

# Announcements

- ❖ Please fill out an on-line course evaluation
- ❖ Final Exam: Wednesday, 3/22, 7:30pm
  - 3 hours
  - same format, rules as midterm: multiple choice with formula sheet, closed book and notes, bring a #2 pencil and a non-internet-enabled calculator
- ❖ Cumulative: study the midterm material, too
- ❖ Review sessions:
  - Marie Friday 3/17, 4-5pm NatSci2 Annex 101
  - Plato Sunday, 3/20, 3-4pm, NatSci2 Annex 101

# Final Exam Review

- ❖ Review the midterm. There will be material from the first half of the class on the final. The best way to study for that is to review the midterm.
- ❖ Review the homework and reading assignments
- ❖ Go to the TA review sessions
- ❖ Formula sheet will be available and posted on the course web pages.

# Properties of Thermal Radiation

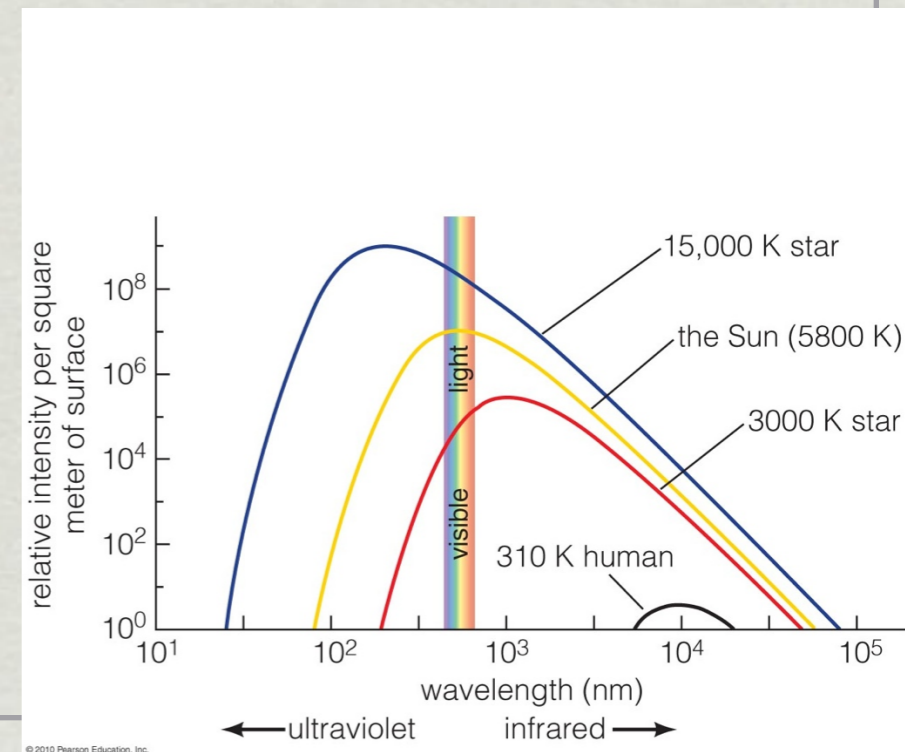
**Stefan - Boltzmann Law:** Energy output per second per meter<sup>2</sup> on the surface of an object =  $\sigma T^4$

**Luminosity** = total energy flow into or out of an object  
Units: Energy/sec or Power, Joules/s or Watts

**Stefan - Boltzmann Law:**

**Luminosity** = Total surface area of object  $\times \sigma T^4$

Universal constant:  
 $\sigma = 5.7 \times 10^{-8} \text{ J}/(\text{s m}^2 \text{ K}^4)$



# Kirchoff's Laws

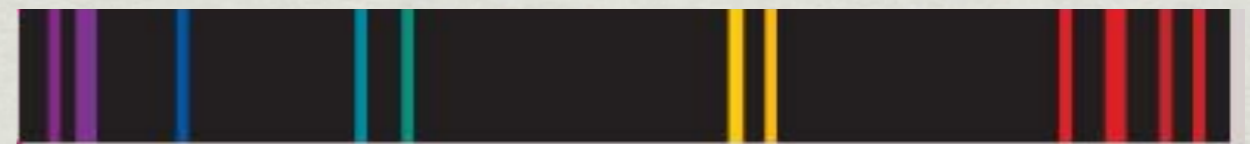
1) Dense, opaque objects: continuous spectrum, thermal emission



Wavelength

2) Transparent, low-density matter:

If too hot to re-absorb photons: emission line spectrum



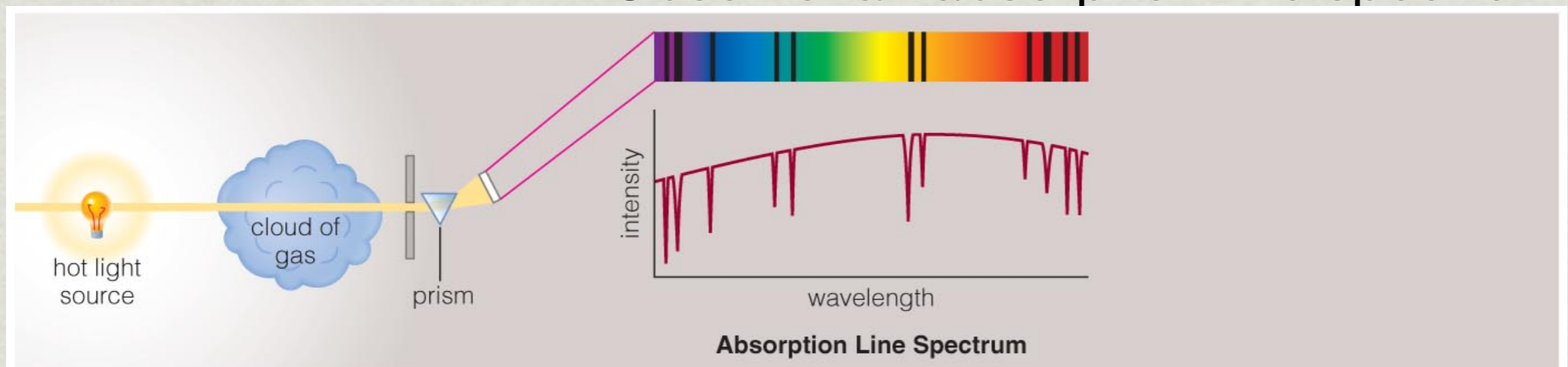
Wavelength

# Kirchoff's Laws

3) Thermal emission traveling through some other matter?

Transparent, cool matter: atoms can re-absorb the light

Observe: an absorption line spectrum



This is often the spectrum we observe from stars and galaxies (which are made of stars)

Exception: hot gas in galaxies, where stars are born or black holes live.

# Doppler Shift

$$\frac{V_{\text{radial}}}{c} = \frac{\Delta\lambda}{\lambda_{\text{rest}}} = \frac{\lambda_{\text{shift}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}}$$

We can measure the velocity (direction and speed) of a light source from its Doppler Shift.

Laboratory spectrum



Rest

Object 1



Red shift, moving away

Object 2



Larger red shift, moving away faster

Object 3



Blue shift, moving toward you

Object 4



Larger blue shift, moving toward you faster

# Recap: Nucleosynthesis

How do you make energy out of  $2 \times 10^{30}$  kg of Hydrogen?

One Helium atom is **less** massive than 4 Hydrogen atoms?

4 H atoms = 4 protons:  $6.693 \times 10^{-27}$  kg

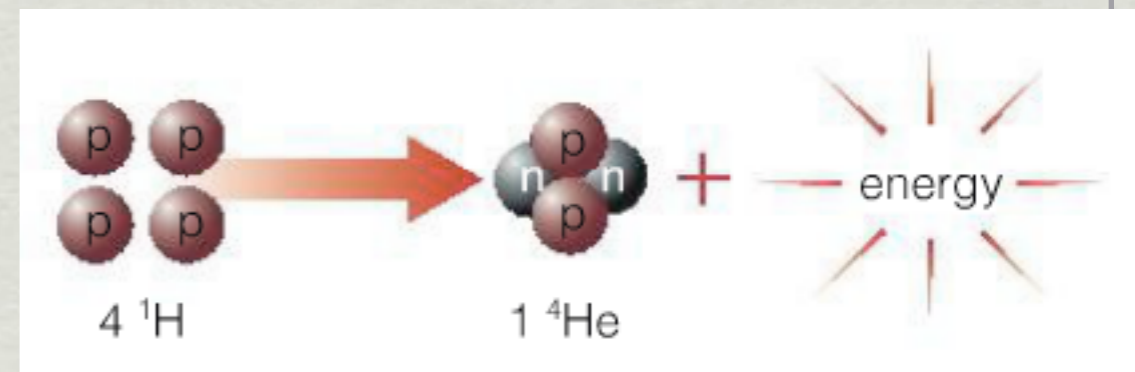
1 He atom = 2 protons + 2 neutrons:  $6.645 \times 10^{-27}$  kg (less massive!  
by 0.7%)

If you stick together Hydrogen atoms to make Helium, the extra mass has to go somewhere.

