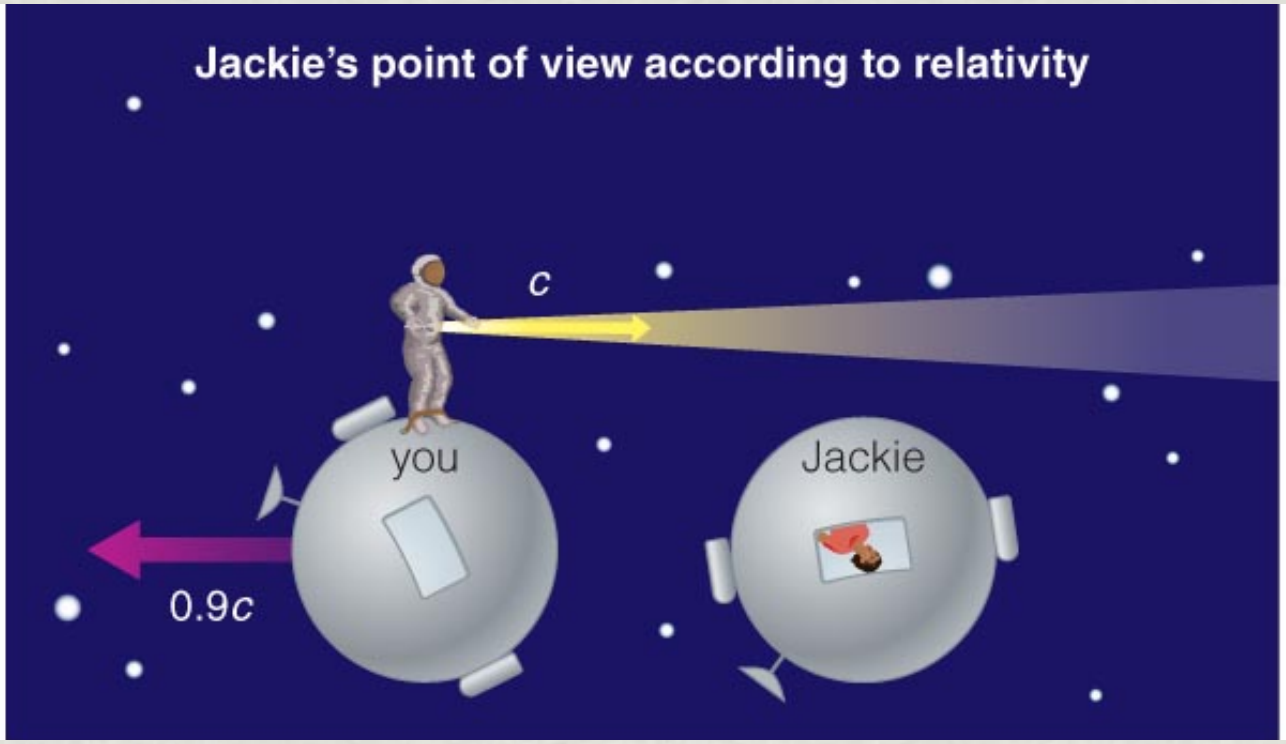


Your point of view



Jackie's point of view according to relativity

Jackie thinks she is stationary.

Jackie sees you moving away at 90 km/hr.

Jackie sees the light moving at speed:

A   0.1 c
B   c
C   1.1 c

Special Relativity

Fact #3:
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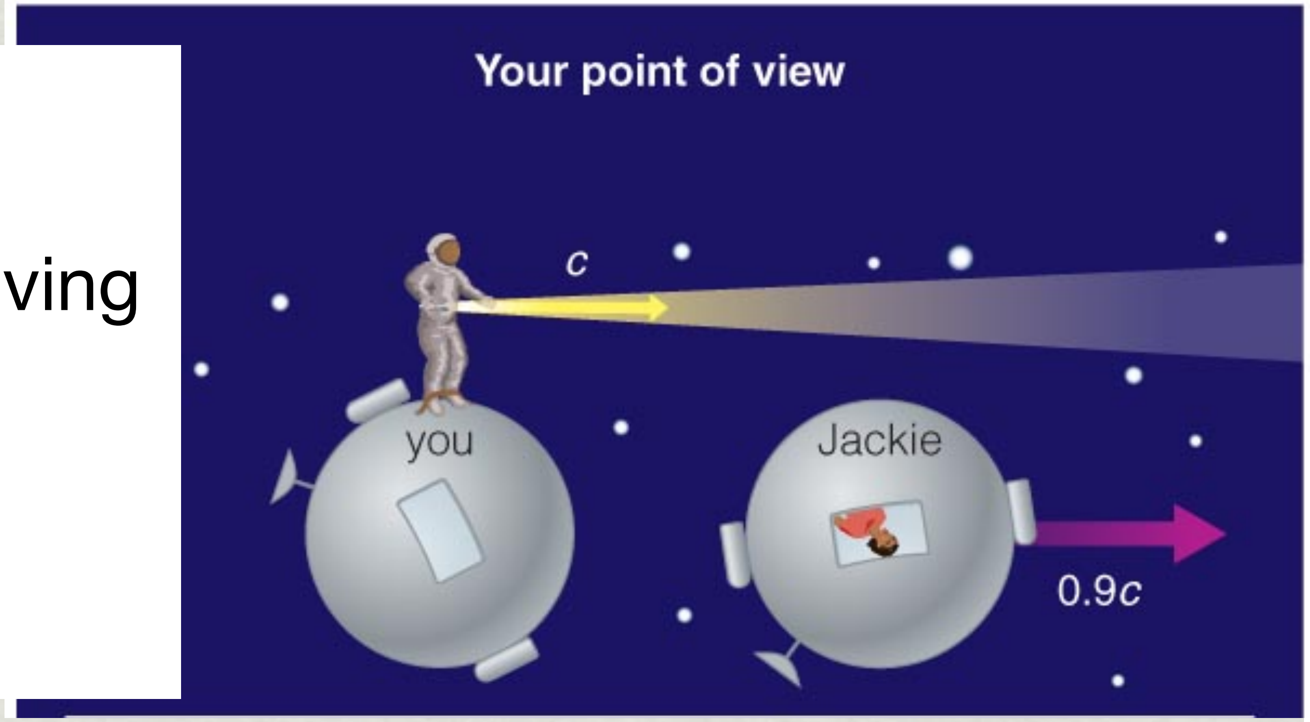
You see Jackie moving away at 0.9c

You see the light moving at c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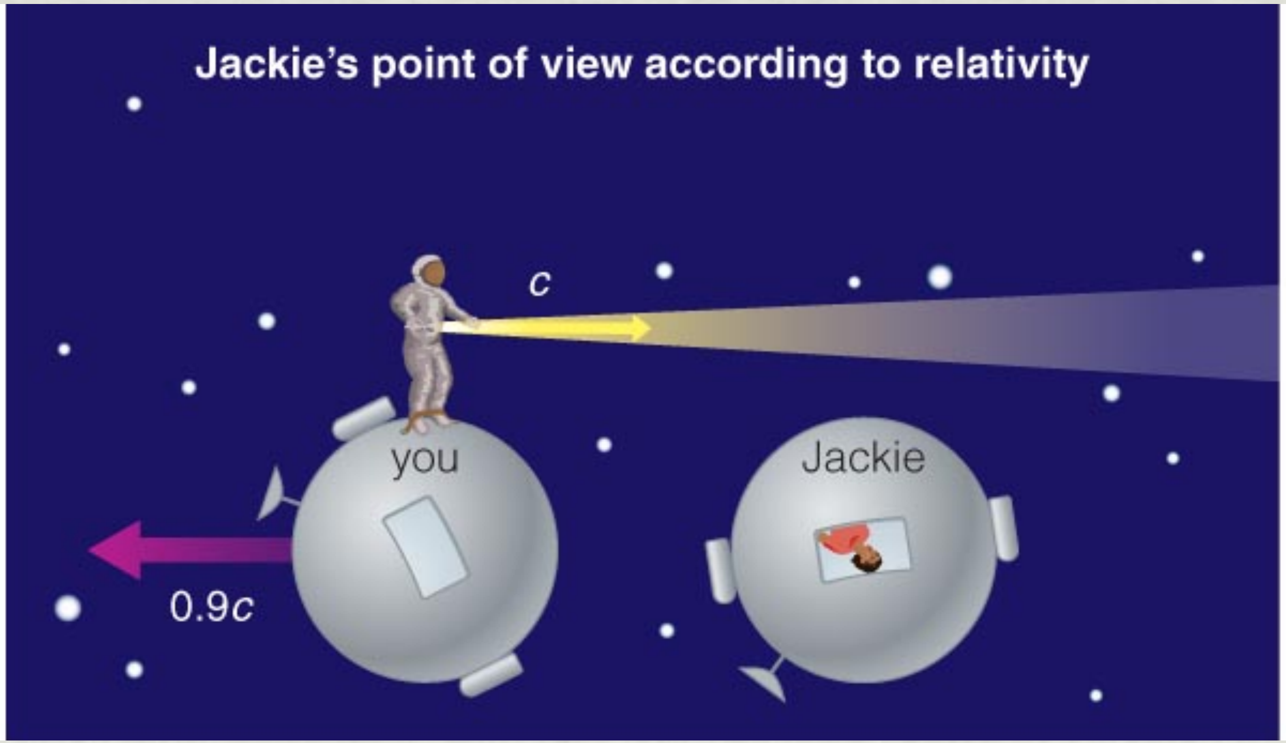
Jackie's point of view according to relativity

Jackie thinks she is stationary.

Jackie sees you moving away at 90 km/hr.

Jackie sees the light moving at c, too

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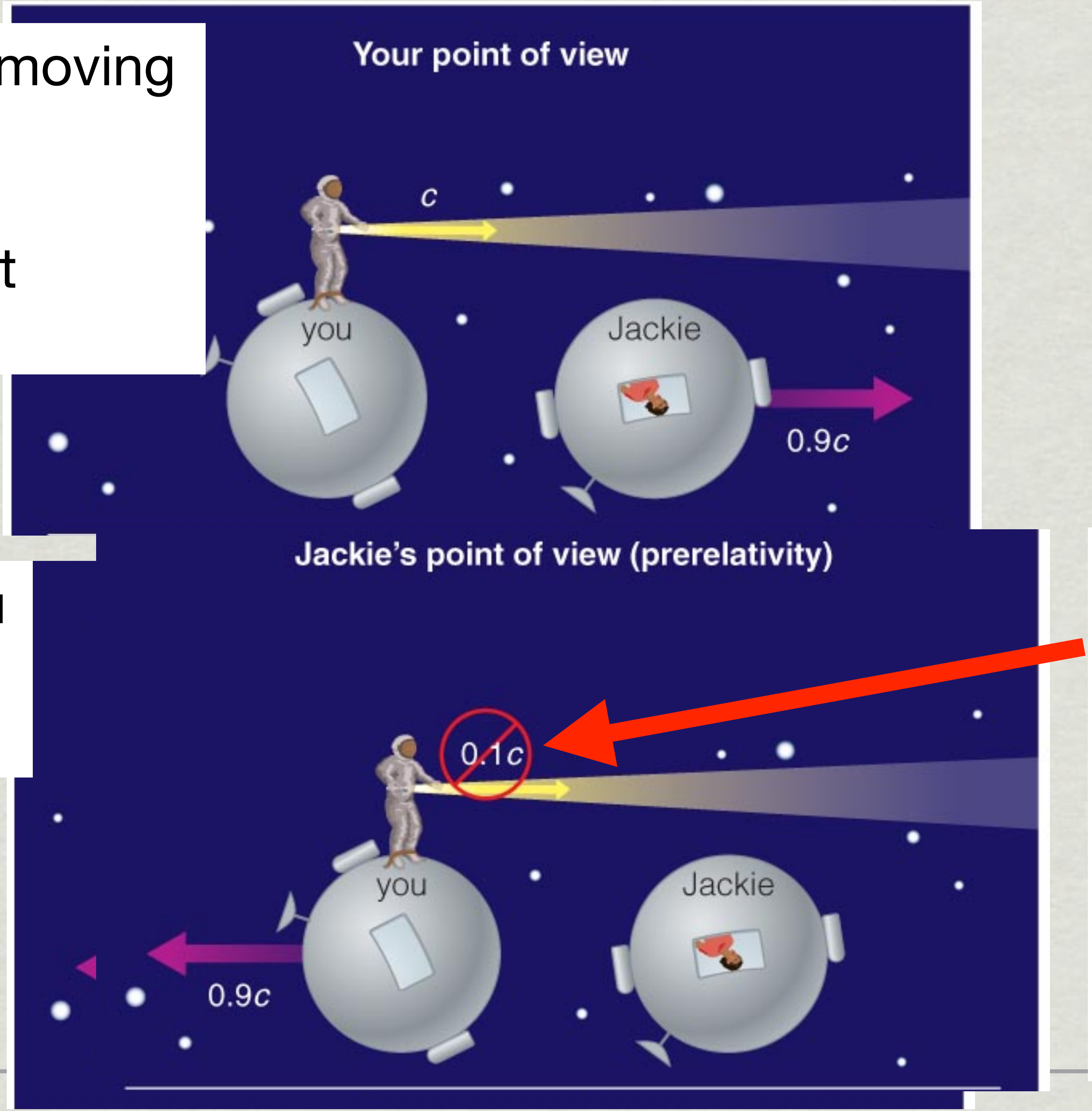


You see Jackie moving away at 0.9c

You see the light moving at c

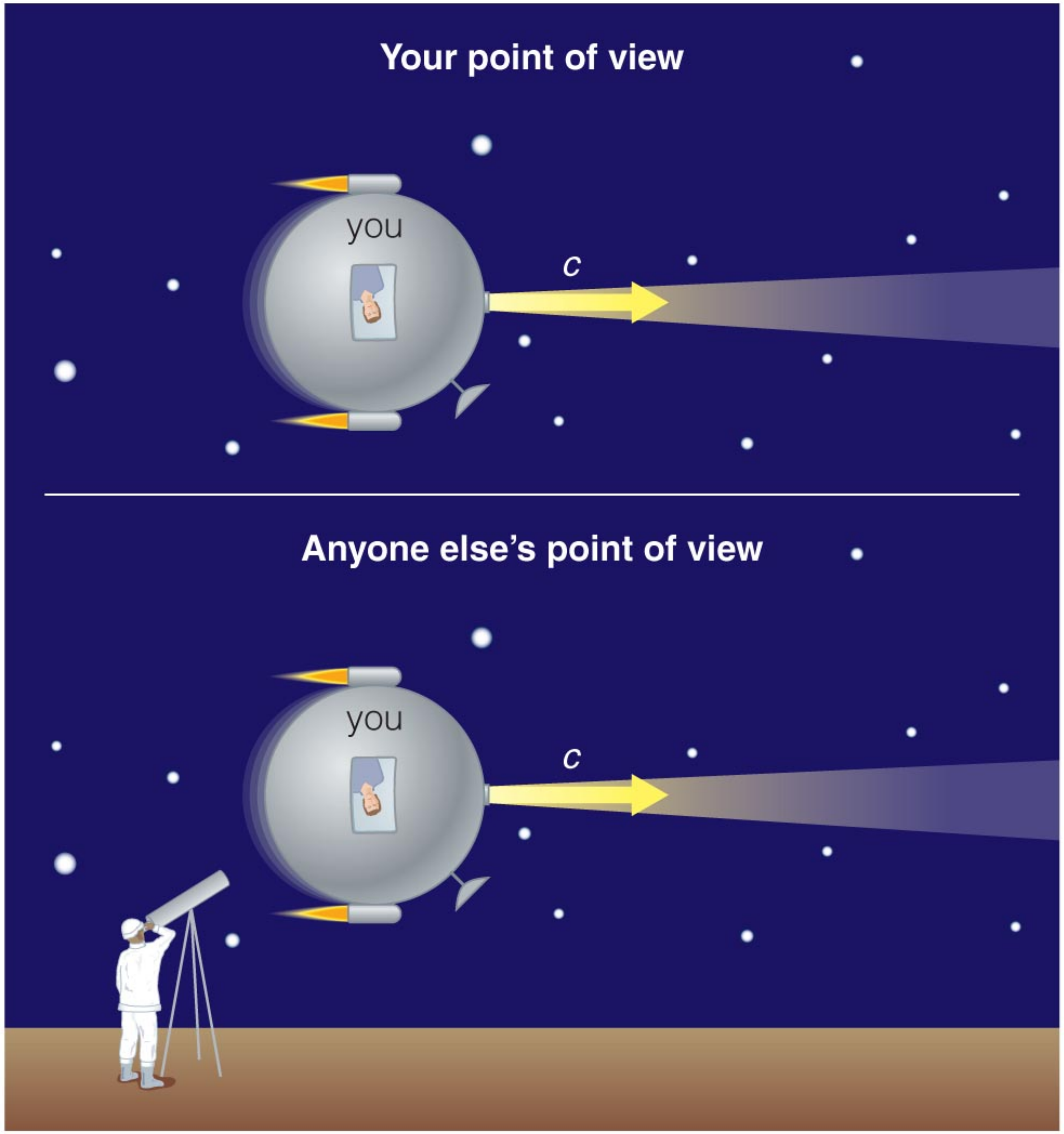
Jackie sees you moving away at 90 km/hr.

If light were like the ball, Jackie would see this instead.

But she doesn't!

Jackie sees the light moving at c

No Going Faster Than Light



You see your headlight beam moving away from you at speed = c

The person with the telescope also sees your headlight beam moving away from you at speed = c

Everyone sees your headlight beam moving faster than you are.