It becomes energy:  **$E = mc^2$**

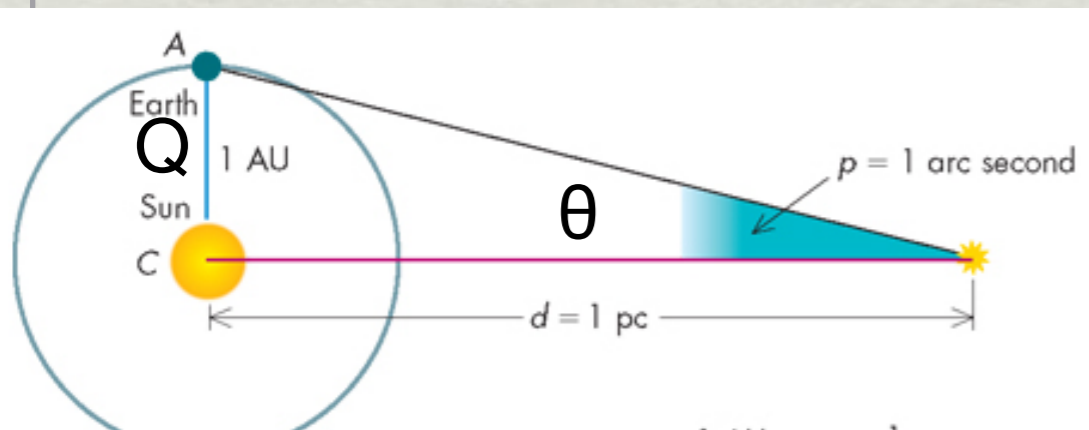
Mass and energy are the same thing, and transform back and forth using this equation.

**Fusing** Hydrogen into Helium must release energy



# Measuring Distance

Distance  $d$  to a star with a parallax measurement  $p$  of 1 arc second



Units for angles:

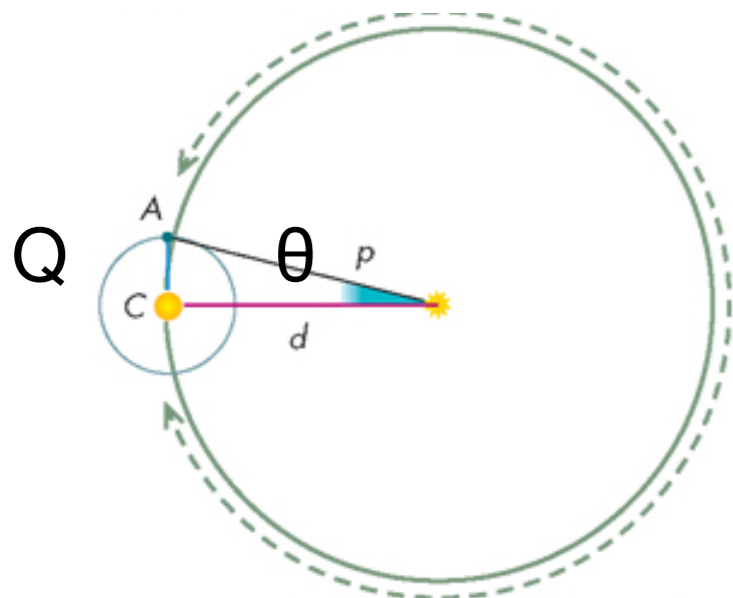
1 degree = 60 arcminutes = 3600 arcseconds

In symbols:  $1^\circ = 60' = 3600''$

$1' = 60''$

$$d = \frac{Q}{p''} \times 206,265$$

$$d = \frac{(1 \text{ AU}) \times 206,265}{p''}$$



Circumference =  $2\pi \times 1 \text{ pc} = 2\pi d$

Define a REALLY convenient unit:  $\Rightarrow$

**1 Parsec** = 206265 AU

=  $3 \times 10^{16}$  m

$$d \text{ (parsec)} = \frac{1}{p \text{ (arcseconds)}}$$

$d$  = distance in parsec,  $p$  = parallax angle in arcseconds

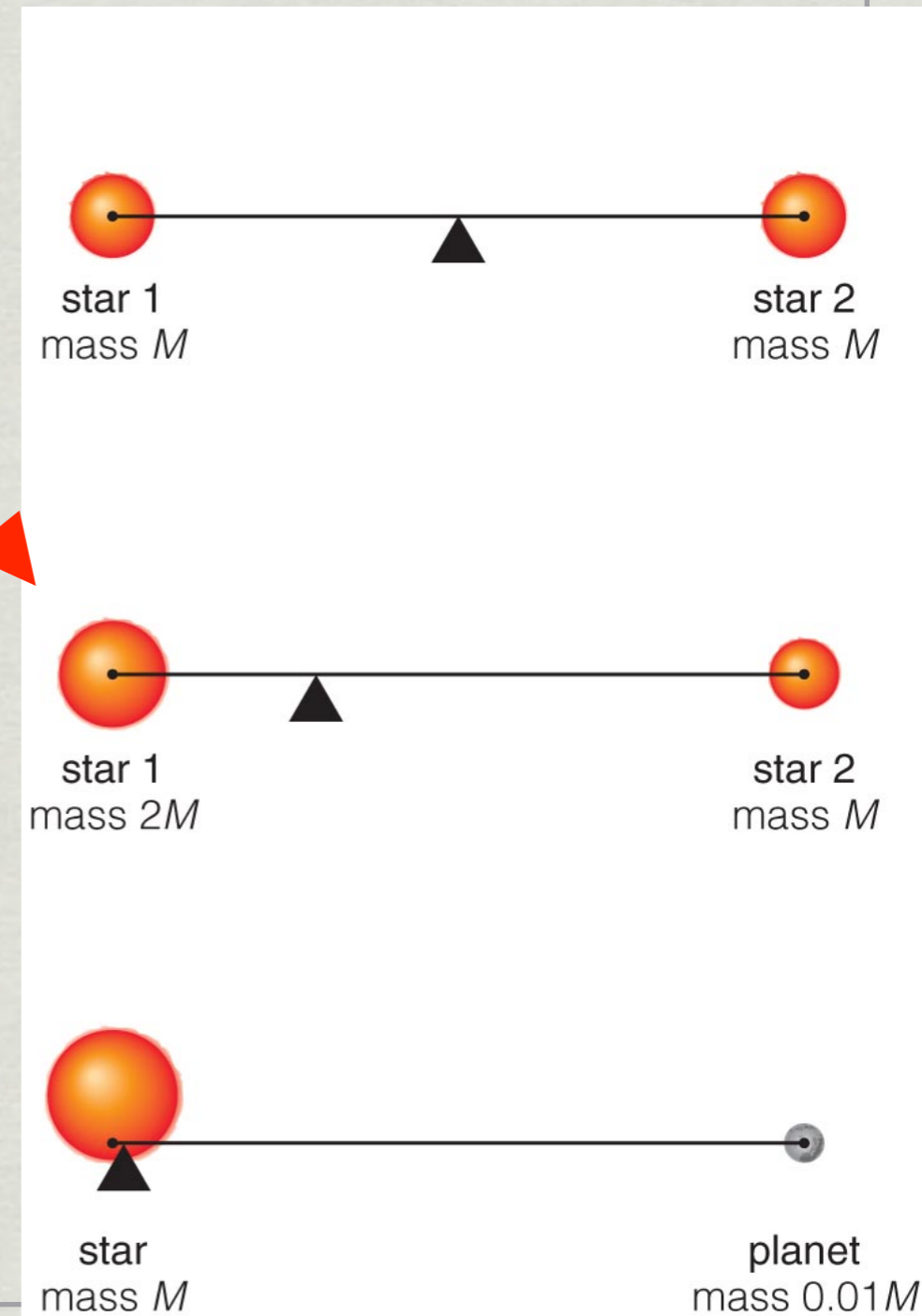


# Measuring Masses of Stars

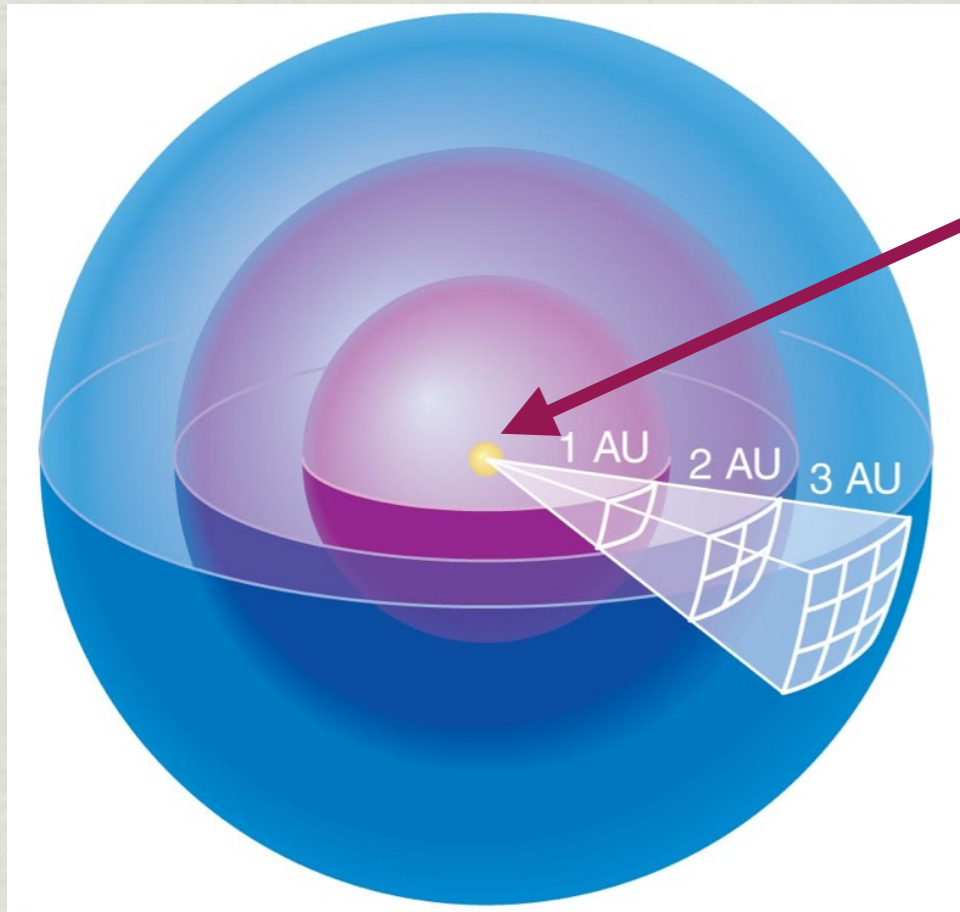
What if  $M_1$ ,  $M_2$  are both large, like binary stars?  
Objects always orbit the Center of Mass.  
CM stays still.  $M_1$  and  $M_2$  orbit around it.