So you must be moving at a speed < c in all reference frames

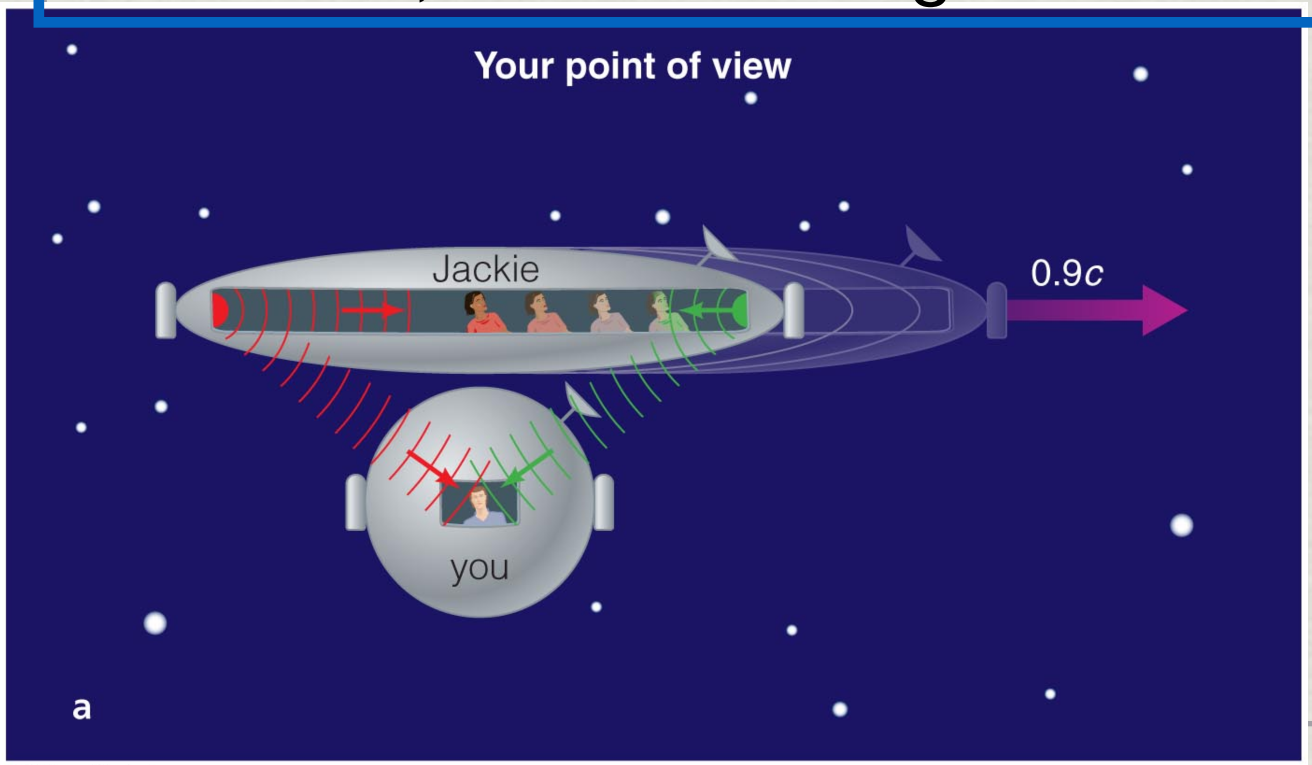
Simultaneity is Relative

All observers must agree on the order of events that occur ***in any one place***.
“OK, so what could get weird about that?”

You see the red and green flashes at the same time.

You see Jackie moving at 0.9c in the direction the green light is coming from.

You see the green light illuminate Jackie first, then the red light.



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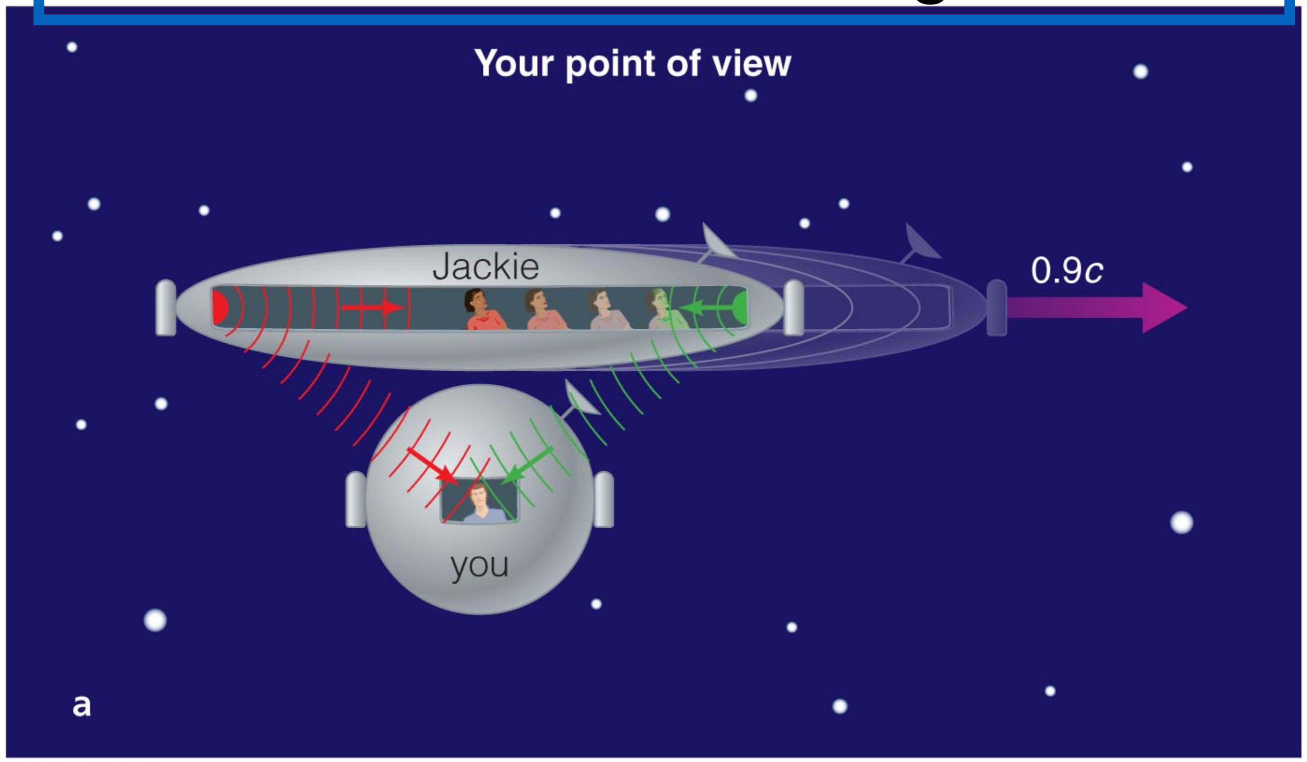
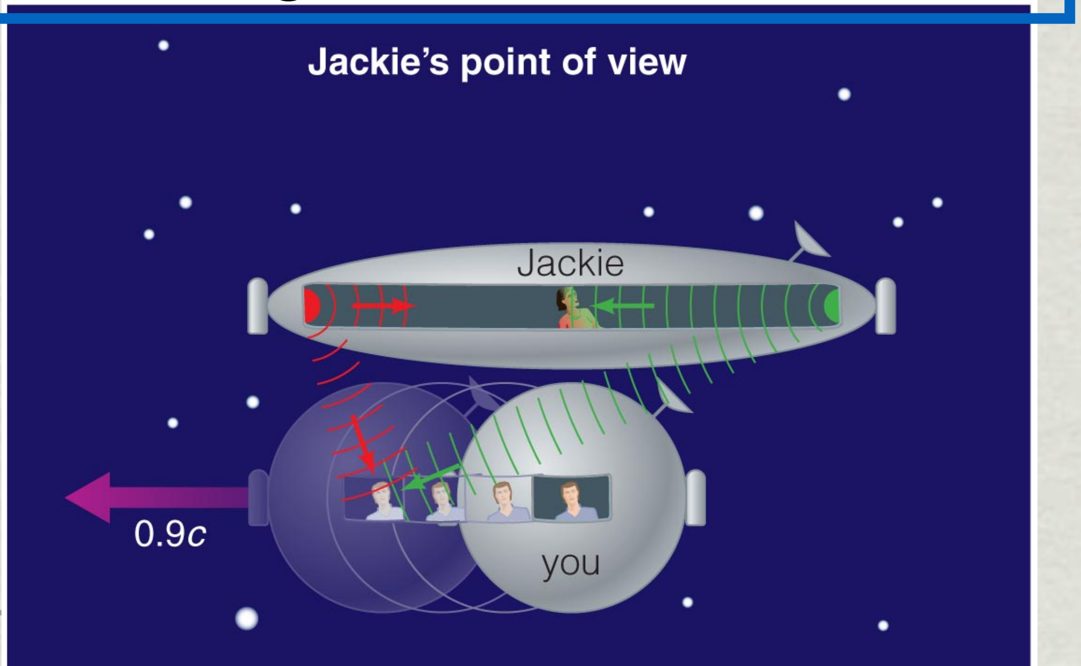


Jackie sees you moving away from her at 0.9c

She thinks she is standing still.

Jackie also sees the green light first.

She thinks that the green light flashed first. She thinks you saw them at the same time because you are moving toward the red light.



Simultaneity is Relative

All observers must agree on the order of events that occur in any one place.
“OK, so what could get weird about that?”

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You see
Jackie f

Both you and Jackie agree that the green light got to Jackie first. You agree on order of events in a single place – Jackie’s location.

But you each infer a different reason that the green light got to Jackie first:

- You think it was because Jackie was moving toward the green light.
- Jackie thinks it was because the green light flashed first.

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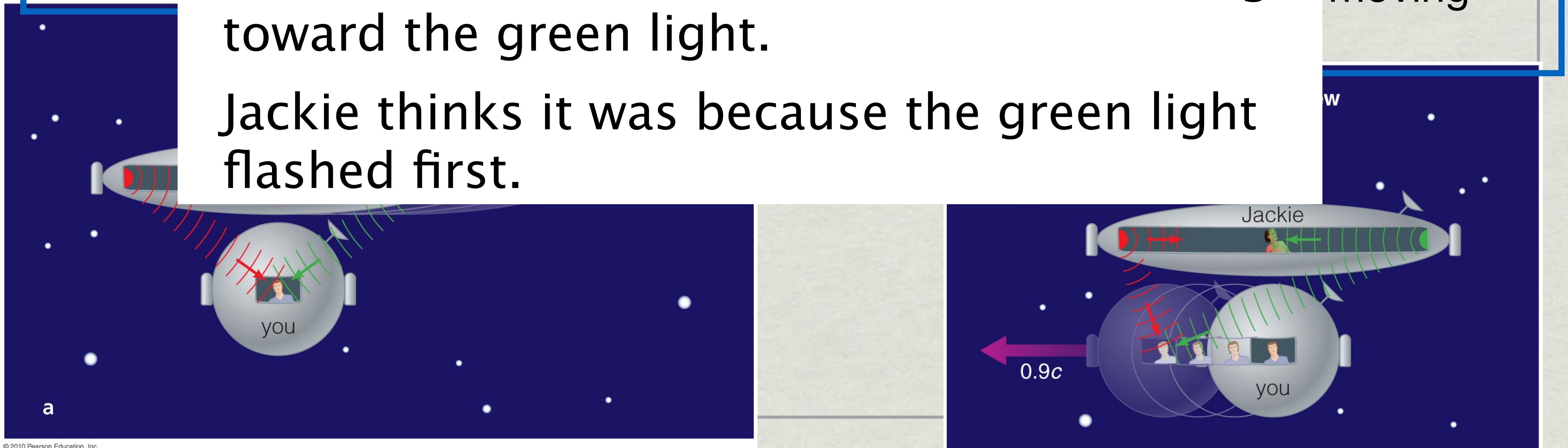
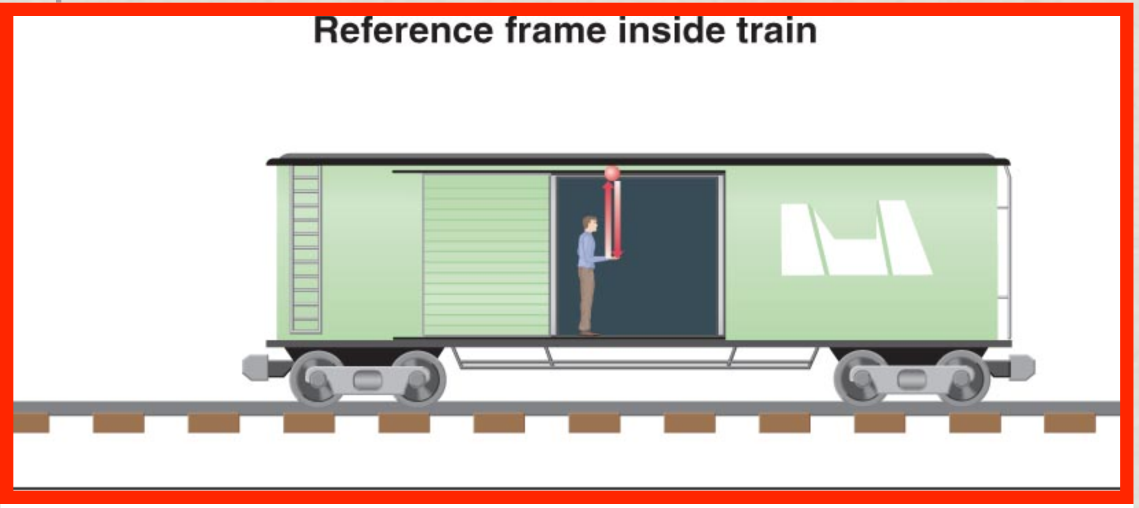
moving



Special Relativity: Time Dilation

Implication #1:



Jackie stands on a moving train.

She tosses a ball straight in the air and catches it.

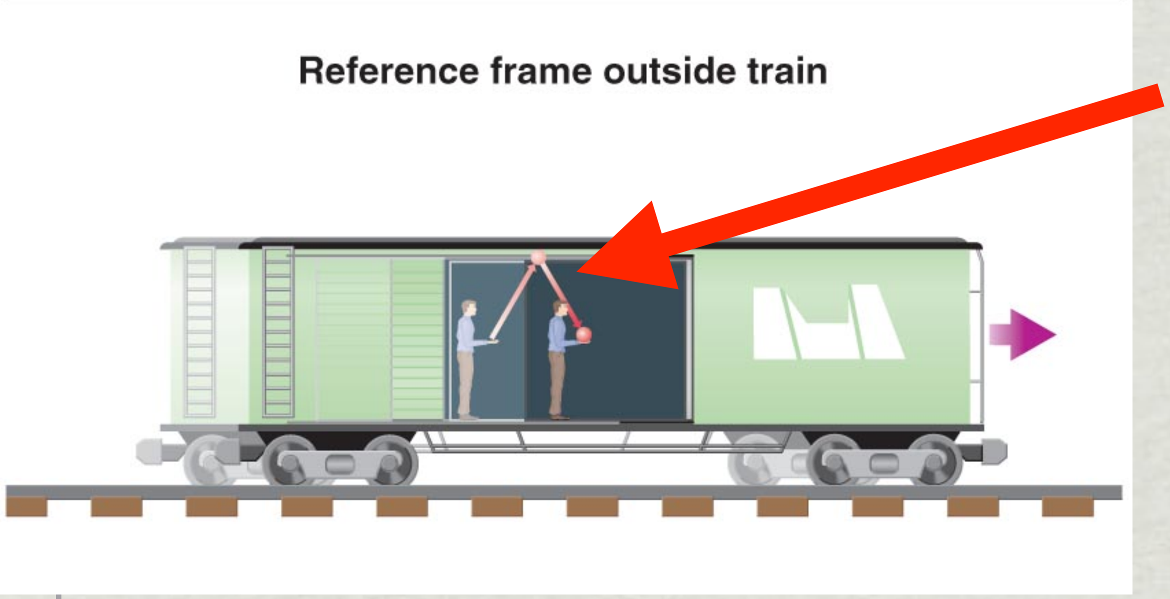
She measures:
- the distance the ball travels up to the roof and back down again
- the time it takes to travel up and back again (its “travel time”)
- the speed of the ball

Relate distance, speed and time:
Distance = Speed x Time → $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

Special Relativity: Time Dilation

Implication #1:

You stand by the tracks and watch.



You see the ball move a larger distance: up +down+motion of train

You also measure a faster speed: speed of the ball+speed of the train

You watch your clock, and measure the same travel time for the ball as Jackie does

You measured a larger distance and a larger speed but the same travel time.

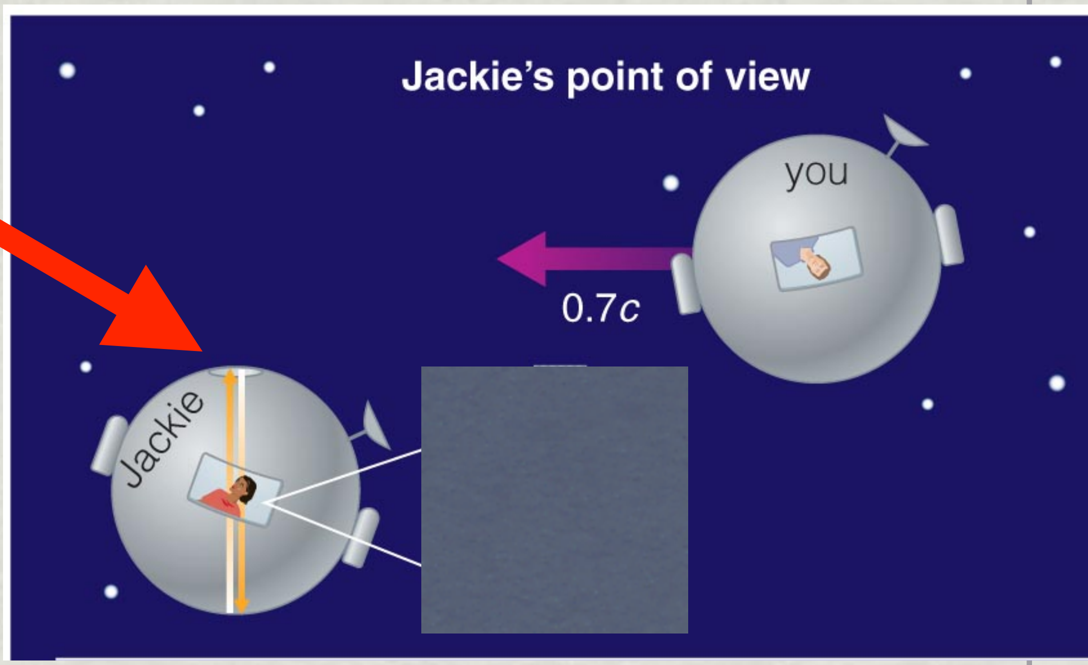
Again, nothing weird, yet.

You and Jackie both agree that
Distance = Speed x Time

Special Relativity: Time Dilation

Implication #1:

Flashlight bounces off mirror on ceiling. Just like the ball toss in the train.



But light is different!

Jackie measures:
- the distance the light travels
- the time the light takes to travel from her flashlight to the ceiling and back down to her camera
- the speed of light, c

Just like with the ball on the train.

Special Relativity: Time Dilation

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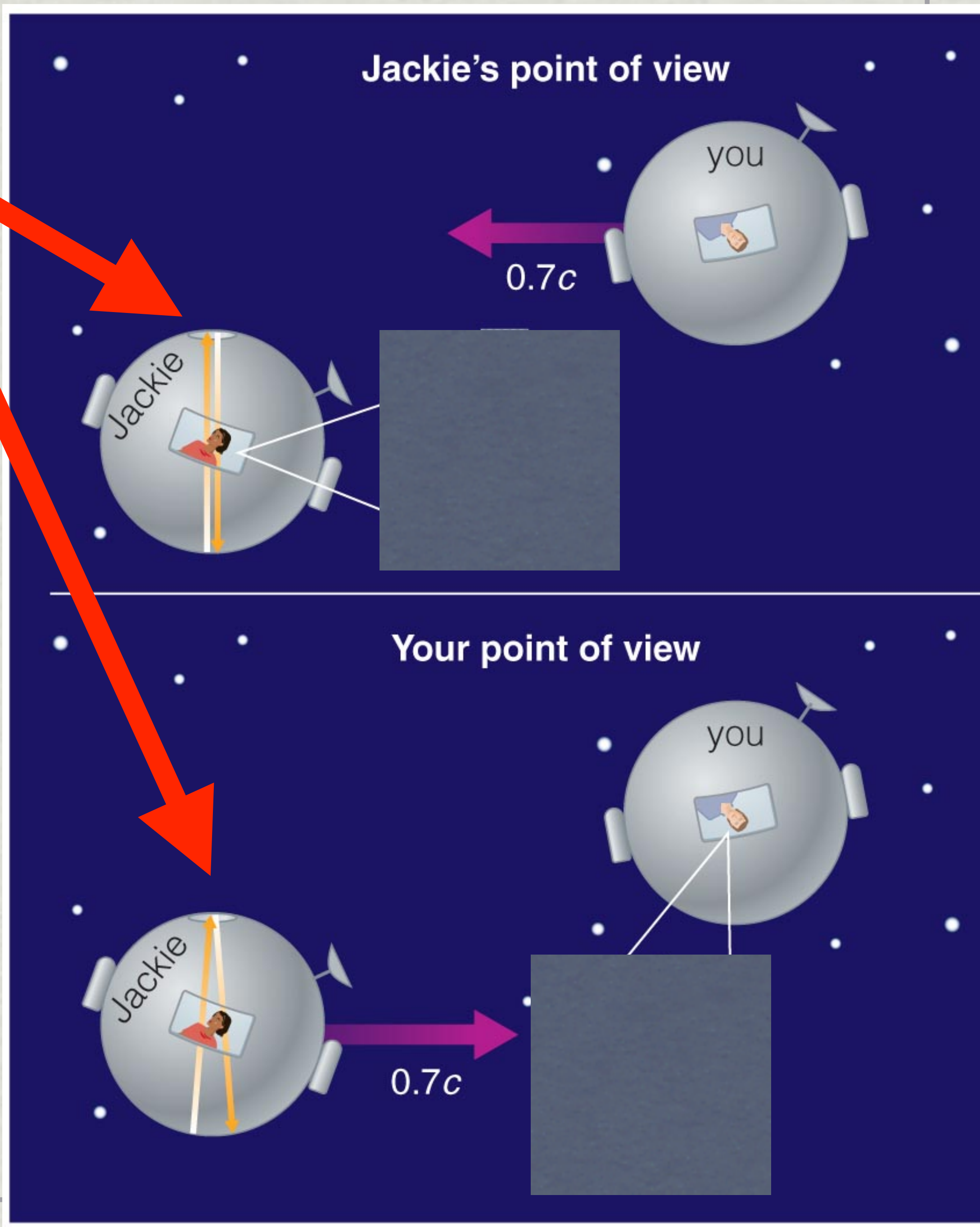


But light is different!

You watch your clock and measure the travel time of the light.

You see the light travel a larger distance because Jackie is moving, just like on the train.

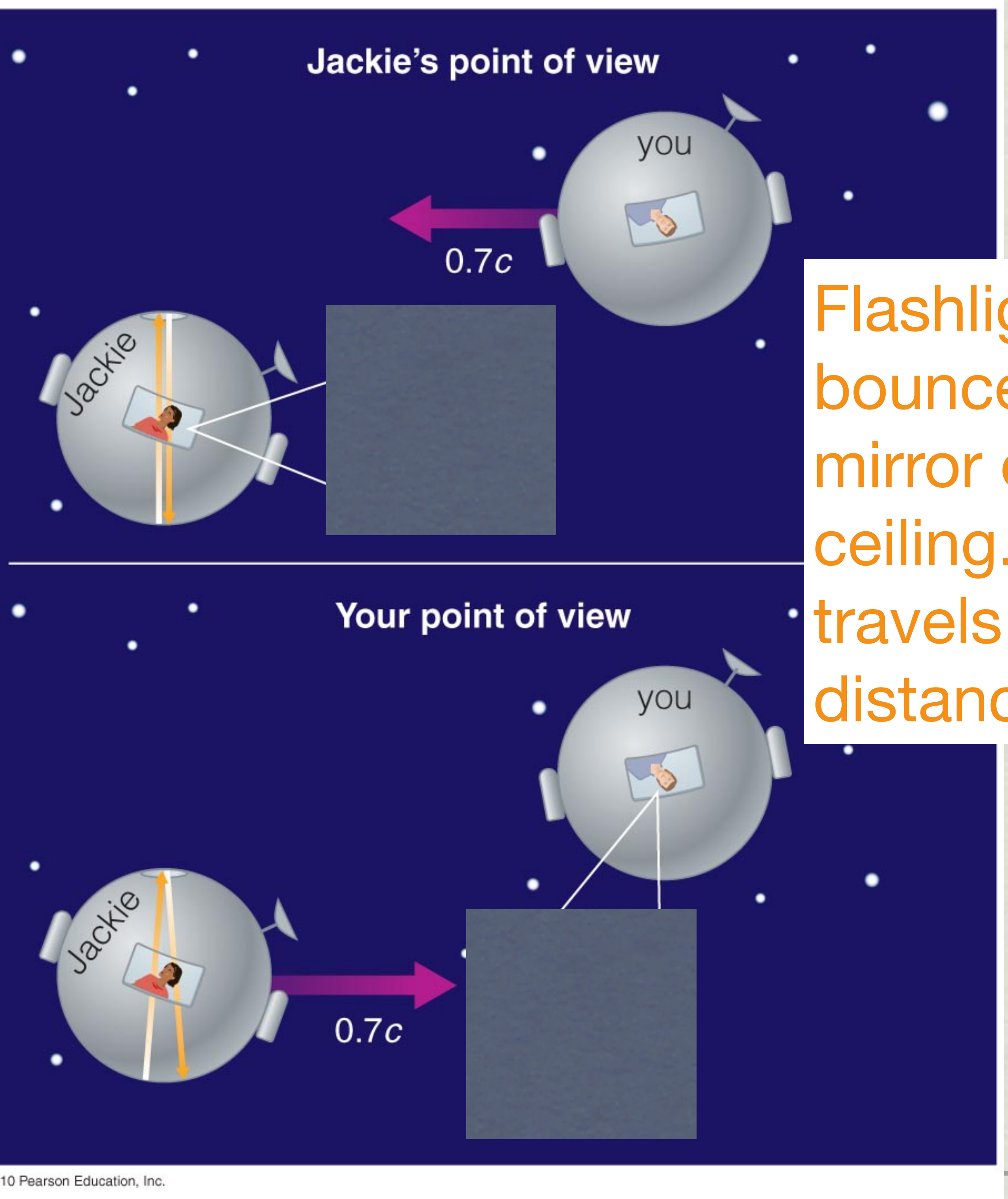
But ***both*** you ***and*** Jackie see the light travel at speed c.

Special Relativity: Time Dilation

Both you and Jackie see the light travel at speed c.

You watch your clock and measure the travel time of the light. Jackie watches her clock and measures the travel time, too.

You see the light travel a larger distance because Jackie is moving.



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Flashlight bounces off mirror on ceiling. It travels distance D.

You and Jackie both know:
Distance = Speed x Time

$$\text{Rearrange: Time} = \frac{\text{Distance}}{\text{Speed}}$$

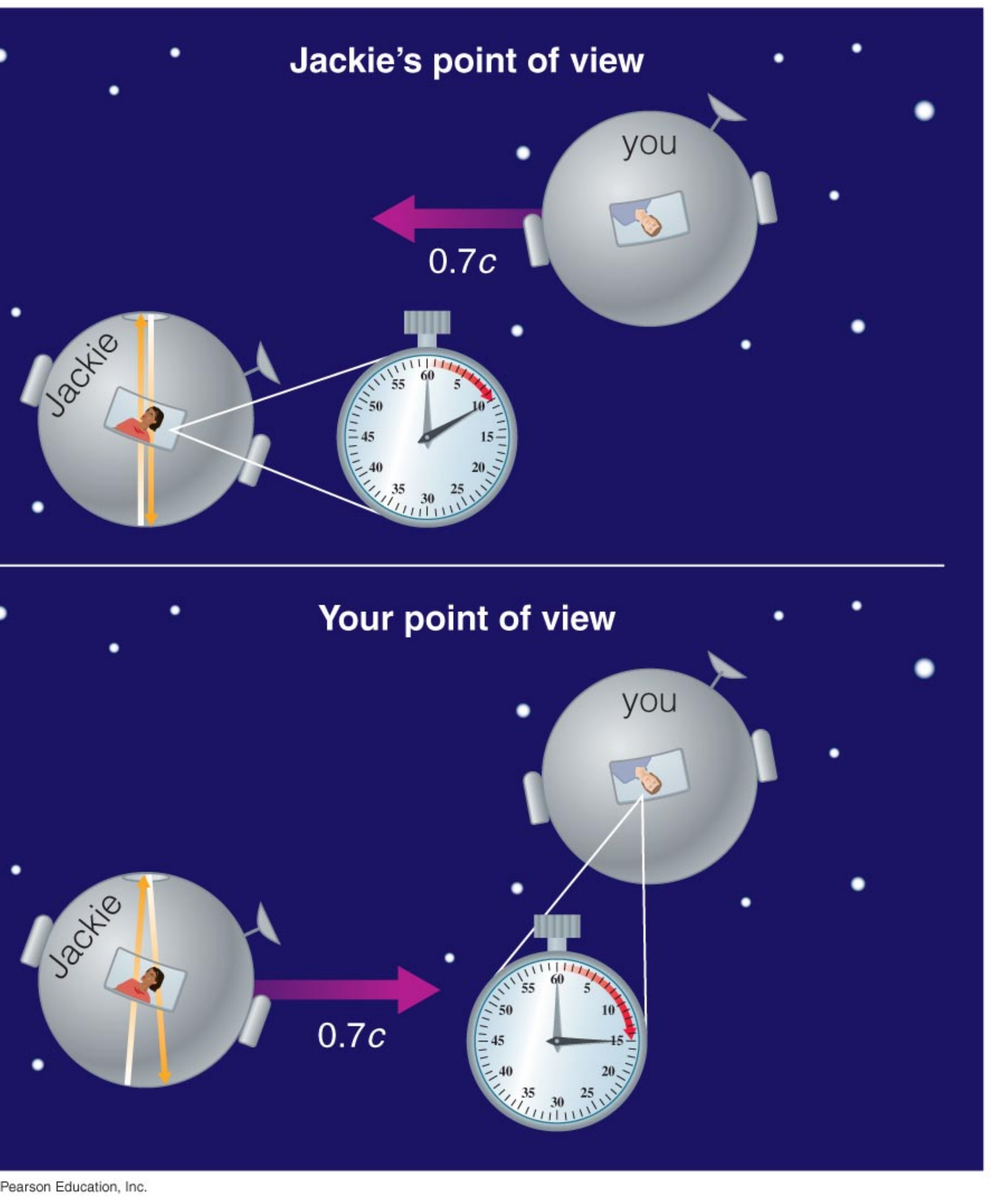
Speed: you and Jackie both measure c

Distance: you measure a larger value than Jackie does

Special Relativity: Time Dilation

Implication #1:

In any reference frame moving at velocity close to c, an outside observer sees TIME SLOW DOWN in that frame.



Both you and Jackie see the light travel at speed c.

You watch your clock and measure the travel time of the light.

You see the light travel a larger distance because Jackie is moving, just like on the train.

You and Jackie both know:

Distance = Speed x Time

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

But if that is true, Jackie's clock measures less time than yours: you must see Jackie's clock run slow!

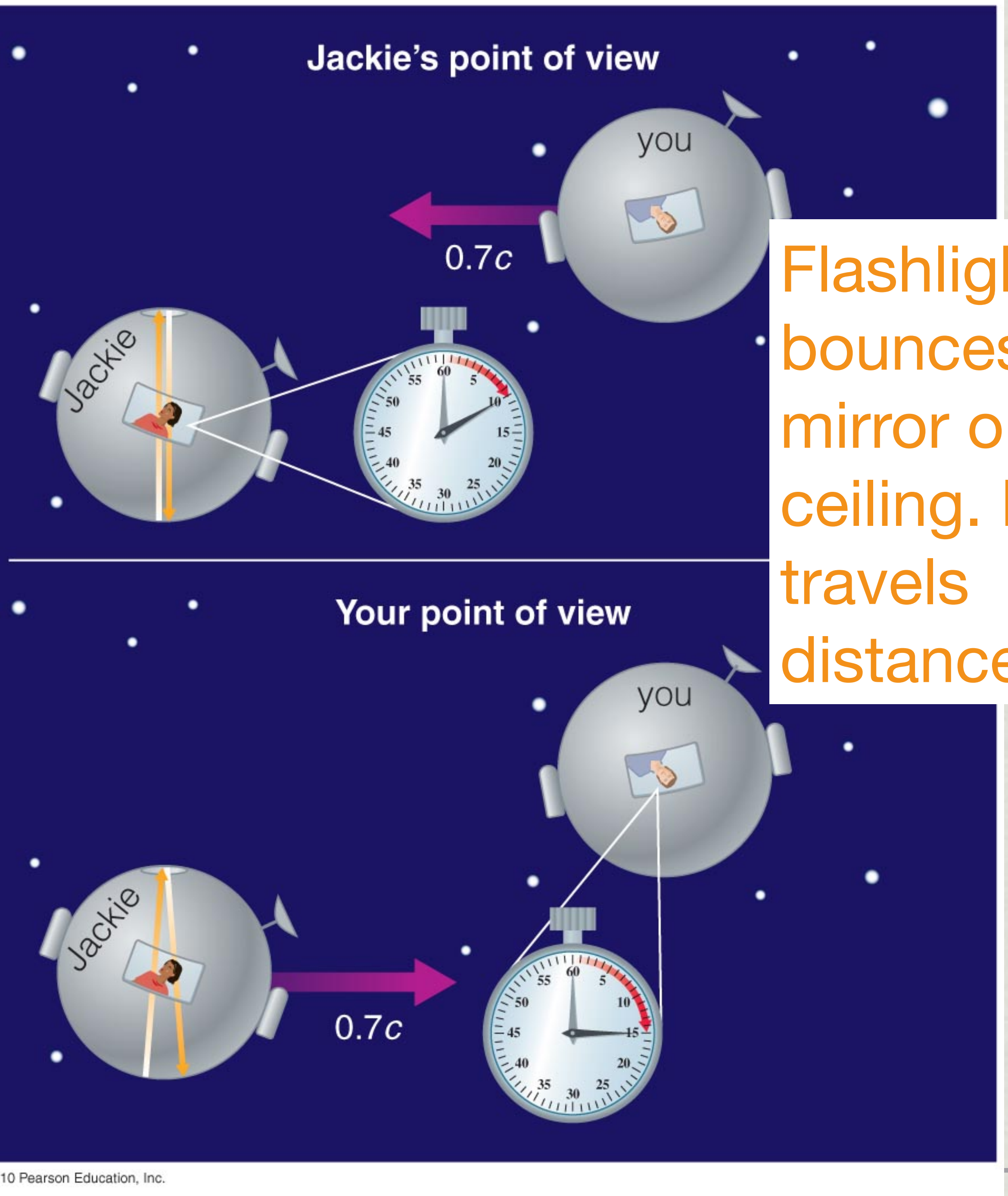
Special Relativity: Time Dilation

Both you and Jackie see the light travel at speed c.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Speed: you and Jackie both measure c

Distance: you measure a larger value D_{you} than Jackie measures D_J



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Flashlight bounces off mirror on ceiling. It travels distance D.