Use: 
$$P^2 = \frac{4\pi^2}{G(M_1+M_2)} A^3$$

Measure  $P$ ,  $A$ , solve for  $M_1+M_2$



# Measuring Luminosity



Luminosity: total amount of energy put out by an object. Units: Joules/sec

Always the same, a property of an object (table, me, star, ...)

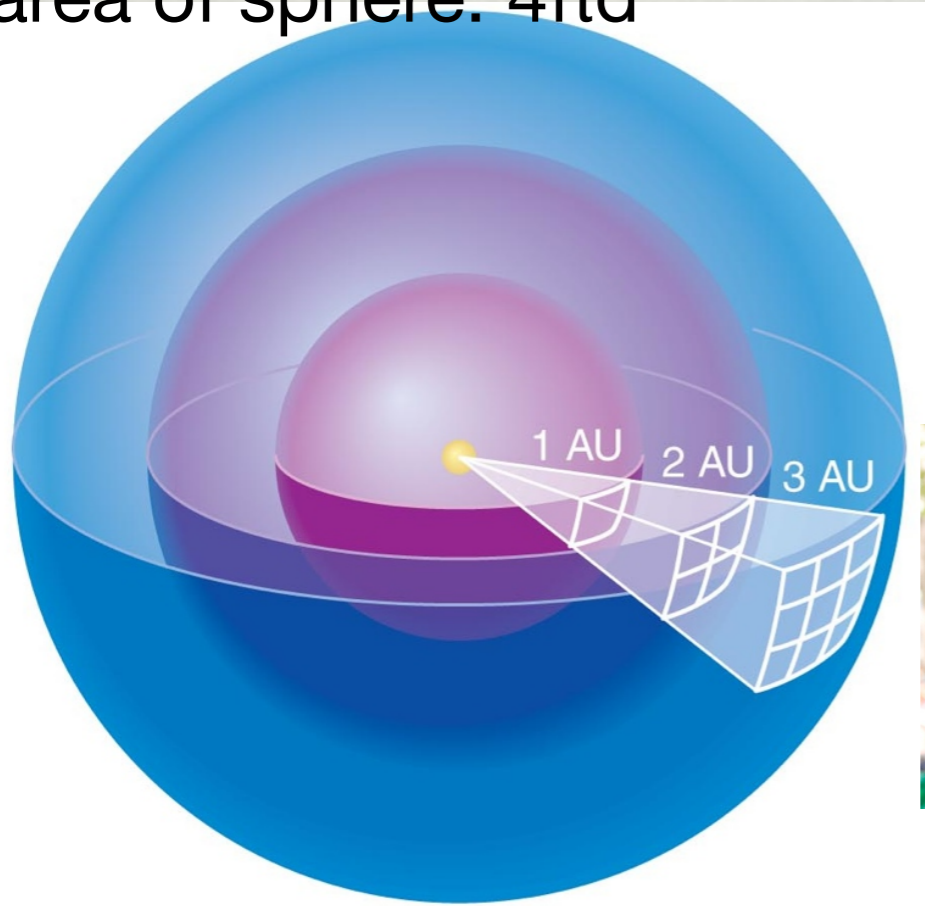
Depends on temperature and surface area of the emitting object

Stefan - Boltzmann Law:

$$\text{Luminosity} = \text{Total surface area} \times \sigma T^4$$

# Measuring Luminosity

area of sphere:  $4\pi d^2$



What can we measure?

Light passing through a patch on the sphere



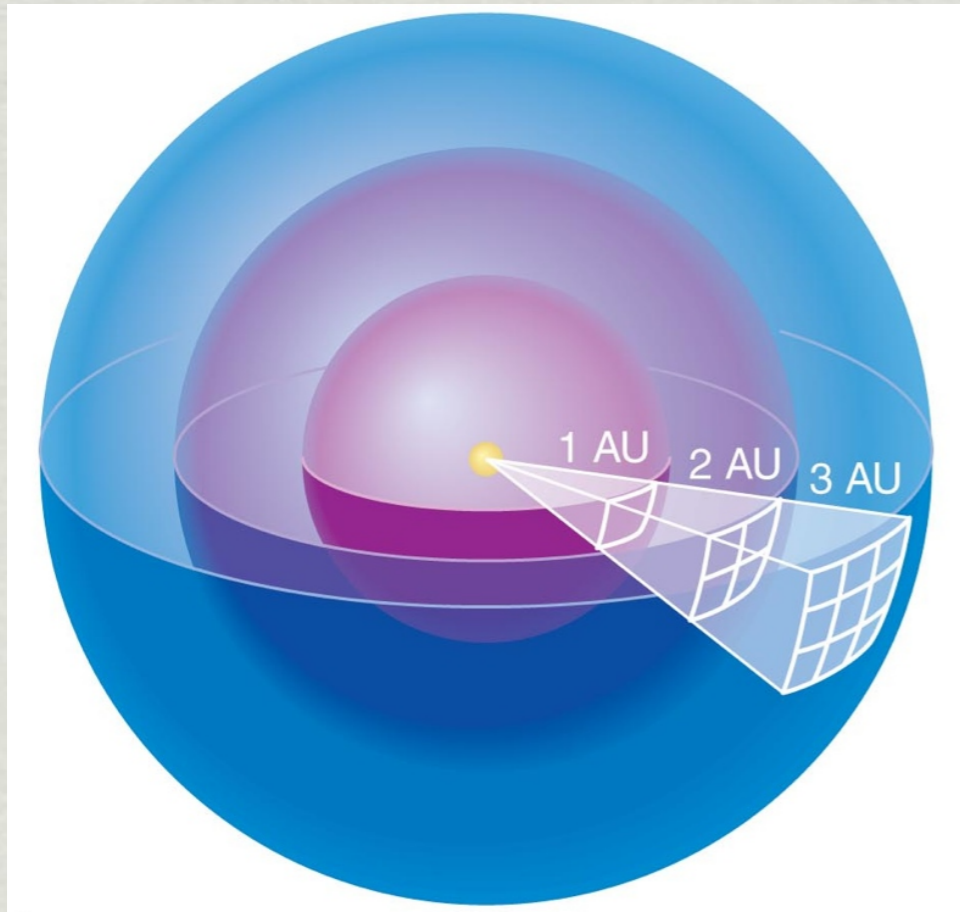
**Apparent brightness** of an object like a star, etc. =  $\frac{\text{Luminosity}}{\text{Surface Area}} = \frac{L}{4\pi d^2}$

**Apparent brightness** depends on distance to the object.

Objects that are farther away appear fainter: we collect less energy/sec in our eyes, camera, etc. because our eyes, camera are the same size no matter how far away the star is

**Luminosity** is always the same. It's a property of the star.

# Measuring Luminosity



If we can measure the apparent brightness  $B$  and the distance to the star  $d$ , we can measure the Luminosity of the star  $L$

Measure apparent brightness in some patch of area on the sphere: your eye, a telescope, a camera, ...

$$\text{Apparent brightness } B = \frac{L}{4\pi d^2} \quad \text{rearrange:} \quad \rightarrow L = B \times (4\pi d^2)$$

# Measuring the Stars

- ❖ Measure Luminosity of a star: total energy output per second, by measuring the apparent brightness and distance

$$\text{Apparent Brightness} = \frac{L}{4\pi d^2}$$

- ❖ Temperature: from Wein's Law (wavelength of max photon emission)
- ❖ Chemical composition: matching chemical fingerprints of different atoms to lines in the spectrum
- ❖ Mass: period and distance separation of binary star orbits
- ❖ Distance: parallax
- ❖ Size: from Luminosity and Temperature, using Stefan-Boltzmann law  
 $\text{Luminosity} = \text{Total surface area} \times \sigma T^4$

# Gravitational Equilibrium in the Sun

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

High temperature (hot):

molecules and atoms have lots of kinetic energy ( $\frac{1}{2} mv^2$ ), so every time an atom changes direction it gets a big acceleration. Lots of force per bounce → high pressure.

Low temperature (cool): less kinetic energy, less acceleration from every bounce → lower pressure

Many molecules (high density)  
many collisions = high  
pressure

Few molecules (low density):  
few collisions = low pressure

Perfect Gas Law:

Pressure =  $k \times$  density  $\times$  temperature

$k$  = universal constant

“perfect”: assumes molecules and atoms bounce perfectly in every collision. Chemistry tells us that doesn't always happen: sometimes they stick together and make molecules!

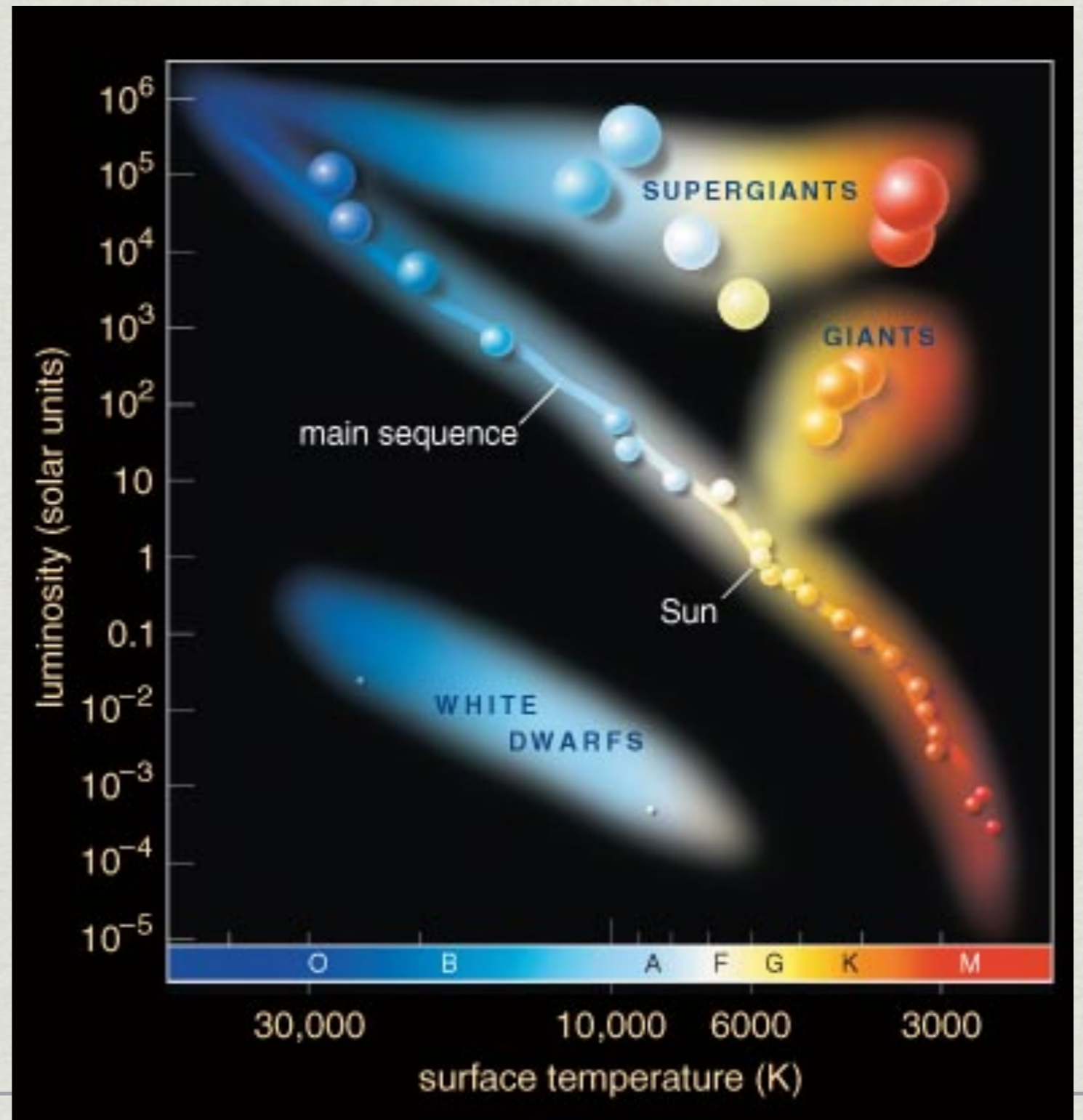
# Hertzsprung-Russell (H-R) Diagram

Relations between luminosity, mass, size, temperature and lifetime of stars.

Where are most of the stars?

On the Main Sequence.

Maintain gravitational equilibrium, hold themselves up against gravity by generating energy in their core from nuclear fusion of hydrogen to helium:  
 $4\text{H} \rightarrow \text{He} + \text{Energy}$



# Hertzprung-Russell (H-R) Diagram

Relations between luminosity, mass, size, temperature of stars.

Where are all the stars?

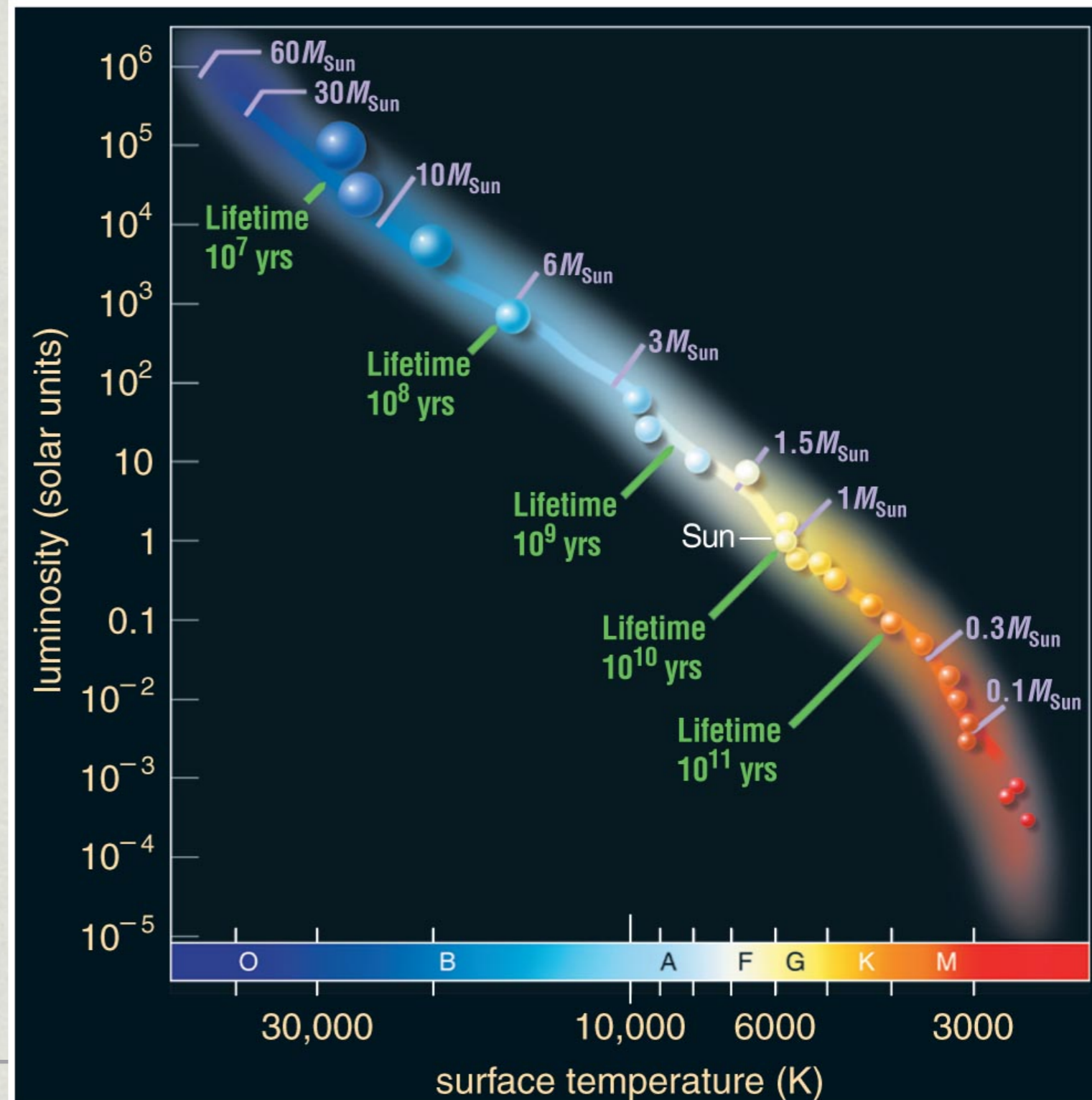
On the Main Sequence.

Main sequence is a sequence in mass:

Hot, bright massive stars  
at top left

Cool, faint low-mass stars  
at bottom right

Why?