You measure:= $\frac{D_{you}}{c} = \text{Time}_{you}$

Jackie measures:= $\frac{D_J}{c} = \text{Time}_J$

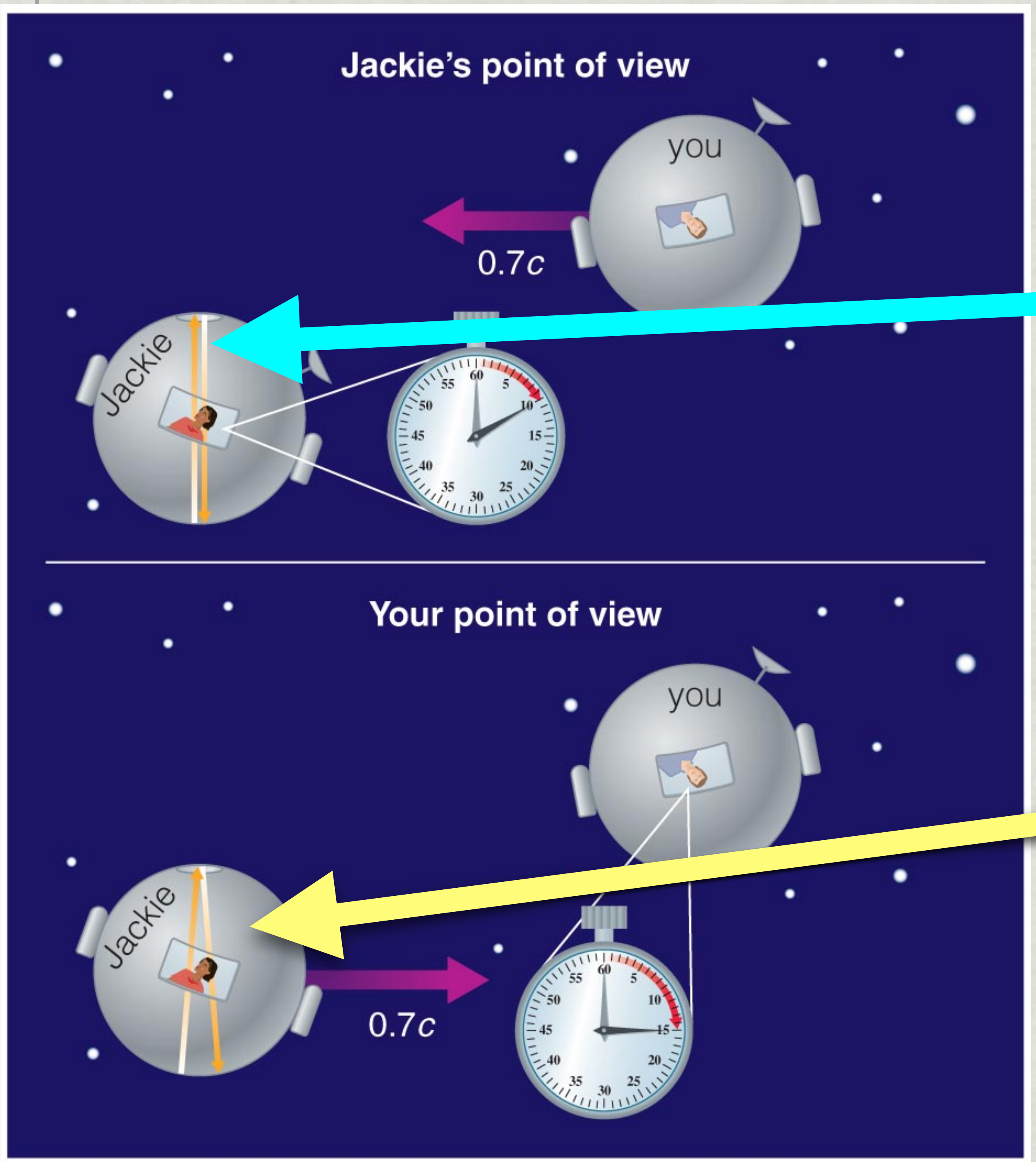
Since $D_{you} > D_J$

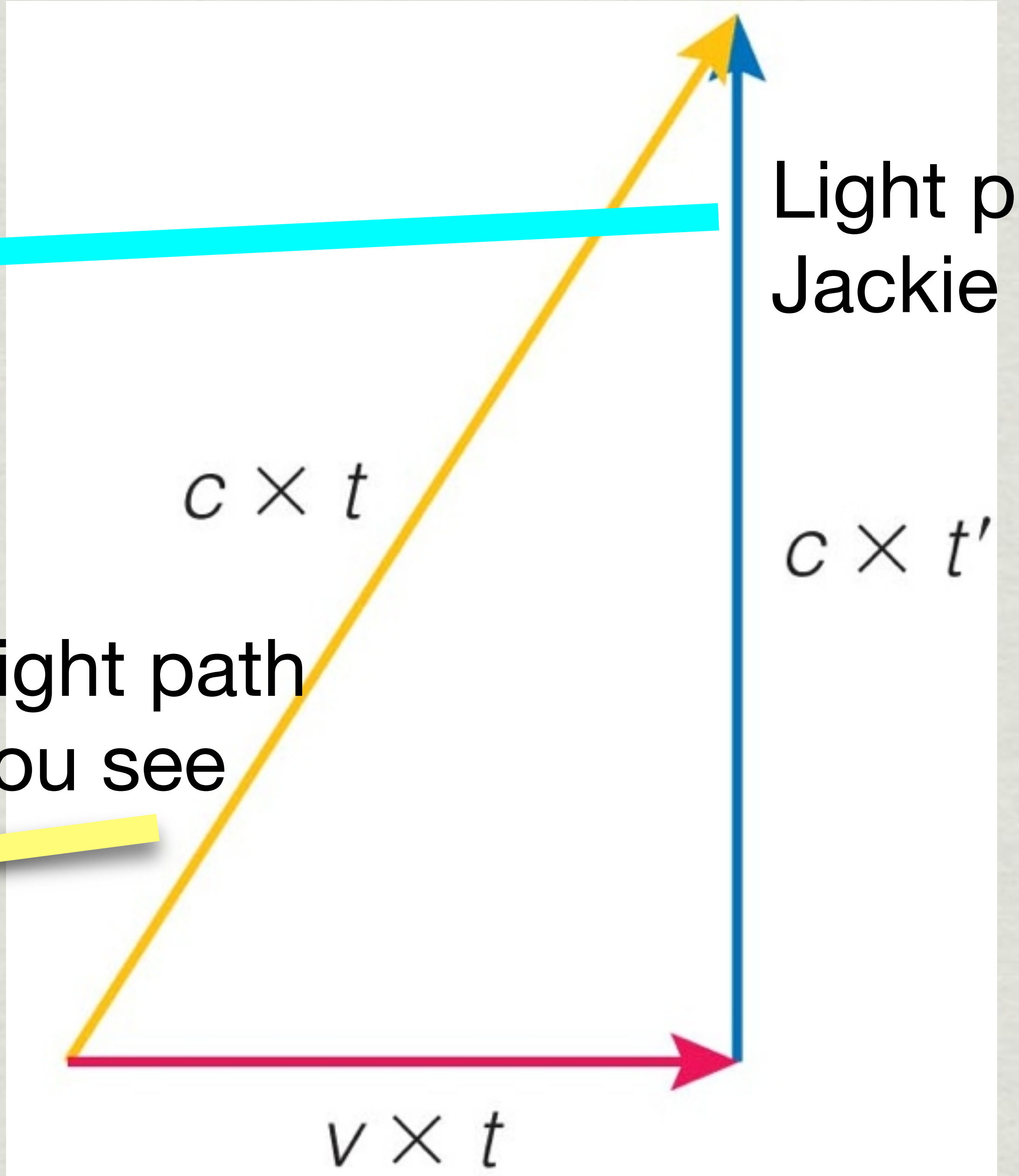
then $\text{Time}_{you} > \text{Time}_J$

Less time passes for Jackie as the light beam travels to the ceiling and back again than for you.

Less time passes for Jackie: time runs more slowly in a moving reference frame!

Special Relativity





Light path
Jackie sees

Light path
you see

Distance that you see Jackie's spaceship travel while the light travels up and down.

Special Relativity

Use Pythagorean theorem: $(ct')^2 + (vt)^2 = (ct)^2$

Solve for t'

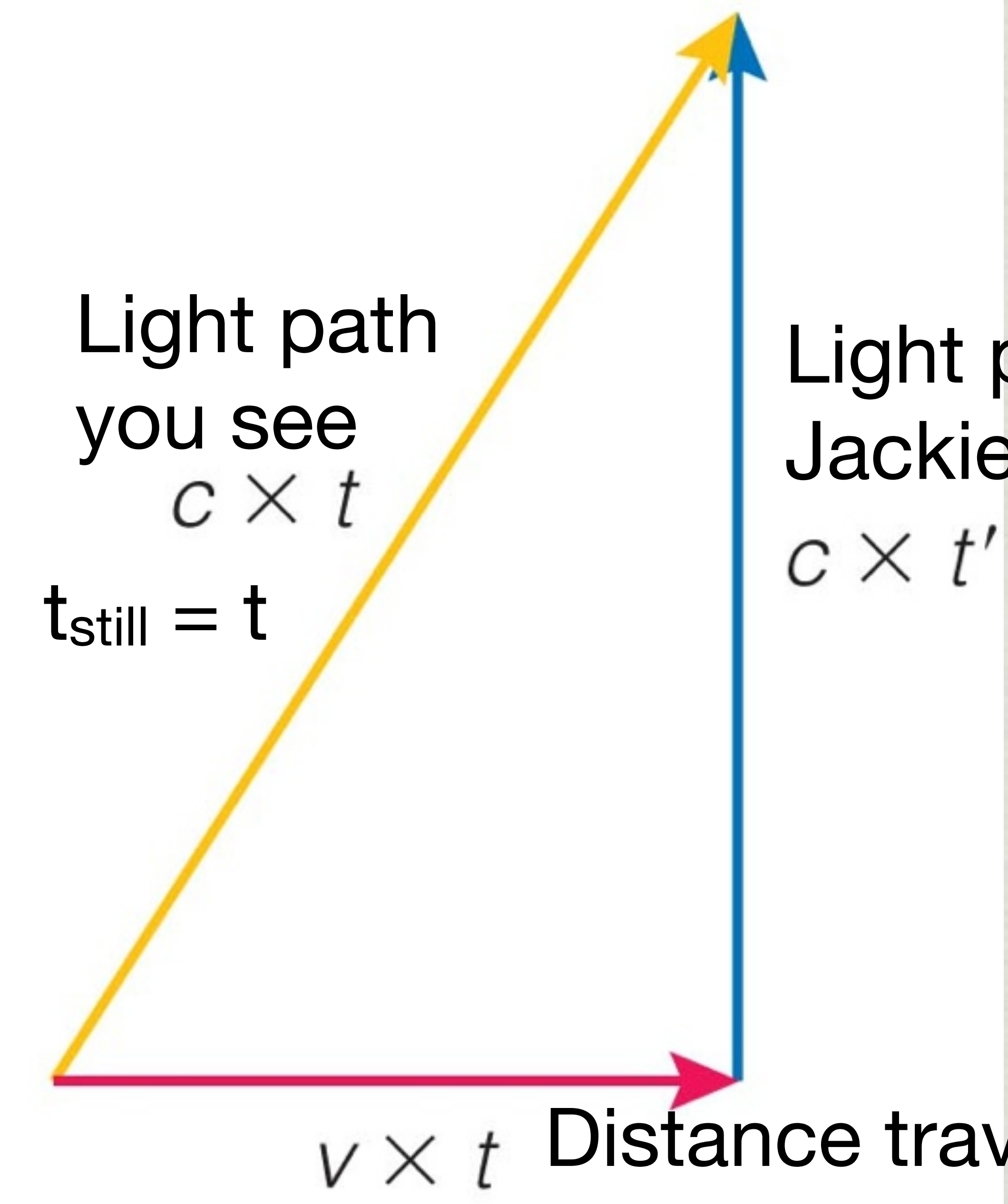
$$\boxed{t_{\text{moving}} = t_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$$

$t_{\text{moving}} = t'$

$t_{\text{still}} = t$

Difference between t_{moving} and t_{still}:

Lorentz factor



Light path you see

$t_{\text{still}} = t$

Light path Jackie sees

$t_{\text{moving}} = t'$

Distance traveled by Jackie's spaceship while the light travels up and down.

$$\boxed{t_{\text{moving}} = t_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$$

v is always less than c.

The Lorentz factor $\boxed{\sqrt{1-\frac{v^2}{c^2}}}$ is always:

- A about equal to 1
- B less than 1
- C greater than 1

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v is always less than c.

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A about equal to 1
B less than 1
C greater than 1

Special Relativity

If v=0.9c, $\sqrt{1-\frac{v^2}{c^2}}$ = 0.44

If v= 0.0001 c, is $\sqrt{1-\frac{v^2}{c^2}}$

A huge!
B tiny!
C about = 1

Special Relativity

At v=0.9c, $\sqrt{1-\frac{v^2}{c^2}}$ = 0.44

At 0.0001 c, is $\sqrt{1-\frac{v^2}{c^2}}$

A huge!
B tiny!
C about = 1

At 0.0001 c, $\sqrt{1-\frac{v^2}{c^2}}$ = 0.999999995

Special Relativity: Length Contraction

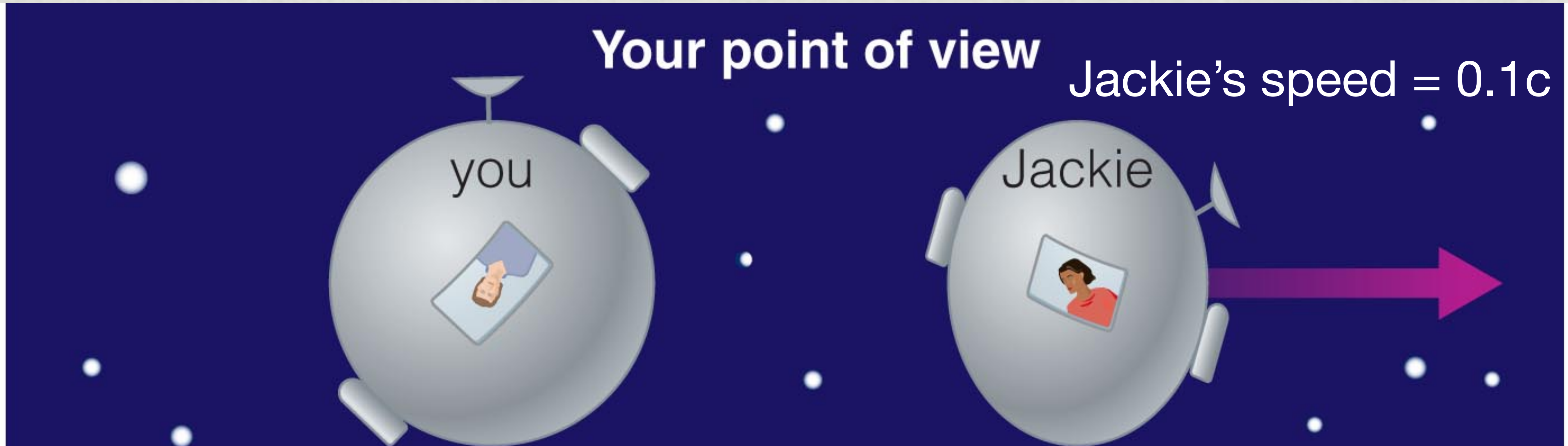
You and Jackie are in identical spaceships.

You each see yourself standing still and the other moving by at speed 0.1c

Jackie measures the size of your spaceship.

How?

She measures the time T on her clock that it takes your ship to pass hers, front to back.



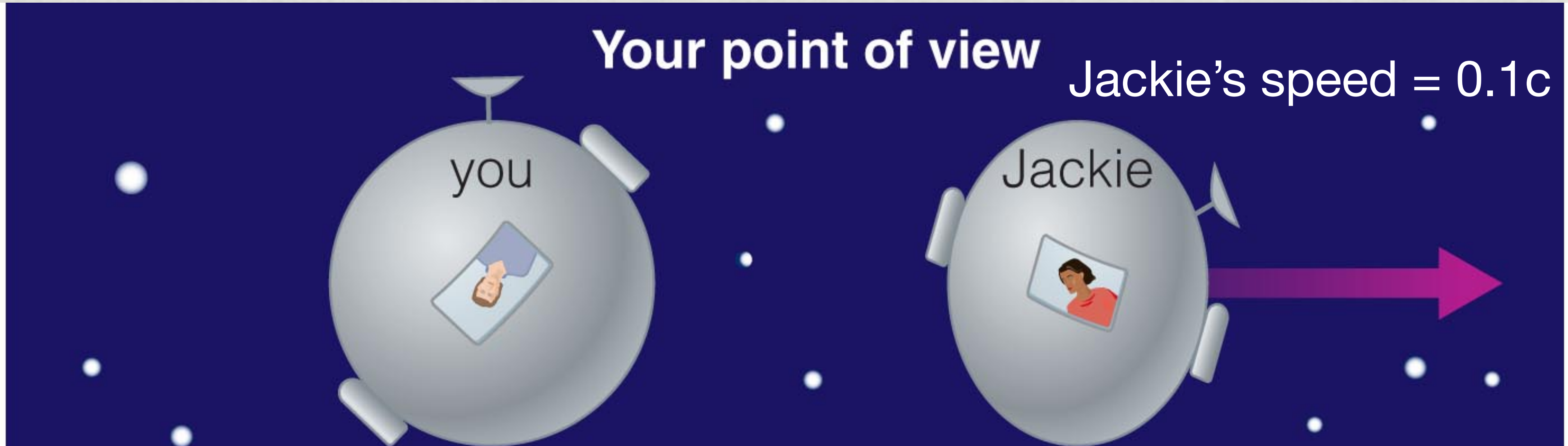
Special Relativity: Length Contraction

Jackie measures the size of your spaceship:
She measures the time T_J on her clock that it takes your ship to pass hers, front to back.