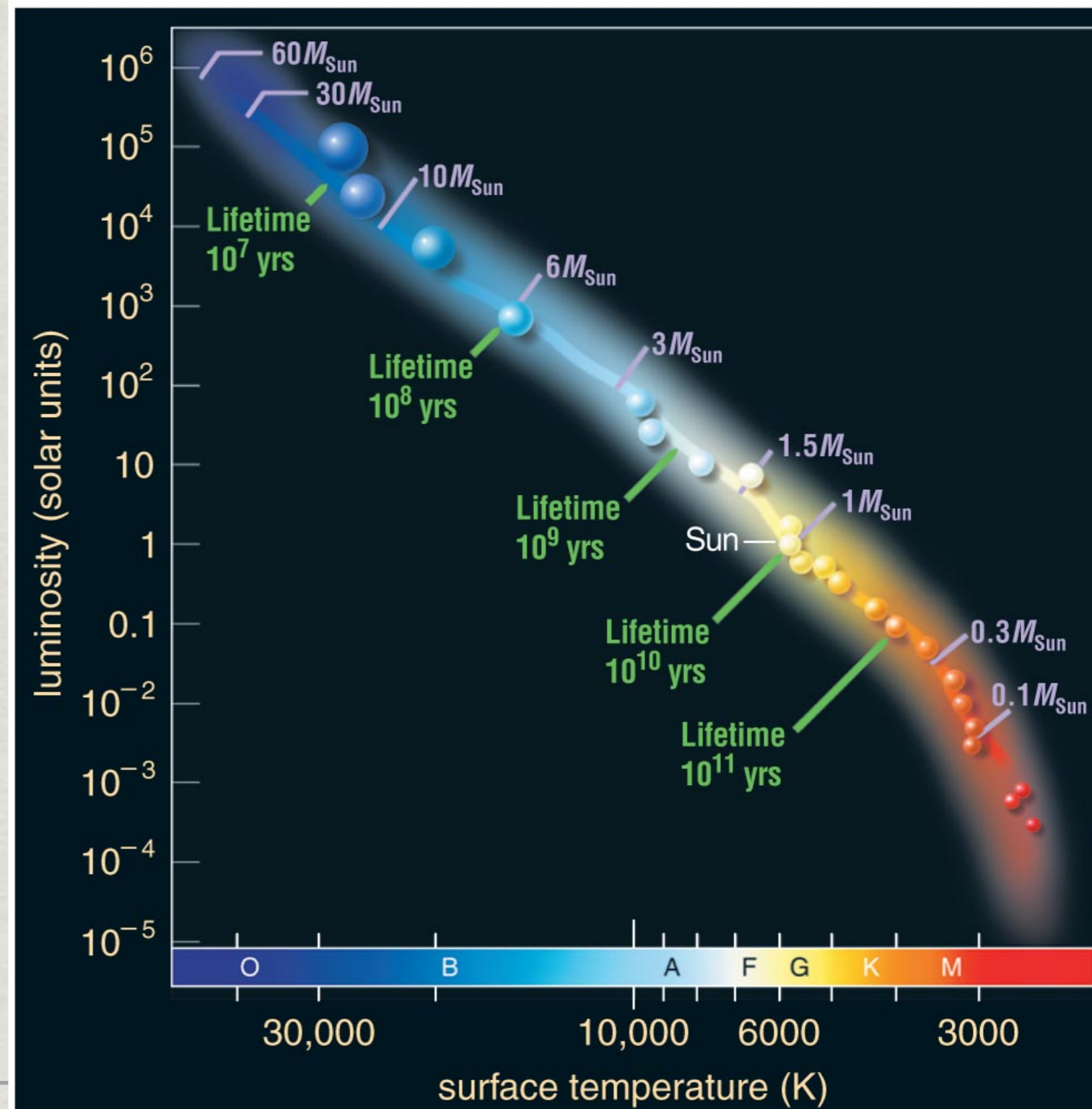
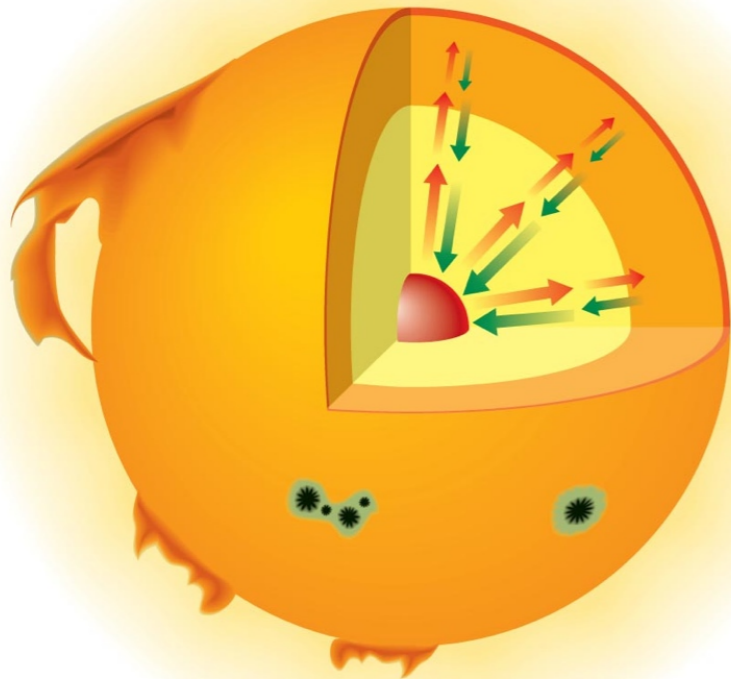


# Hertzprung-Russell (H-R) Diagram

Higher mass = more pressure from gravity

Star has to generate more energy from nuclear fusion to create more thermal pressure to hold itself up against gravity

pressure   
gravity 

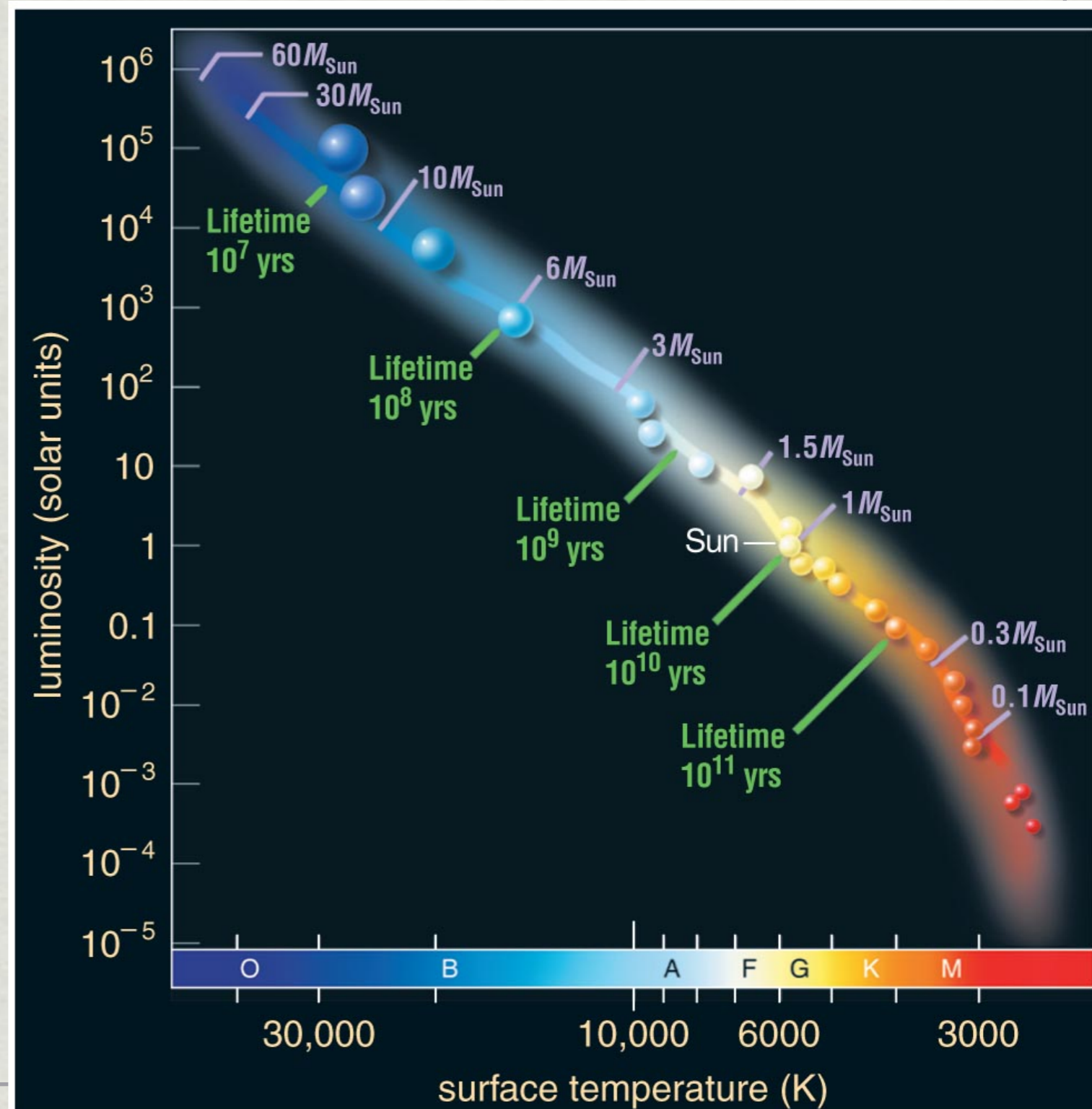


# Hertzsprung-Russell (H-R) Diagram

Higher mass = more pressure from gravity

Higher mass:  
Higher core pressure, need  
Higher core temperature.  
Need larger rate of energy  
generation from fusion to  
hold the star up

Lower mass: lower core  
pressure, lower core  
temperature. Can  
counter-act the pressure  
from gravity with a lower  
energy generation rate  
from nuclear fusion.



# Hertzsprung-Russell (H-R) Diagram

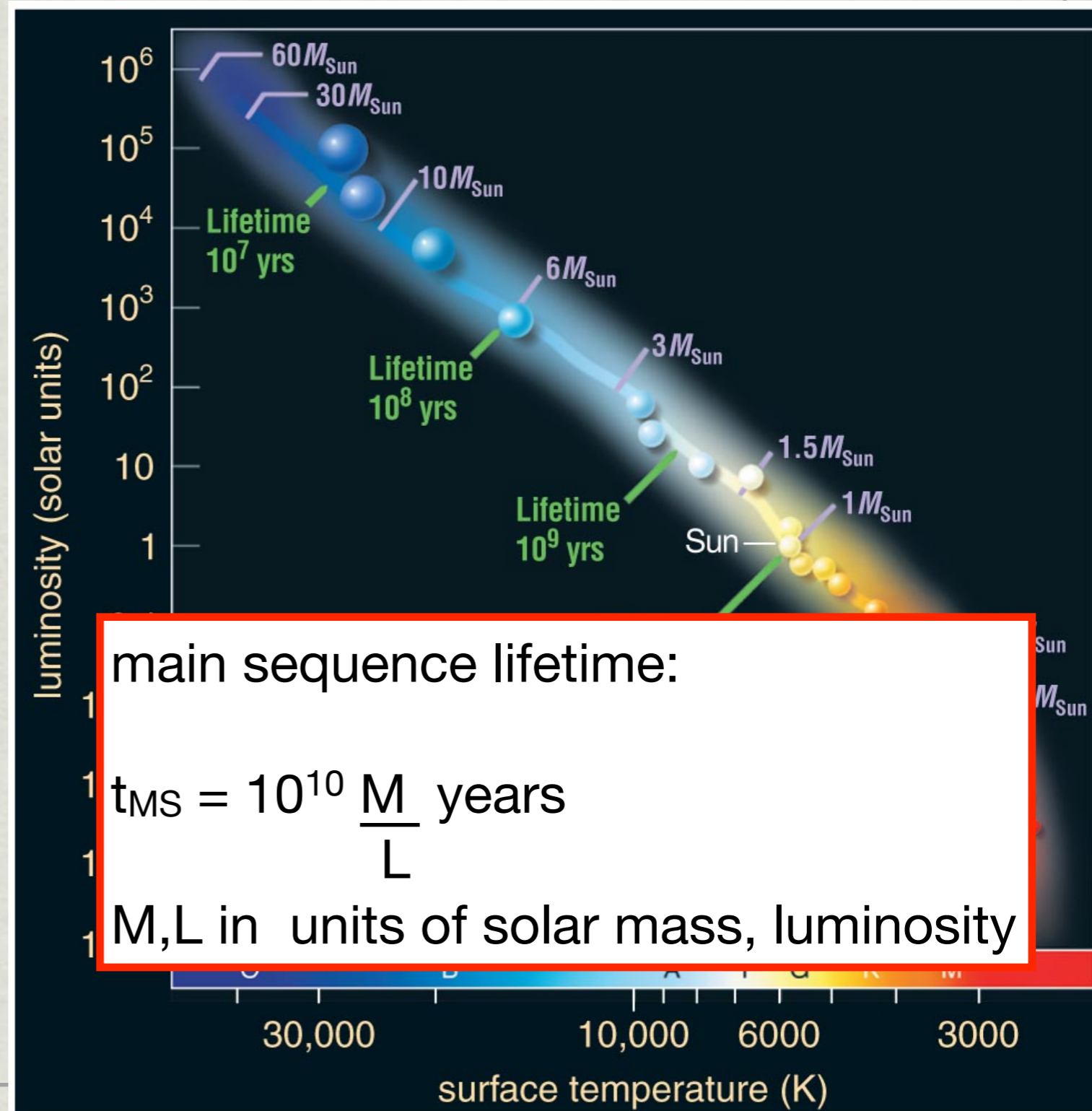
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Need larger rate of energy  
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Lower mass: lower core  
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counter-act the pressure  
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energy generation rate  
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# Hertzsprung-Russell (H-R) Diagram

Higher mass = more pressure from gravity

Main sequence:

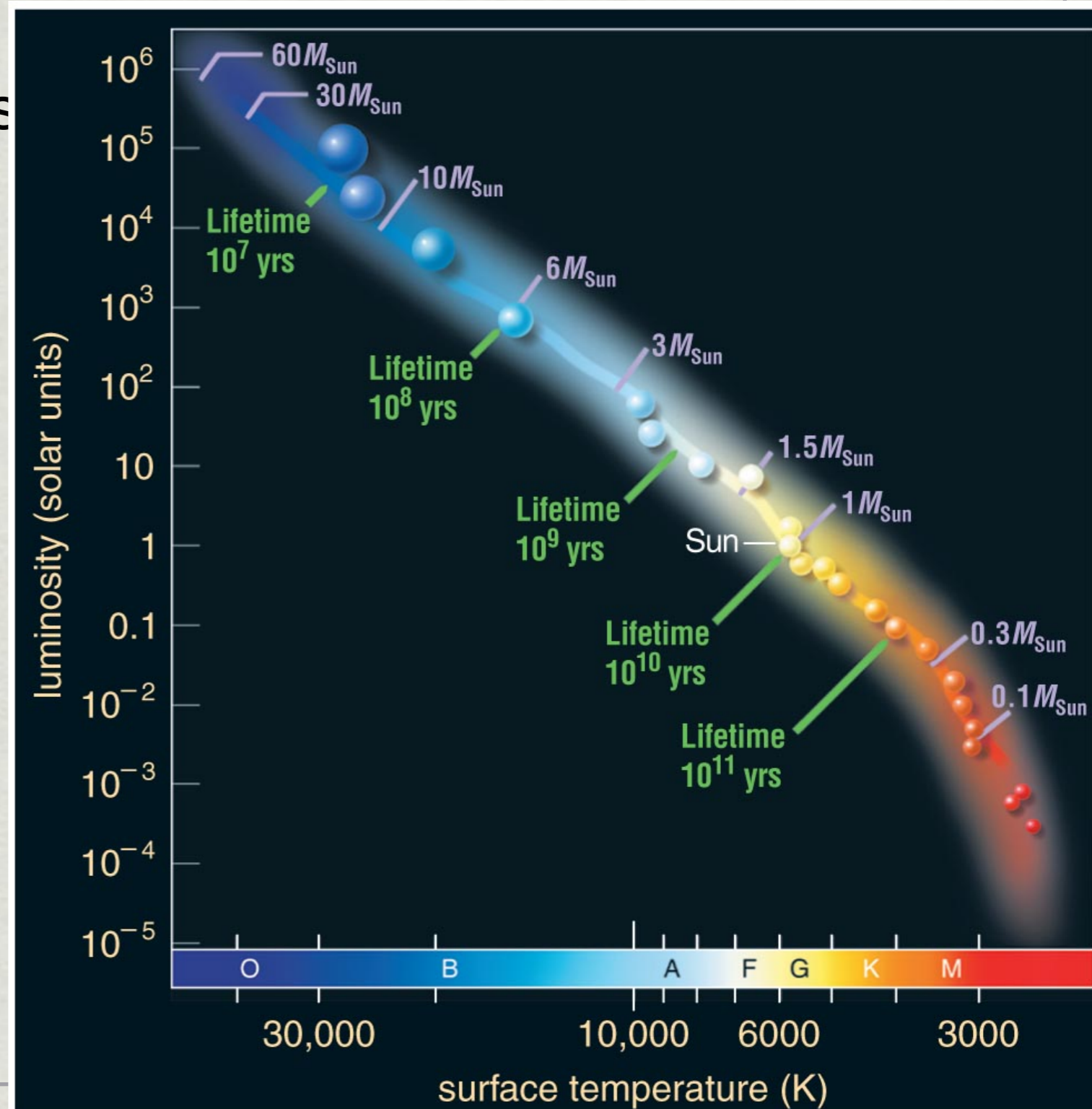
- Powered by nucleosynthesis of H to He
- Properties set by mass.

High mass:

- High luminosity
- Blue = high temperature
- Large radius
- Short lifetime

Low Mass:

- Low luminosity
- Red = cool temperature
- Small radius
- Long lifetime



# High Mass Stars: Post Main Sequence Evolution

New fusion cycles initiated as core collapses, never becomes degenerate.

Q: When does it stop?

A: When fusion of new elements doesn't release energy

Mass per particle in the nucleus decreases from H to Fe.

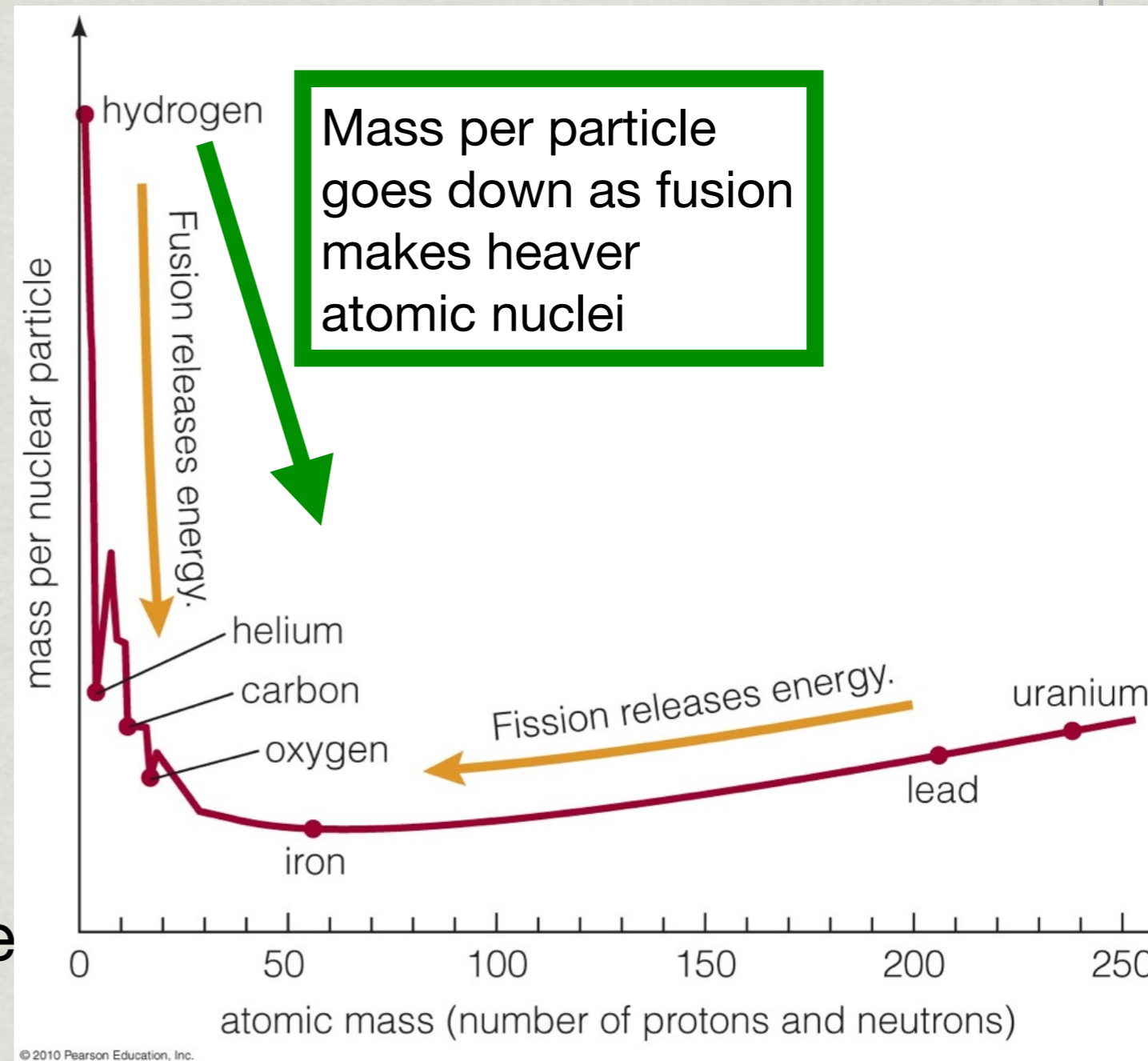
Where does it go?

It is converted to energy!