She knows: Distance = Speed x Time

Distance = L, length of your spaceship
Speed = 0.1c

Then she computes
$L_{\text{your spaceship, by Jackie}} = T_J \times 0.1c$



Jackie's speed = 0.1c

Special Relativity: Length Contraction

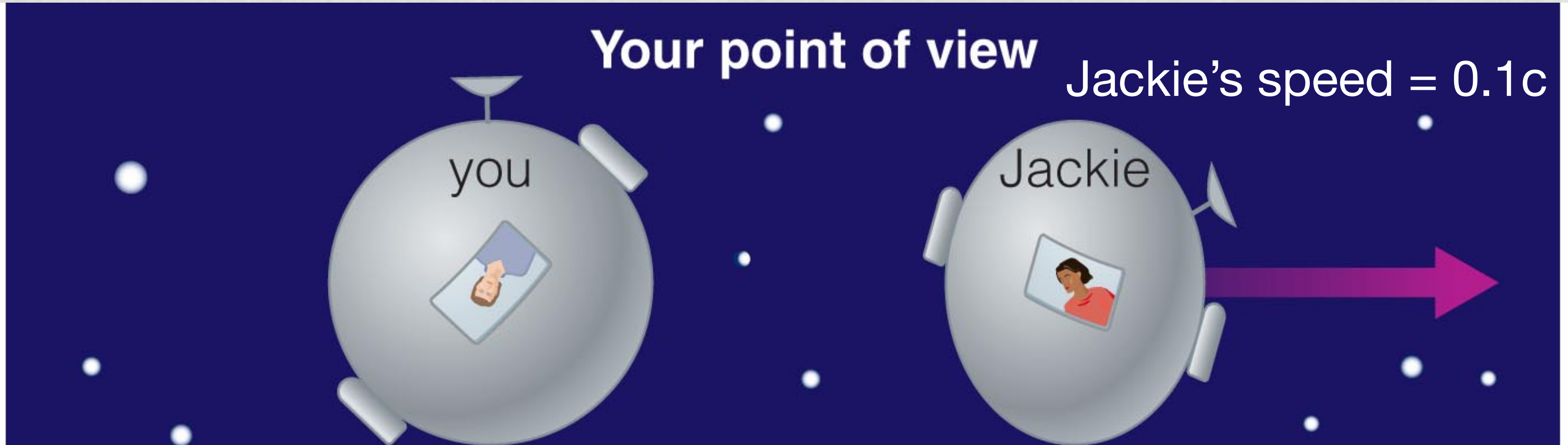
You measure the length L of your spaceship, too.

You measure the time T_{you} on your clock that it takes Jackie's ship to pass by yours, front to back.

You see Jackie move by at speed 0.1c

You know Distance = Speed x Time

Then you compute: $L_{your\ spaceship,you} = T_{you} \times 0.1c$



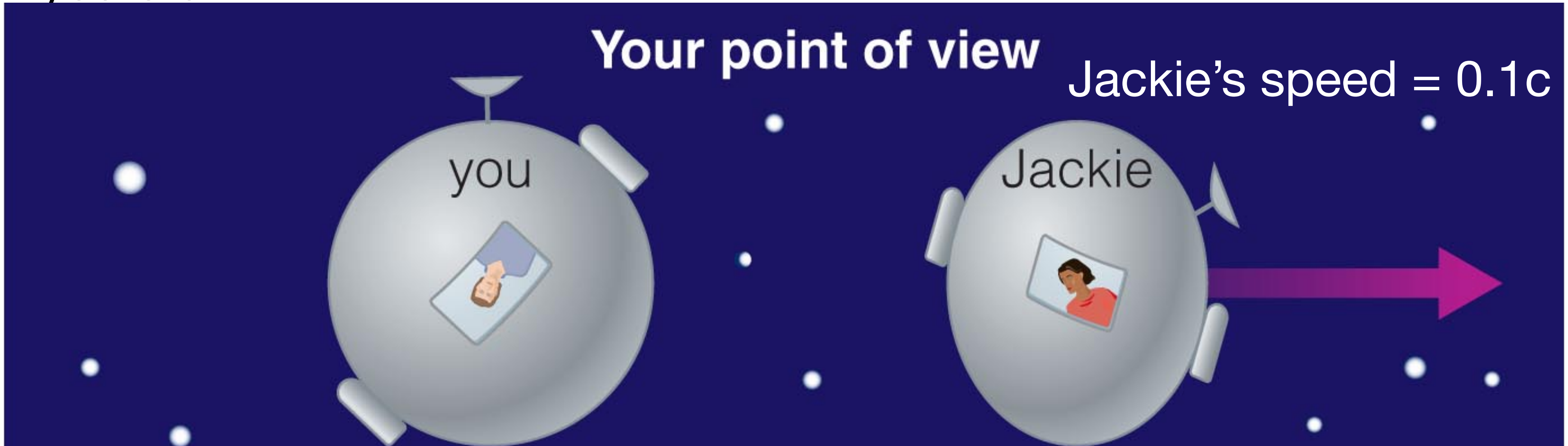
Special Relativity: Length Contraction

Jackie computes the length of your spaceship by looking at her clock as you pass by her spaceship: $L_{\text{your spaceship, by Jackie}} = T_J \times 0.1c$

You measure the length of your spaceship by looking at your clock as her spaceship passes by yours: $L_{\text{your spaceship,you}} = T_{\text{you}} \times 0.1c$

You see Jackie move past you. You see Jackie's clock run slow.
You know the time T_{you} you measured for Jackie's spaceship to pass by you is bigger than T_J, the time she measured for your spaceship to pass by her.

$T_{\text{you}} > T_J$ so you know Jackie computed a smaller size L for your spaceship than you did



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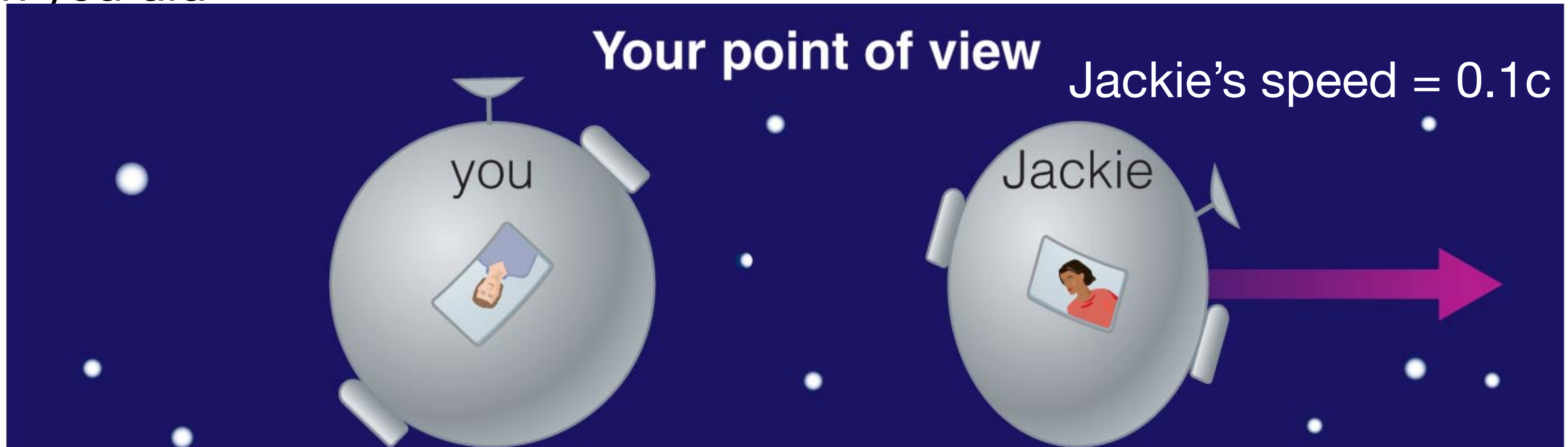

Jackie's speed = 0.1c

Special Relativity: Length Contraction

Jackie computes the length of **her** spaceship by looking at her clock as you pass by her spaceship: $L_{\text{Jackie's spaceship, by Jackie}} = T_J \times 0.1c$

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Jackie sees you move past her. Jackie sees your clock run slow.
She knows the time T_{you} you measured for her spaceship to pass by you is smaller than T_J, the time she measured for your spaceship to pass by her.

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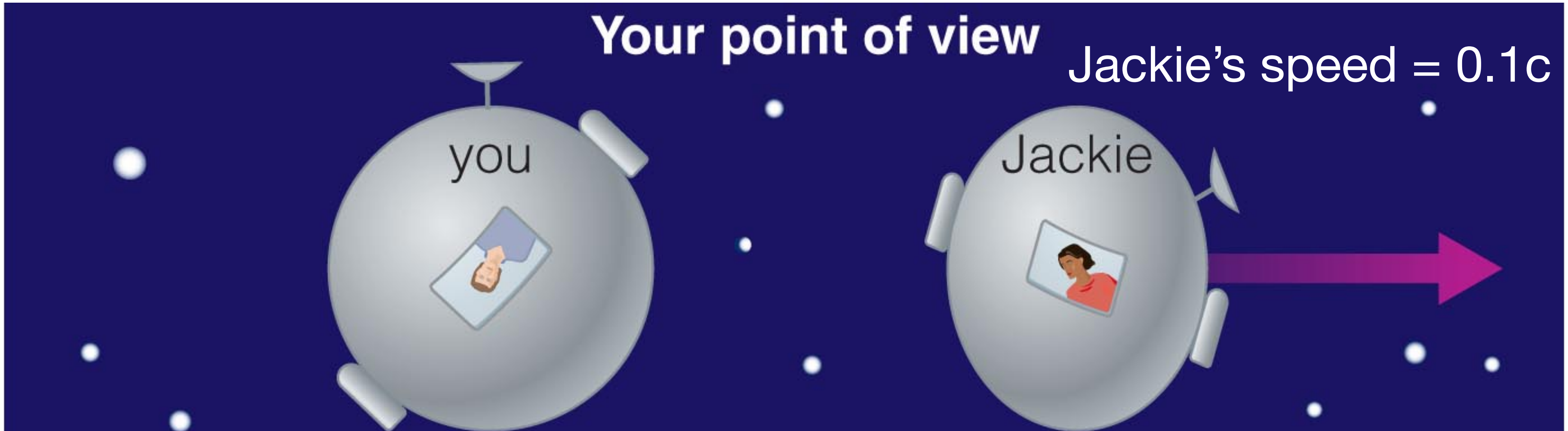


Special Relativity

Implication #1:
In any reference frame moving at velocity close to c (v~c), an outside observer sees TIME SLOW DOWN in the moving frame.

$$\boxed{t_{\text{moving}} = t_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$$

Implication #2:
In a frame moving at v~c, length appears to get shorter

$$\boxed{\text{length}_{\text{moving}} = \text{length}_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$$

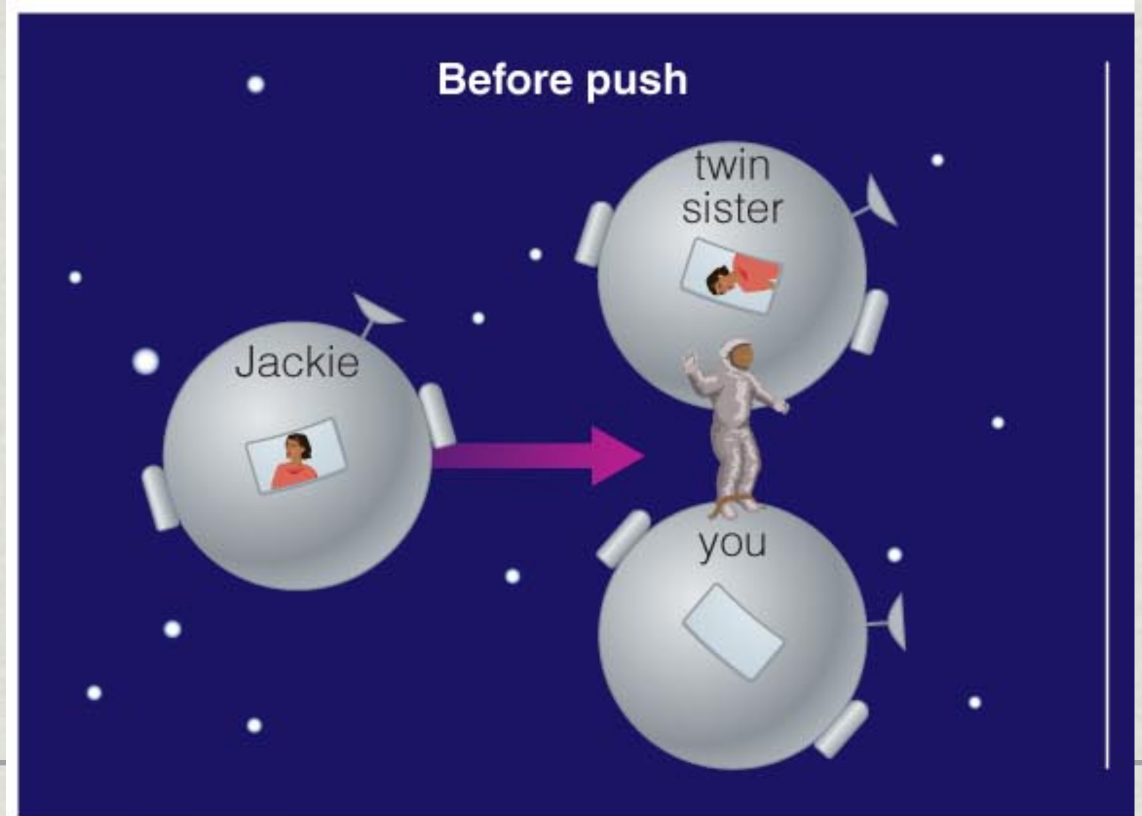
Special Relativity: Mass Increase

Jackie is moving toward you at v = 0.9c

You see her clock run slow.

Jackie’s twin sister is at rest with respect to you.

You give them both an identical push as Jackie passes you by.



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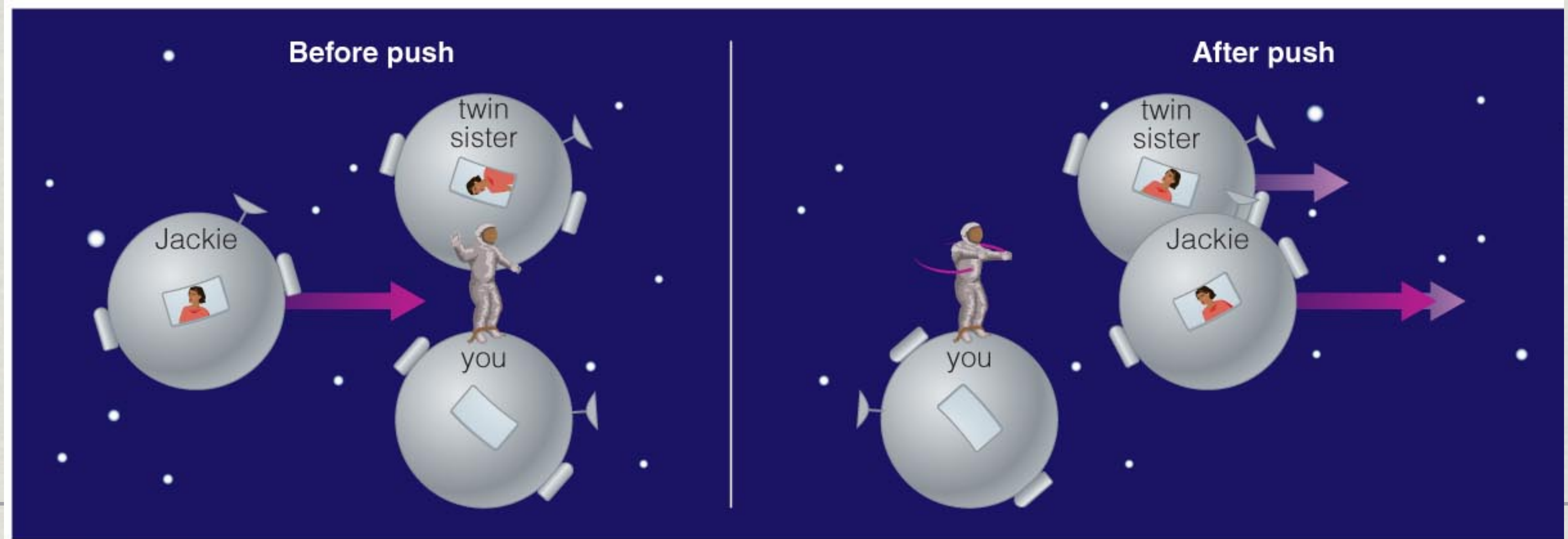


Special Relativity: Mass Increase

Jackie is moving toward you at $v = 0.9c$

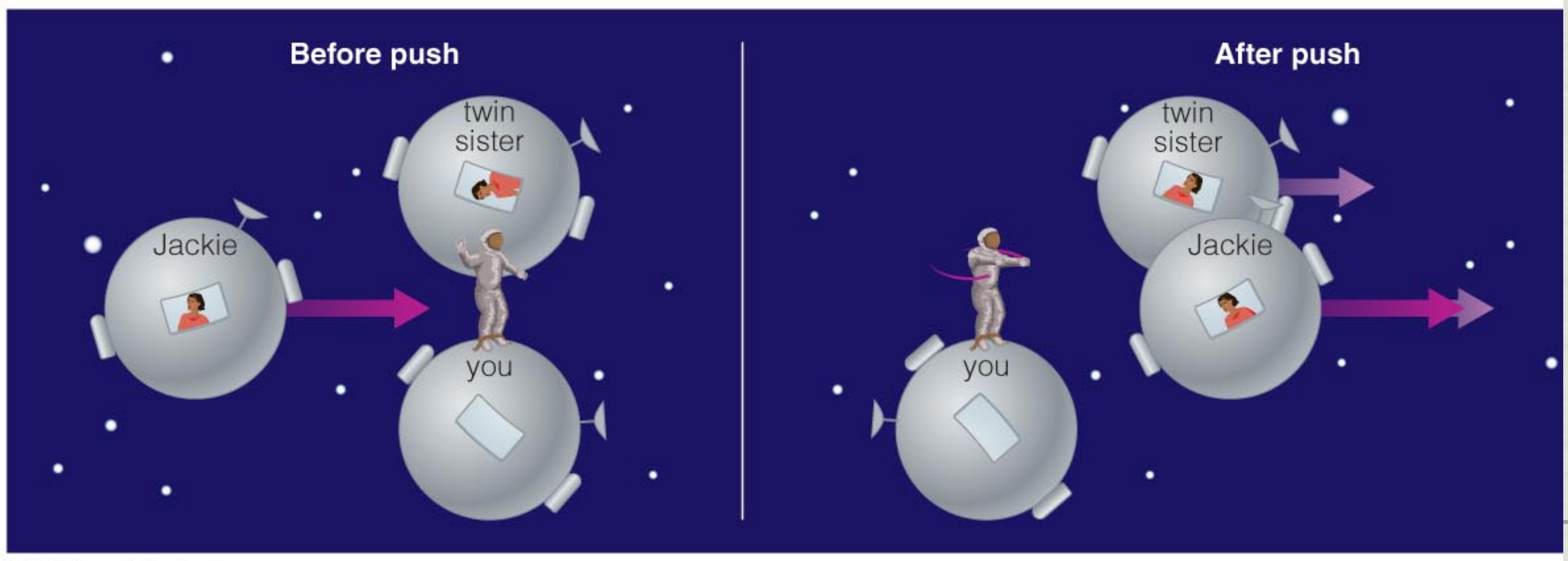
Jackie's twin sister is at rest with respect to you.