Remember: of 4 H nuclei > 1 He nucleus.

$$\Delta m = \text{Mass}_{4\text{H}} - \text{Mass}_{\text{He}}$$

$$E_{\text{out}} = \Delta mc^2$$



H, He, ... C...Fe...Ag...Pb...U → increasing number of particles in nucleus for elements in the periodic table

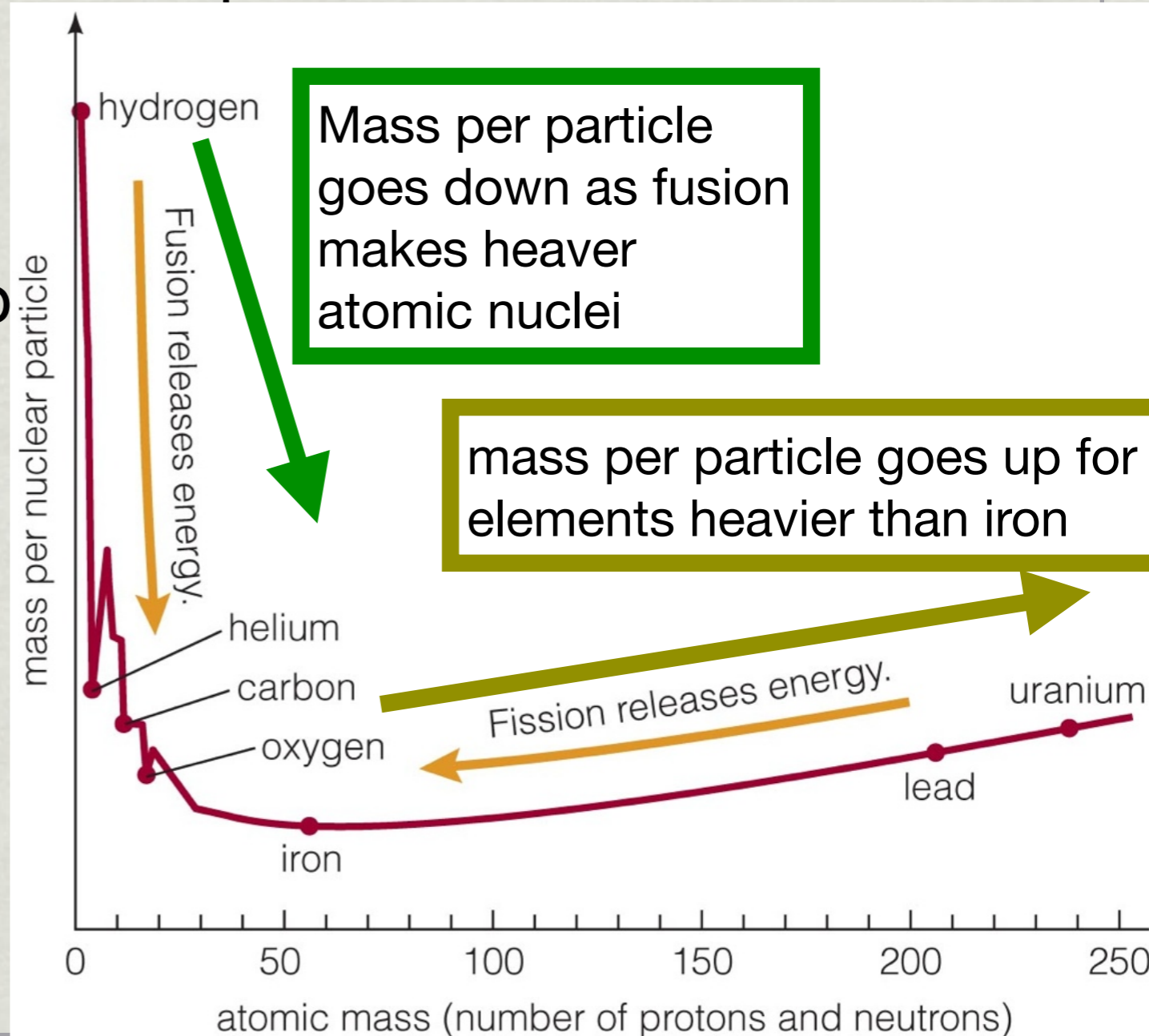
# High Mass Stars: Post Main Sequence Evolution

Iron (Fe) is the end of the line for fusion.

Fe nucleus has lowest mass per particle: no conversion of mass to energy when you add more particles

Now gravity can win as the core collapses. Star can't generate energy to hold itself up with thermal pressure.

1	H																	2	He																
3	Li	4	Be											5	B	6	C	7	N	8	O	9	F	10	Ne										
11	Na	12	Mg											13	Al	14	Si	15	P	16	S	17	Cl	18	Ar										
19	K	20	Ca	21	Sc	22	Ti	23	V	24	Cr	25	Mn	26	Fe	27	Co	28	Ni	29	Cu	30	Zn	31	Ga	32	Ge	33	As	34	Se	35	Br	36	Kr
37	Rb	38	Sr	39	Y	40	Zr	41	Nb	42	Mo	43	Tc	44	Ru	45	Rh	46	Pd	47	Ag	48	Cd	49	In	50	Sn	51	Sb	52	Te	53	I	54	Xe
55	Cs	56	Ba	*	72	Hf	73	Ta	74	W	75	Re	76	Os	77	Ir	78	Pt	79	Au	80	Hg	81	Tl	82	Pb	83	Bi	84	Po	85	At	86	Rn	
87	Fr	88	Ra	+	104	Rf	105	Db	106	Sg	107	Bh	108	Hs	109	Mt	110	Ds	111	Rg	112	Uub	113	Uut	114	Uuq	115	Uup	116	Uuh	117	Uus	118	Uuo	
* Lanthanide Series				57	La	58	Ce	59	Pr	60	Nd	61	Pm	62	Sm	63	Eu	64	Gd	65	Tb	66	Dy	67	Ho	68	Er	69	Tm	70	Yb	71	Lu		
+ Actinide Series				89	Ac	90	Th	91	Pa	92	U	93	Np	94	Pu	95	Am	96	Cm	97	Bk	98	Cf	99	Es	100	Fm	101	Md	102	No	103	Lr		
Alkali metals		Alkaline earth metals		Lanthanoids			Actinoids			Transition metals			Poor metals		Metalloids		Other Nonmetals		Halogens		Noble Gases														



# High Mass Stars: Post Main Sequence Evolution

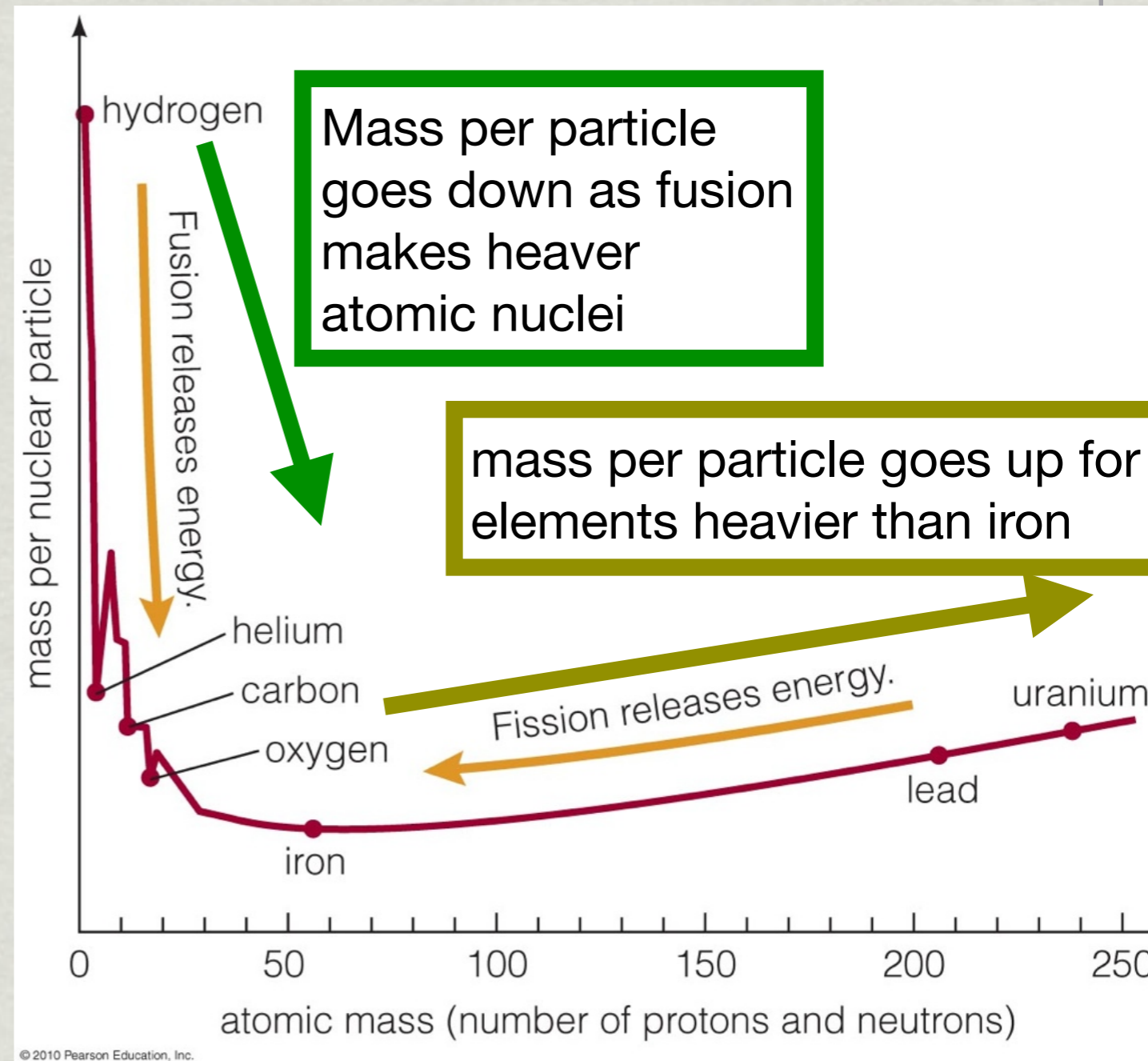
Mass per particle (neutron, proton) in the nucleus increases for elements heavier than Fe.