You give them both an identical push as Jackie passes you by.

That push increases their velocity: when you push you exert a force on their spaceships, so they get an acceleration: $F = ma$

Units of acceleration: m/s^2 Example: gravity, $9.8\ m/s^2$

If the force of your push accelerates them at $2\ m/s^2$, their speed increases by 2 m/s ever second your push lasts



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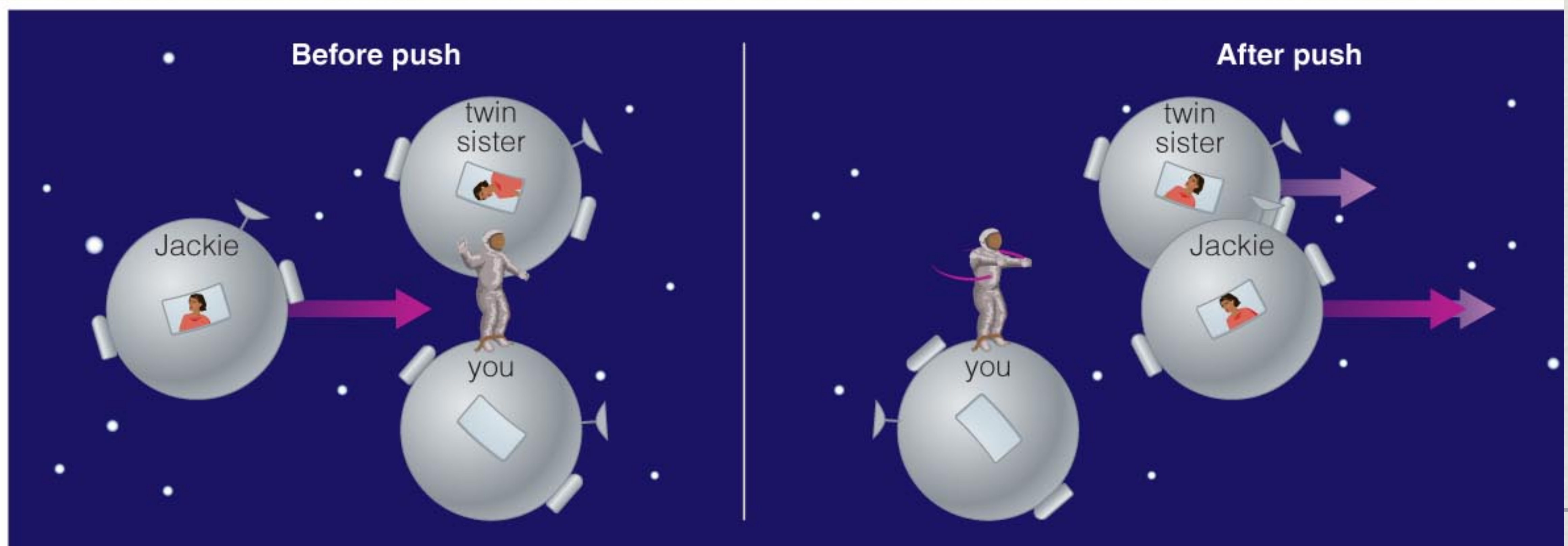
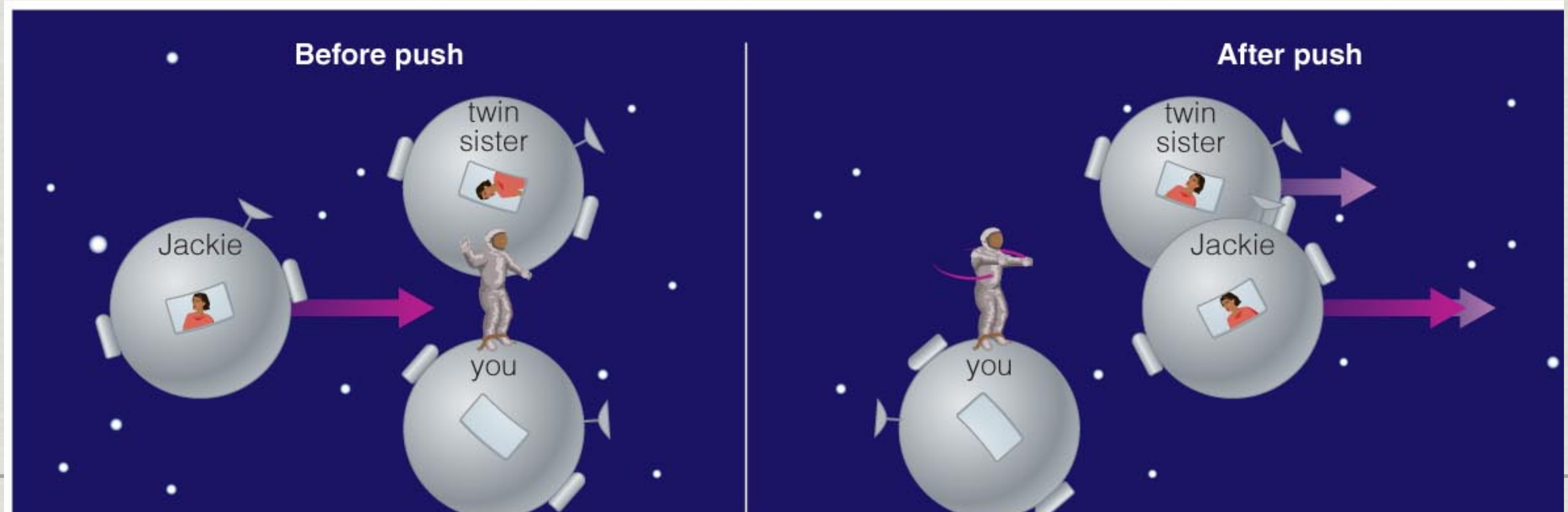


Special Relativity: Mass Increase

You see Jackie's clock run slow, so she gets the push for a shorter time than her sister, so her final speed increase is less.

According to your clock, you gave both sisters the same push (same force for the same amount of time), but you see a smaller increase in Jackie's speed

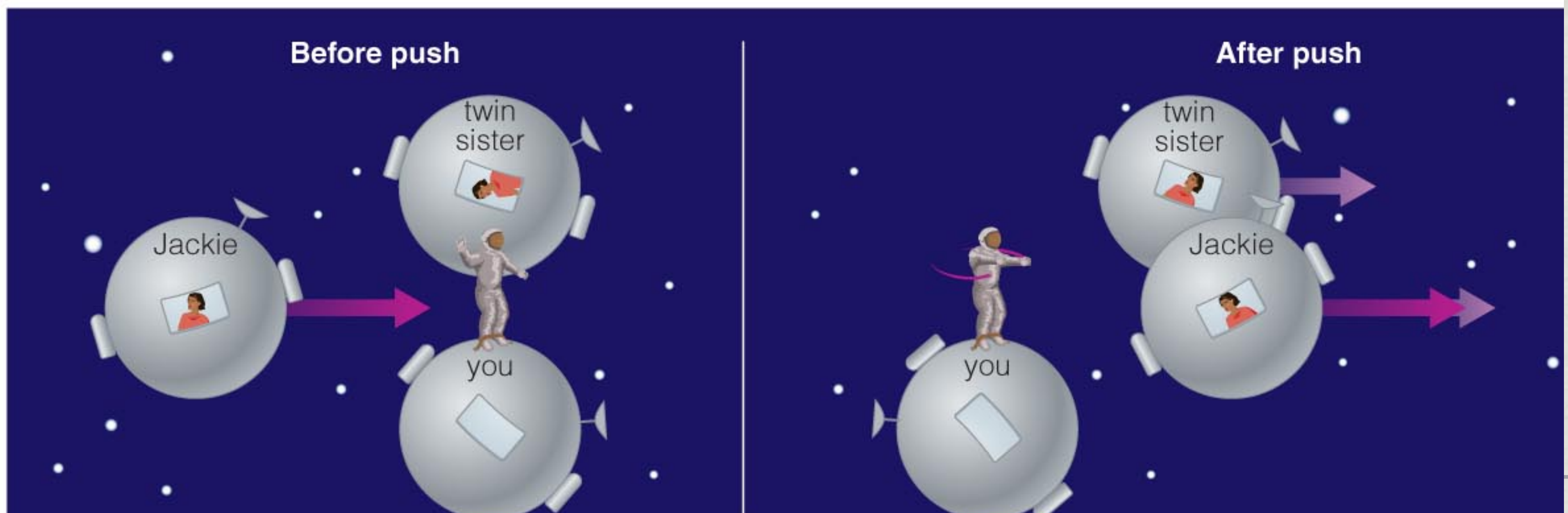


Special Relativity: Mass Increase

According to your clock, you gave both sisters the same push (same force for the same amount of time), but you see a smaller increase in Jackie's speed

Force = mass x acceleration

If you pushed Jackie with identical force, by your clock for the same number of seconds, but her speed increase (acceleration!) was less than her sister's, Jackie's mass must be larger.



Special Relativity

Implication #1:
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Implication #2:
In a frame moving at v~c, length appears to get shorter

$$\boxed{\text{length}_{\text{moving}} = \text{length}_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$$

Implication #3:
In any moving reference frame at v~c, mass* will increase.
It is harder to ACCELERATE in a moving frame.

$$\boxed{m_{\text{moving}} = \frac{m_{\text{still}}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

*really, mass-energy. Remember E = mc^2

Special Relativity

At v=0.9c, $\sqrt{1-\frac{v^2}{c^2}}$ = 0.44

Implication #1:

In any reference frame moving at velocity close to c (v~c), an outside observer sees TIME SLOW DOWN in the moving frame.

$$\boxed{t_{\text{moving}} = t_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}} = 0.44\ t_{\text{still}}$$

Implication #2:

In a frame moving at v~c, length appears to get shorter

$$\boxed{\text{length}_{\text{moving}} = \text{length}_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}} = 0.44\ \text{length}_{\text{still}}$$

Implication #3:

In any moving reference frame at v~c, mass energy will increase.

It is harder to ACCELERATE in a moving frame.

$$\boxed{m_{\text{moving}} = \frac{m_{\text{still}}}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{m_{\text{still}}}{0.44}$$

Special Relativity

At v=0.0001c, $\sqrt{1-\frac{v^2}{c^2}}$ = 0.999999995 ~ 1

At v=0.0001c, $\boxed{t_{\text{moving}} = t_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$ ~ t_{still}

At v=0.0001c, $\boxed{\text{length}_{\text{moving}} = \text{length}_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$ ~ $\text{length}_{\text{still}}$

At v=0.0001c, $\boxed{m_{\text{moving}} = \frac{m_{\text{still}}}{\sqrt{1-\frac{v^2}{c^2}}}}$ ~ $\frac{m_{\text{still}}}{1}$

Special Relativity

“yeah, right...I don’t believe it,” you say.

1) Remember all this matters only if v~c. Otherwise, the effects are tiny.

2) The basic cause of this weirdness is that light always travels at c, no matter what reference frame the measurement is made in. Things would be ***even weirder*** if this fact about light were not true.

Special Relativity

The basic cause of this weirdness is that light always travels at c, no matter what reference frame the measurement is made in. Things would be ***even weirder*** if this fact about light were not true.

If v does not always = c for light, then you would see car A get to the accident before car B.

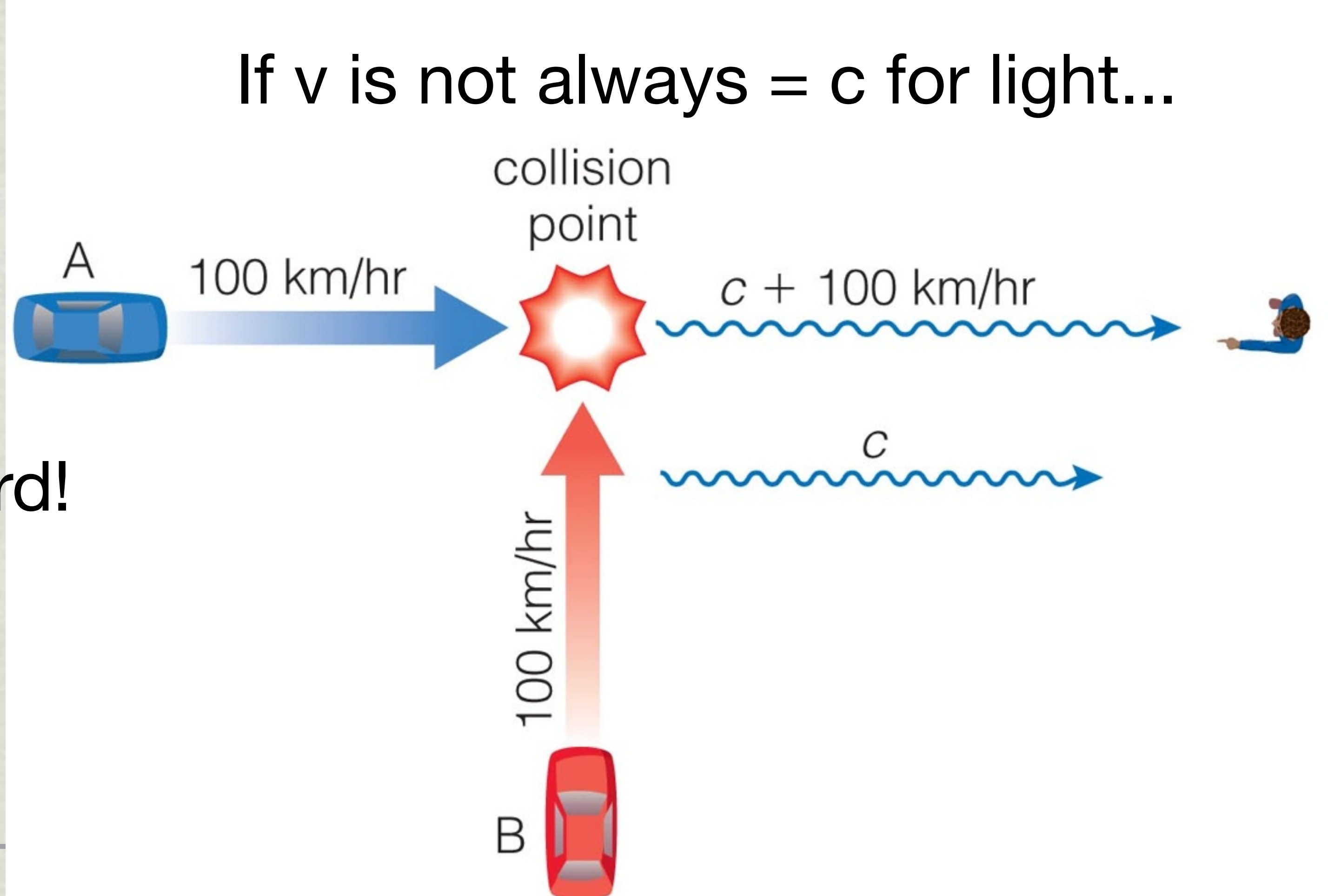
But the passengers experience a crash!

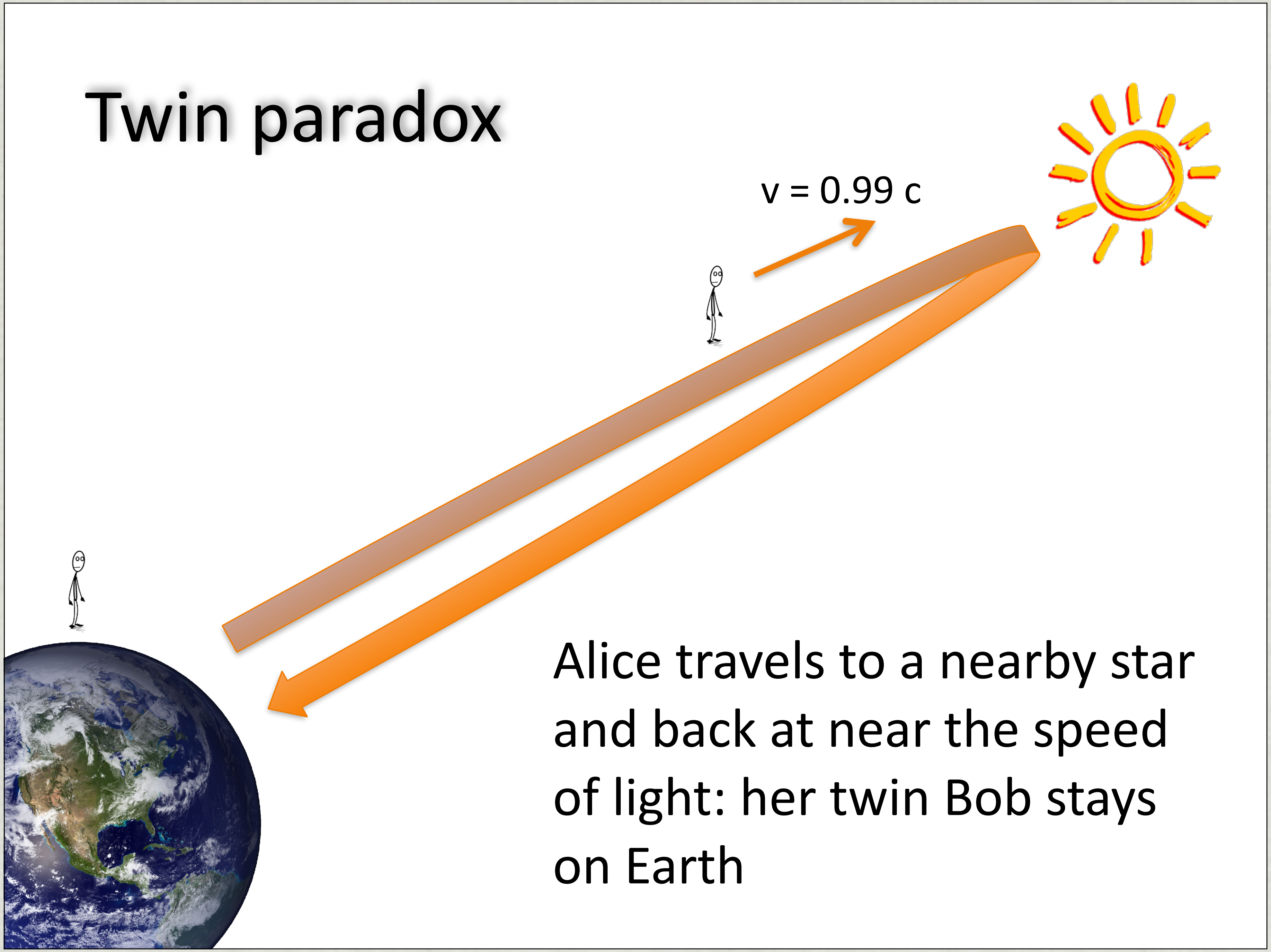
So if v does not always = c, you and the passengers can't agree on what happened.

Crash vs. no crash? That's ***really*** weird!

For 100 mph, $\sqrt{1-\frac{v^2}{c^2}}$

= 0.999999999999989

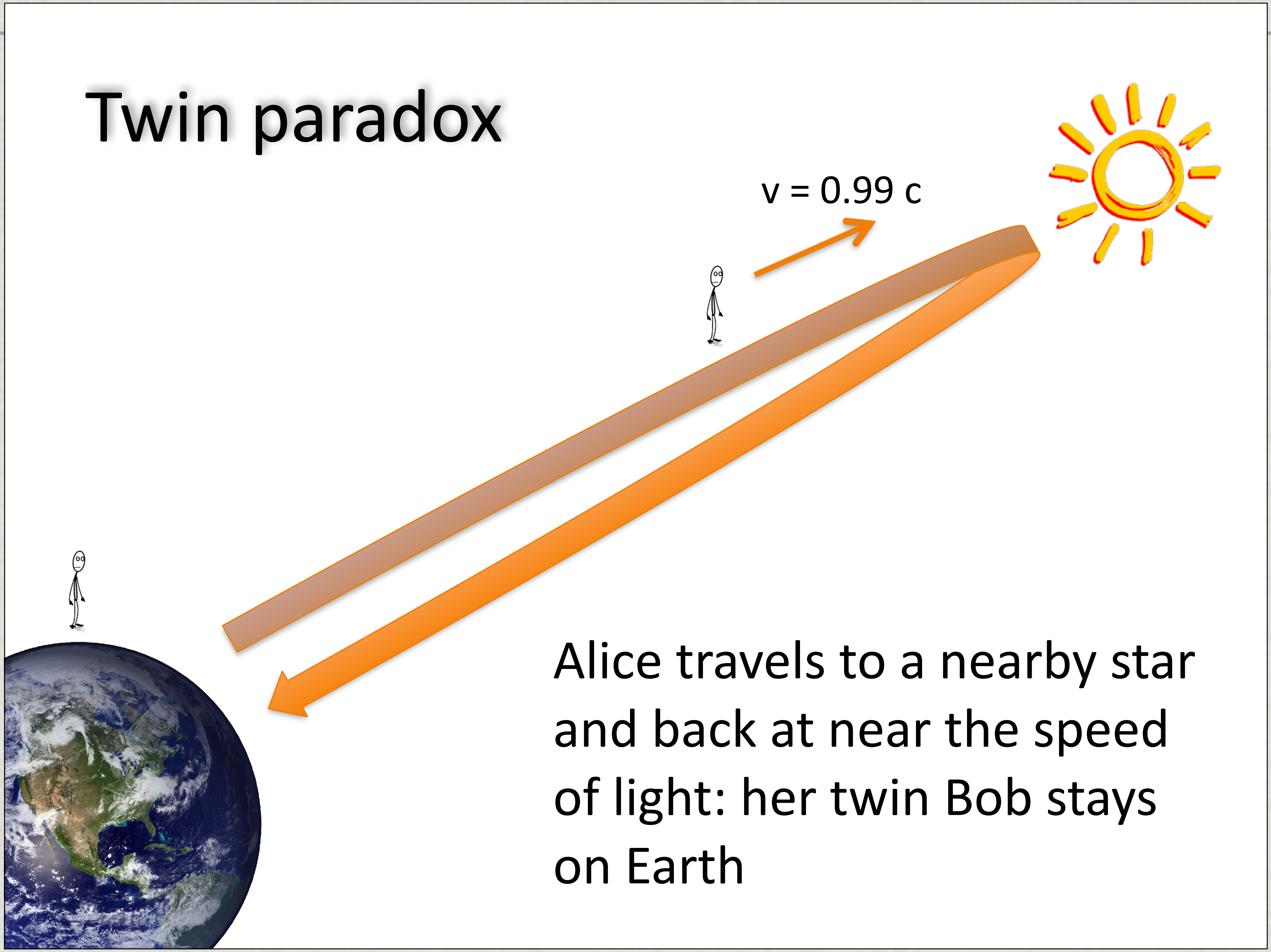

If v is not always = c for light...



Twin paradox

Alice travels to a nearby star and back at near the speed of light: her twin Bob stays on Earth

When Alice returns, who is older?
Don’t they each see the other’s clocks run slow?

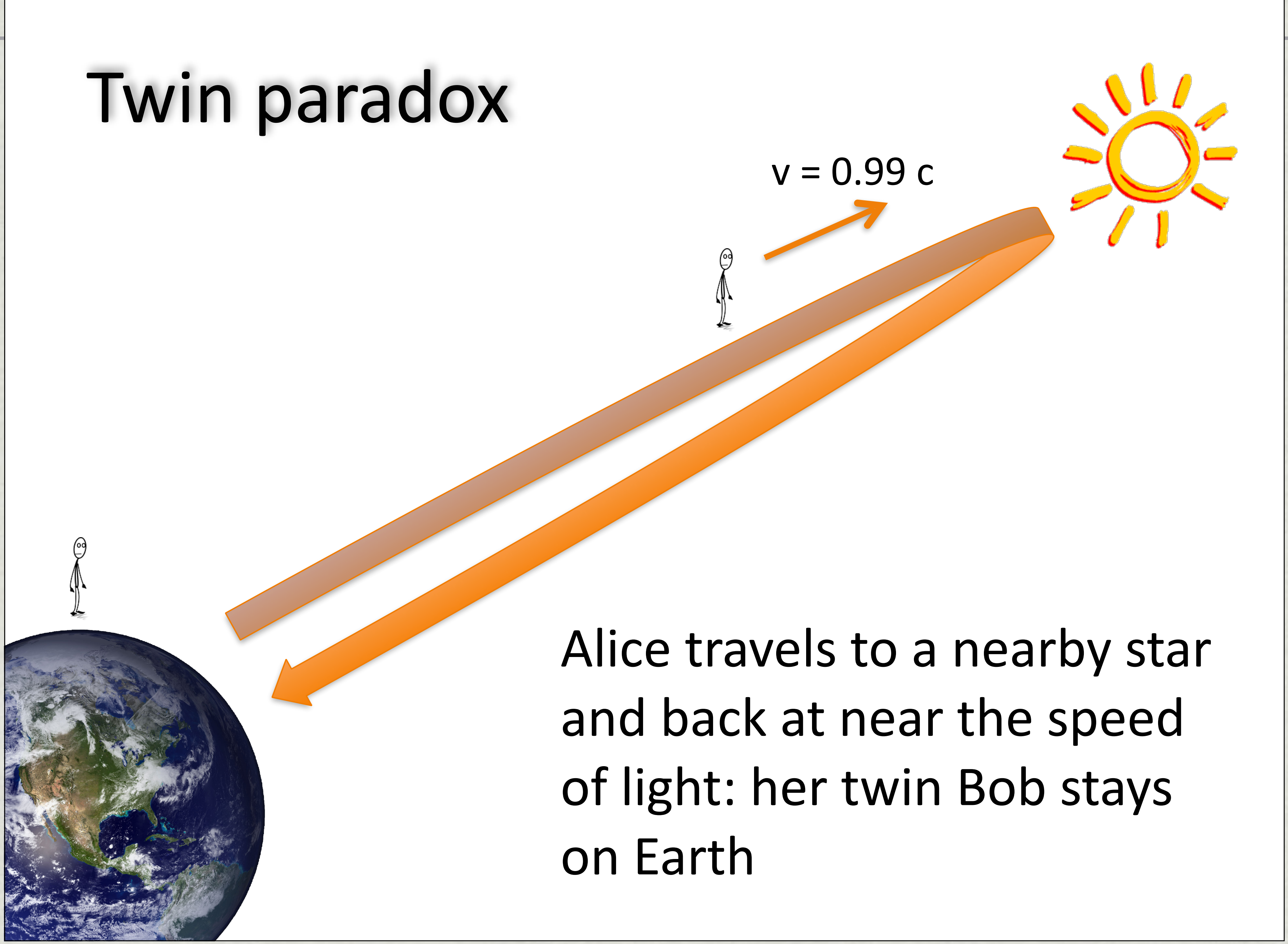


Twin paradox

$v = 0.99\ c$

Alice travels to a nearby star and back at near the speed of light: her twin Bob stays on Earth

Bob sees Alice travel distance D and speed 0.99c, and can use
Distance = speed x time
to compute how long it will take Alis to make the trip.



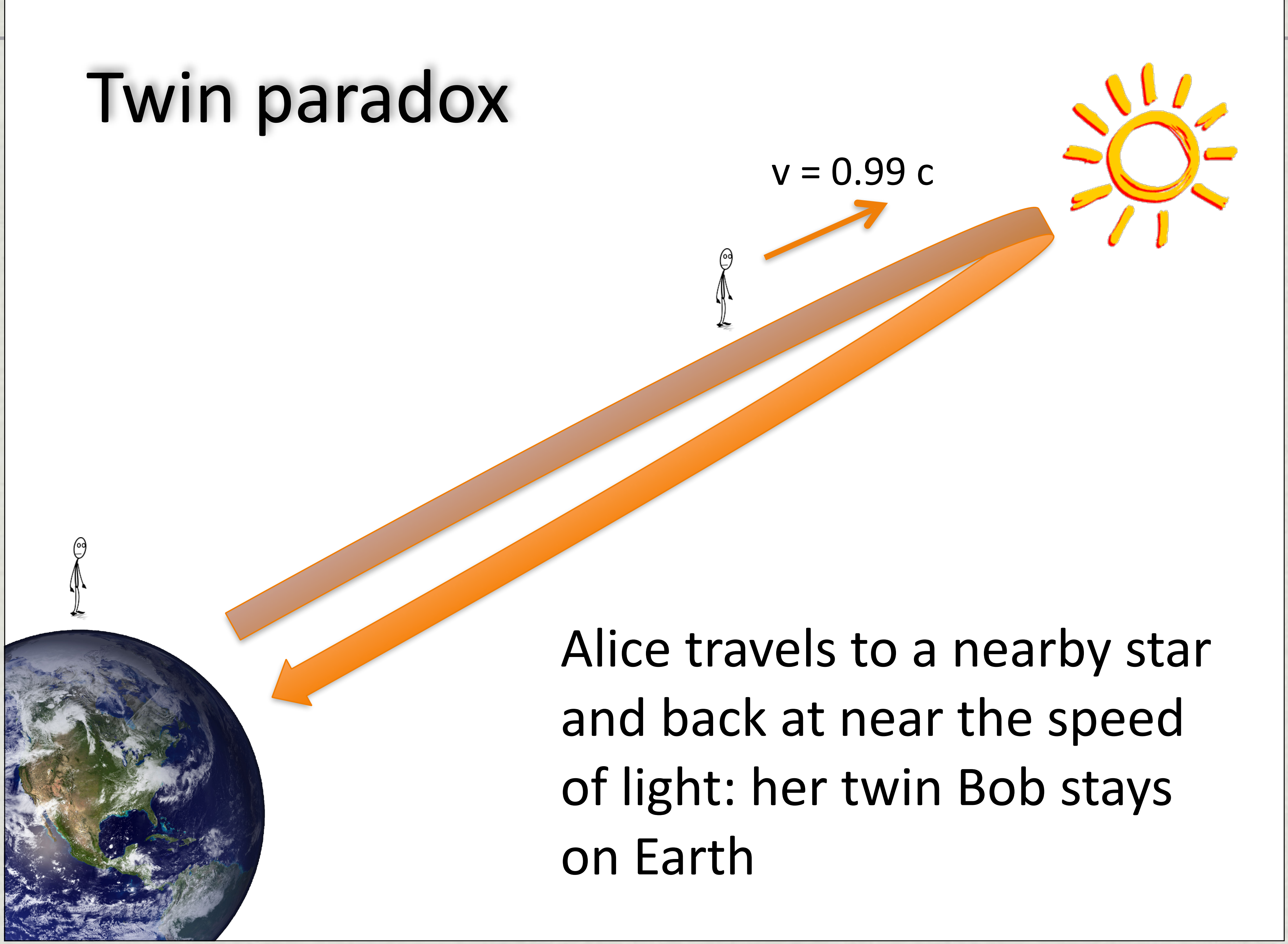
Twin paradox

Alice travels to a nearby star and back at near the speed of light: her twin Bob stays on Earth

Bob knows Alice's clock will run slow:

$$t_{\text{moving}} = t_{\text{still}} \sqrt{1 - \frac{v^2}{c^2}}$$

So less time will pass for Alice during the trip than for him.
He knows Alice will be younger than he is when she returns.

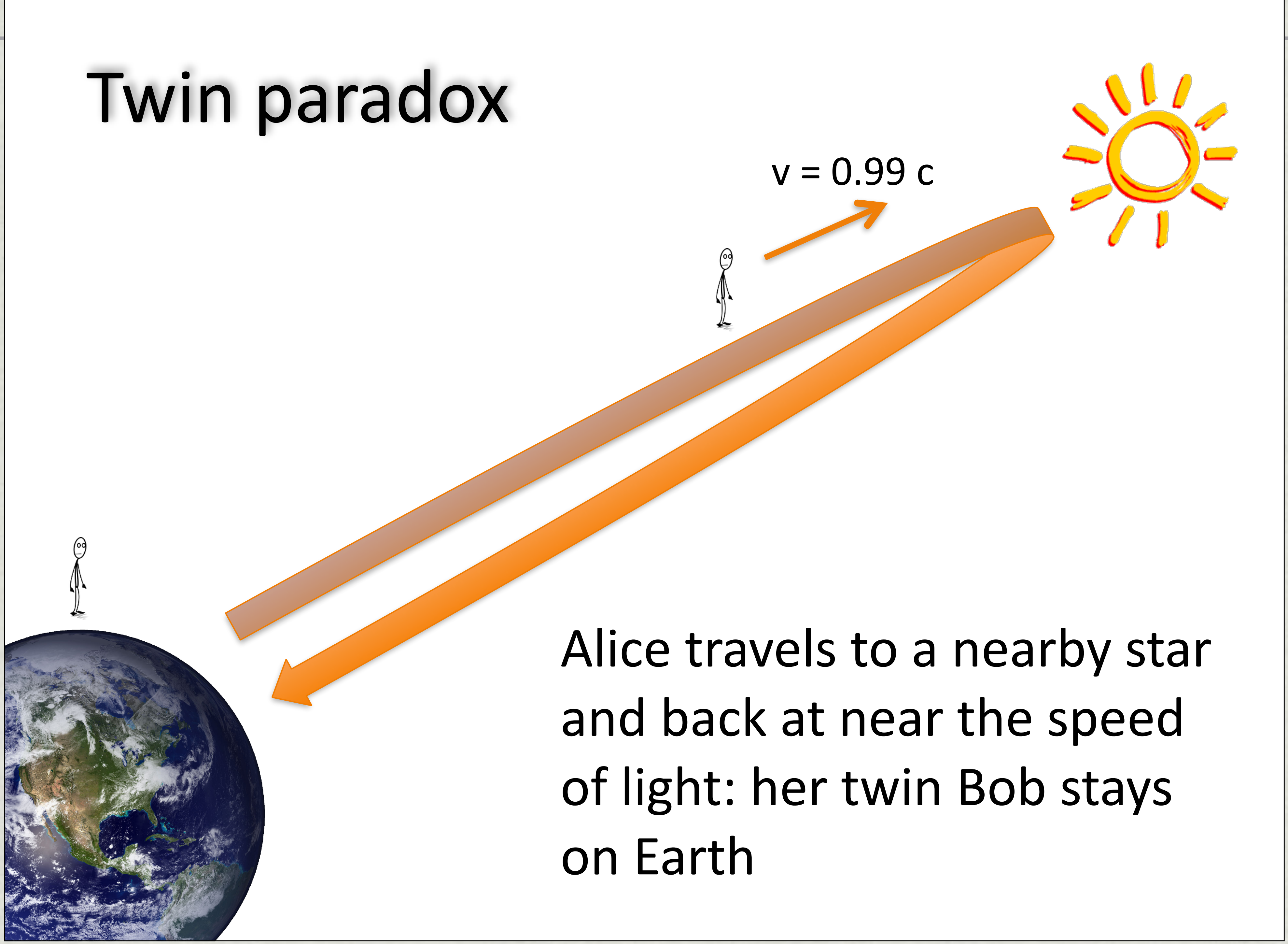


Twin paradox

Alice travels to a nearby star and back at near the speed of light: her twin Bob stays on Earth

What about Alice?

She sees the earth going away from her and the star coming at her at 0.99c



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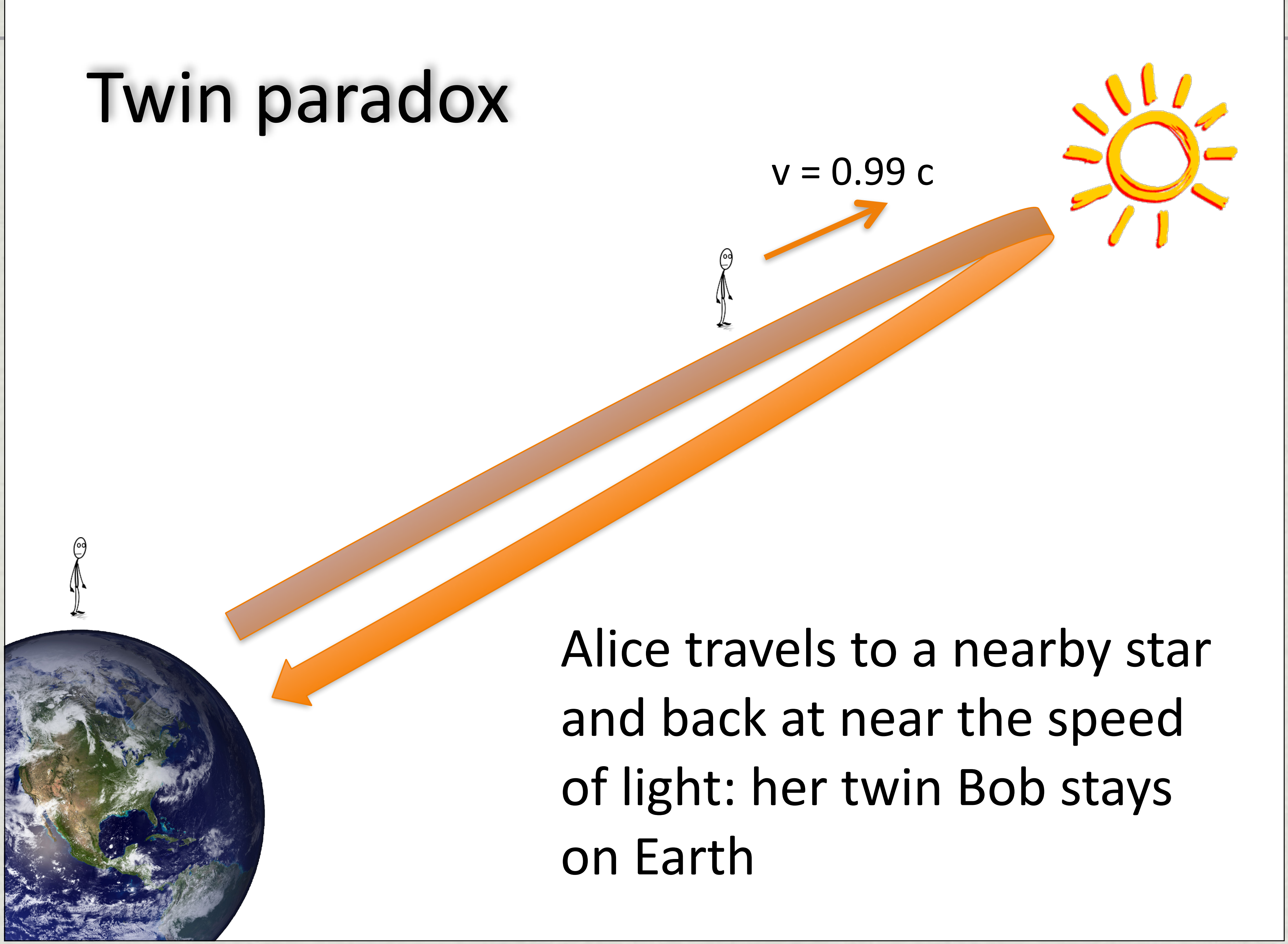


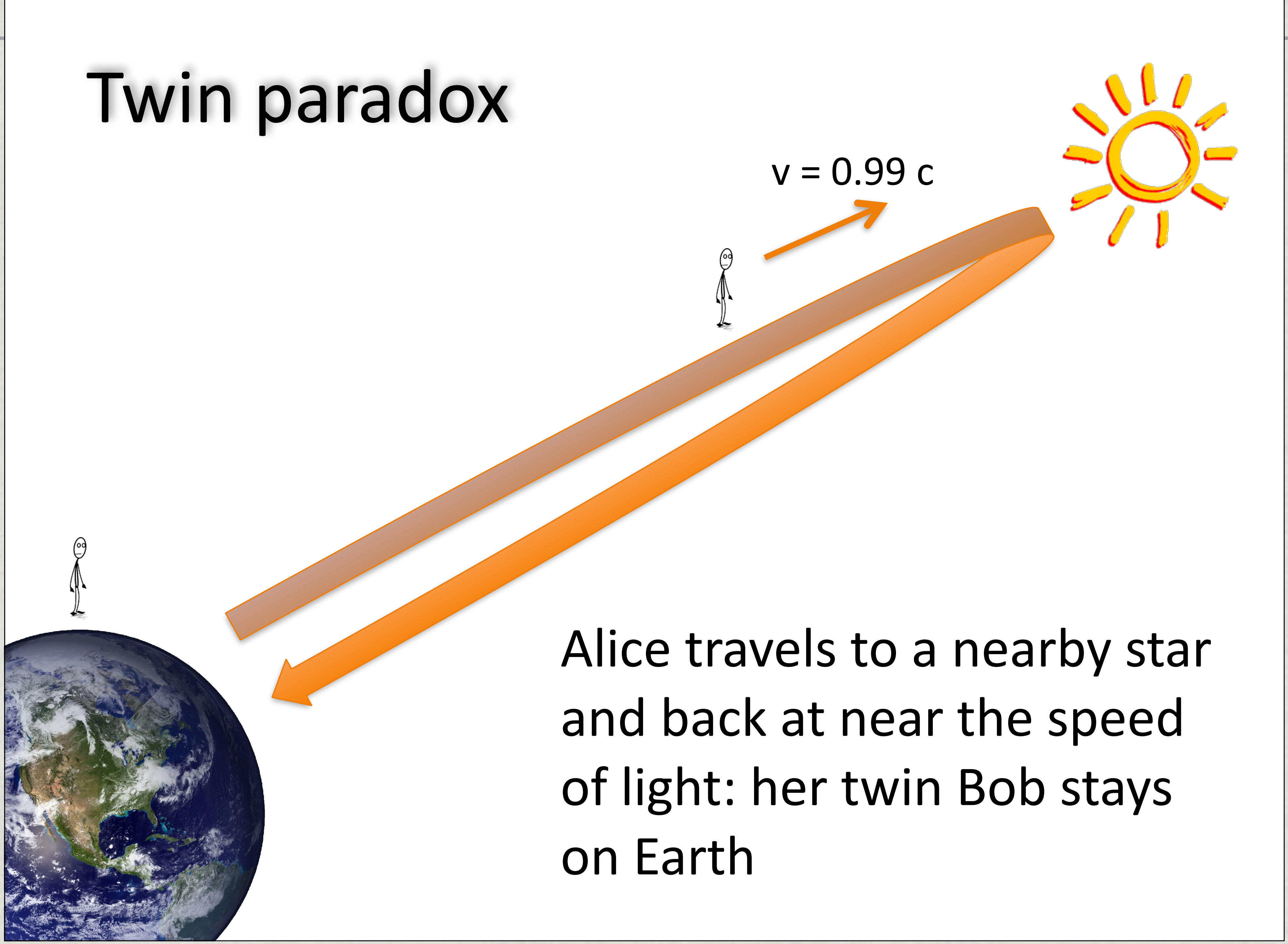
Twin paradox

Alice travels to a nearby star and back at near the speed of light: her twin Bob stays on Earth

She sees the earth going away from her and the star coming at her at 0.99c

She sees the distance between the earth and the star get smaller:

$$\text{length}_{\text{moving}} = \text{length}_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}$$



Twin paradox

Alice travels to a nearby star and back at near the speed of light: her twin Bob stays on Earth

Alice sees the distance between the earth and the star get smaller:

$$\text{length}_{\text{moving}} = \text{length}_{\text{still}}\sqrt{1 - \frac{v^2}{c^2}}$$

She uses Distance = speed x time to compute how long her trip will take.

Twin paradox

Alice sees a smaller distance between the earth and the star than Bob, so she computes a smaller time for her trip than Bob does.

The time she computes for her trip is smaller by exactly the same amount Bob sees her clock run slow.

Everyone agrees: Alice is younger than Bob when she returns.

Why is this a Paradox?

Ali

sm

She uses Distance = speed x time to compute how long her trip will take.

$E = mc^2$

$$\boxed{m_{\text{moving}} = \frac{m_{\text{still}}}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

If x is small compared to 1: $(1 + x)^{-1/2} \sim 1 - \frac{1}{2}x$

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \sim 1 - \frac{1}{2}\left(-\frac{v^2}{c^2}\right) = 1 + \frac{1}{2}\frac{v^2}{c^2}$$

So: $m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \sim m_0\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$

$$m = m_0 + \frac{1}{2}\frac{m_0 v^2}{c^2}$$

$$mc^2 = m_0c^2 + \frac{1}{2}m_0v^2$$

Left side: total energy of a moving particle

Last term: kinetic energy of particle

m_0c^2 = energy of particle even at rest

Applied Relativity

“Bah. Who cares?” you ask.

Applied Relativity

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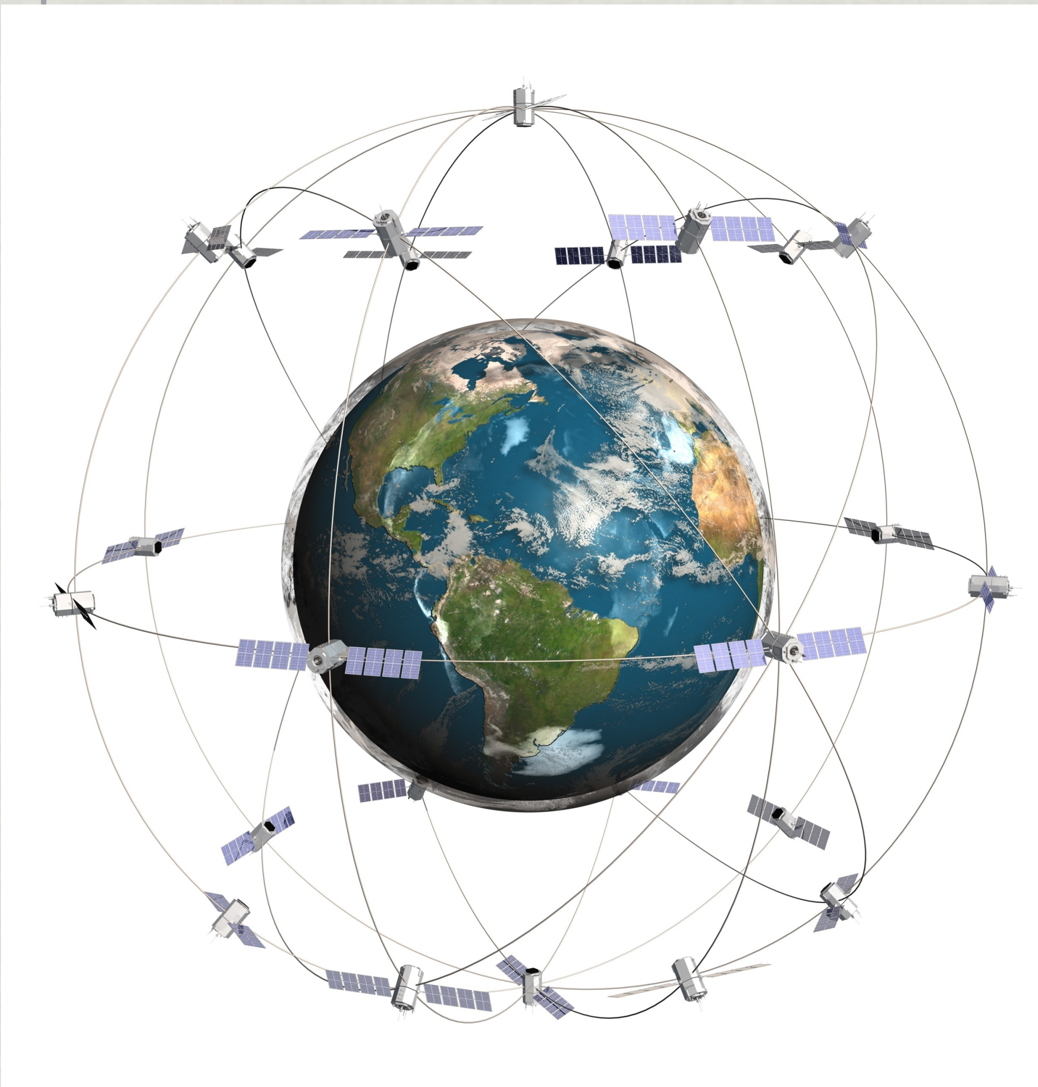
GPS satellites:
Orbital speed 14,000 km/hr = 3.9 km/s = 1.3 x 10^{-5}c

$$\boxed{t_{\text{moving}} = t_{\text{still}}\sqrt{1-\frac{v^2}{c^2}}}$$

→ clocks run slow
by 7 x 10^{-6} seconds/day

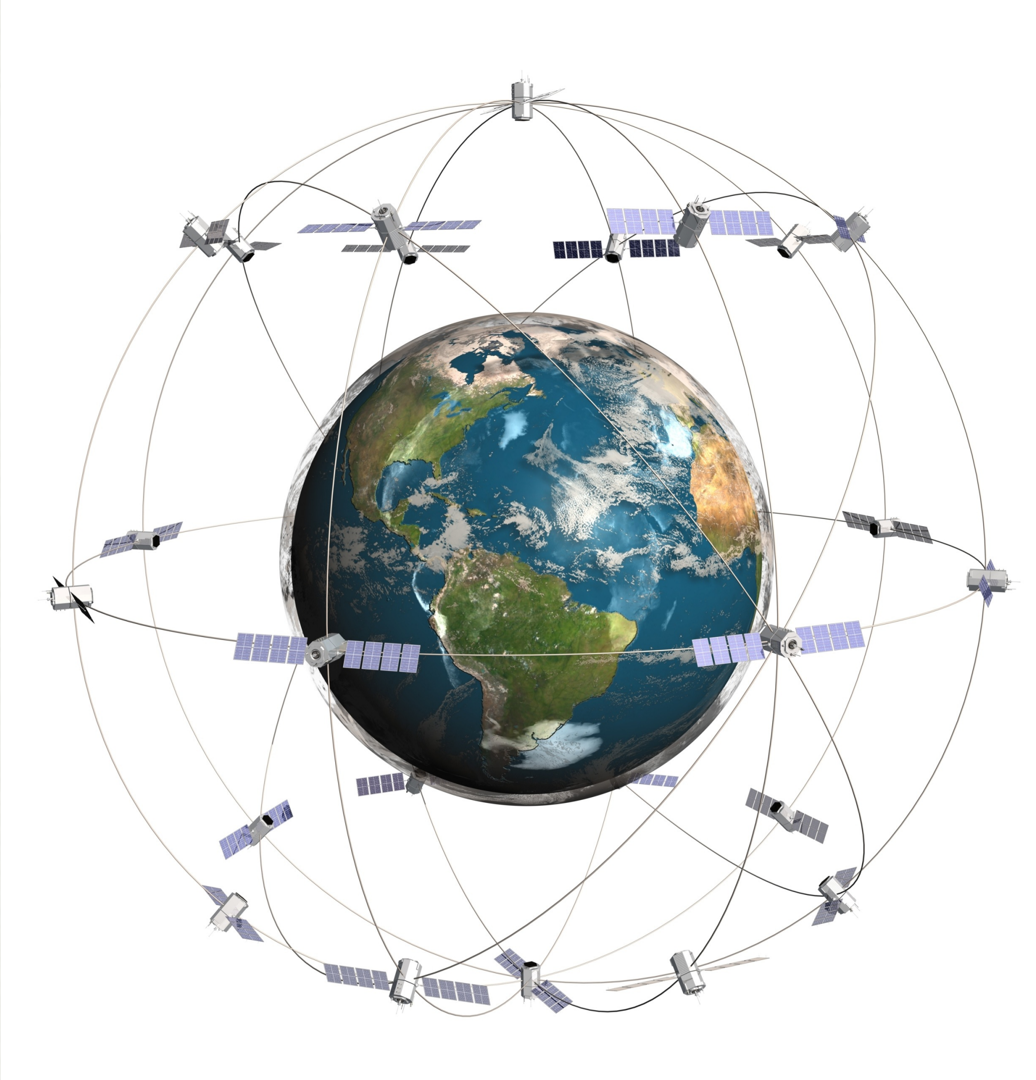


Applied Relativity

“Bah. Who cares?” you ask.

Special Relativity means GPS clocks run slow by 7 x 10^{-6} seconds/day than clocks on earth.

How does GPS work?



Speed of light is constant.

Talk to three GPS satellites with your receiver (or phone, etc.)

Each one knows where it is in its orbit.
Each one tells you what time it replied to you.

Measure time delay for receiving all three replies.

Distance = speed x time

Figure out your distance to all three satellites.
That tells you where you are.