Mass per particle of a lead nucleus > Mass per particle of an iron nucleus.

$$\Delta m = \text{Mass}_{\text{Fe}} - \text{Mass}_{\text{Pb}} < 0$$

No energy out by fusing nuclei to make lead!

How are these elements made?  
Put energy *in* during the explosions that end the lives of massive stars

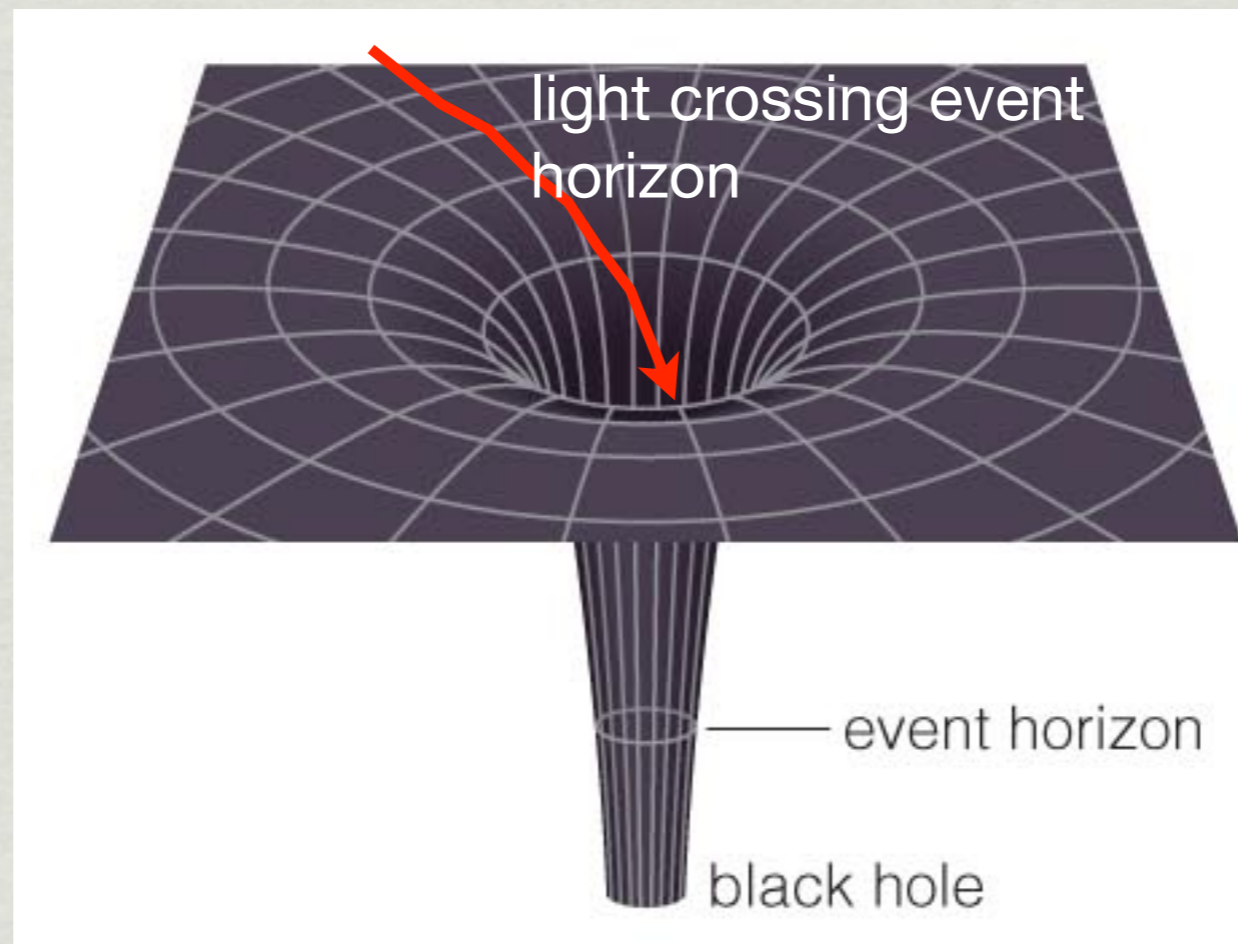


H, He, ... C...Fe...Ag...Pb...U → increasing number of particles in nucleus for elements in the periodic table

# General Relativity

Implication #1:

The path of light is always straight, but space is bent by gravity.



Space curved if the sun were a black hole

If mass is dense enough, there is a radius from which light can't escape.

Radius of this "Event Horizon" = Schwarzschild Radius



# Black Holes

Escape velocity: velocity needed to get free of an object's gravitational pull.

$$V_{\text{escape}} = \sqrt{\frac{2 G M}{d}}$$

M = mass, d = distance from the center of the object you want to escape from

For a star, light is emitted at its surface, so **d** is the radius of the star, **R**

If M is very big and/or R is very small, then  $V_{\text{escape}} > c$ , the speed of light.

NOTHING can have speed  $> c$ , so NOTHING can escape, not even light.

Star becomes a Black Hole



Artist's rendition, not a real picture

# Black Holes

For any mass define a radius, the Schwarzschild radius  $R_s$ , where

$$V_{\text{escape}} = c$$

$$V_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

Then rearrange:

$$R_s = \frac{2GM}{c^2} = \left(\frac{2G}{c^2}\right) M$$

$$\text{If } M = M_{\text{sun}}, R_s = 3 \text{ km}$$

$$\text{So for any star, } R_s = 3 \text{ km} \times \frac{M^*}{M_{\text{sun}}}$$

Artist's rendition, not a real picture



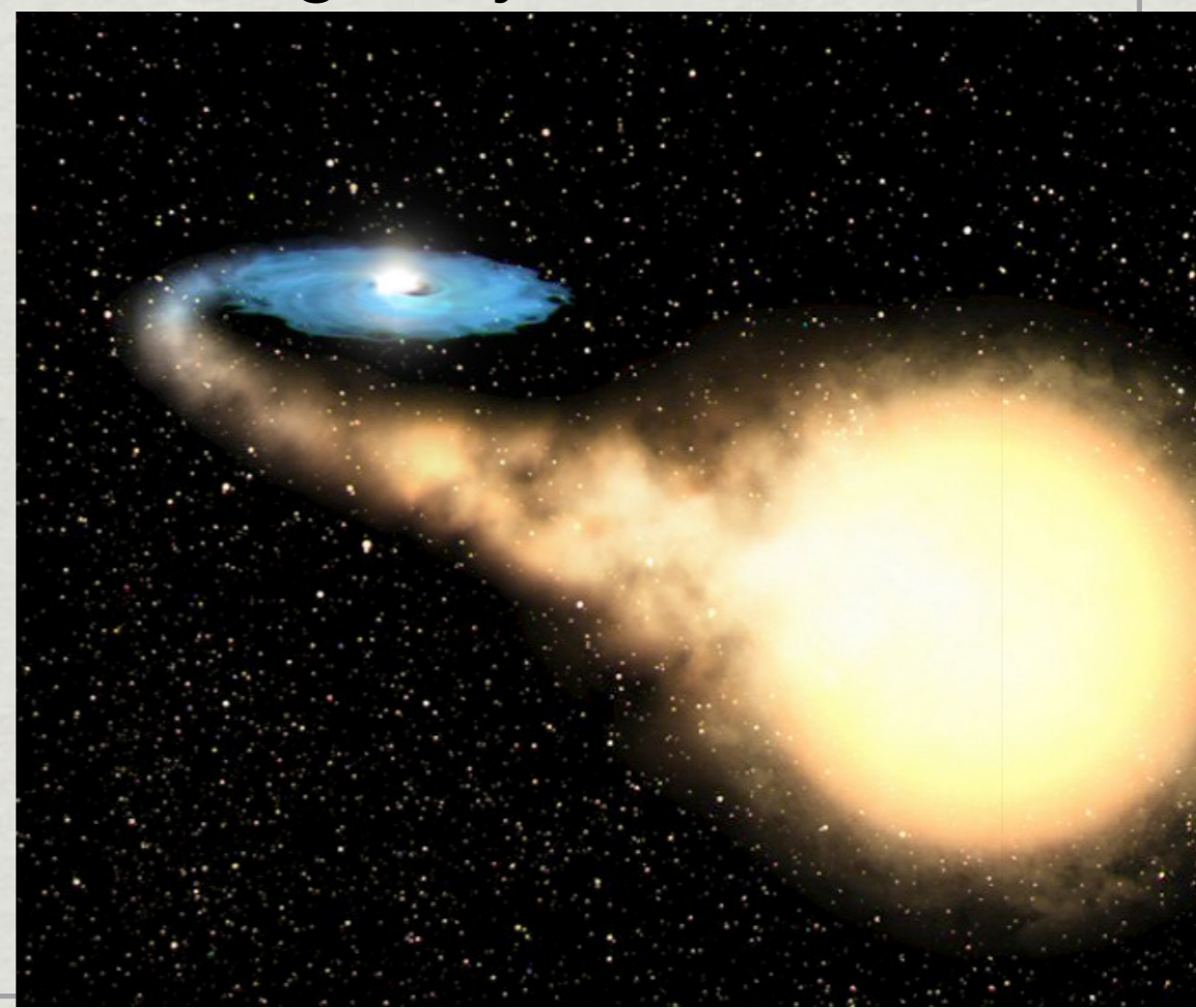
# Black Holes

Black holes in the centers of galaxies can pull in gas from the rest of the galaxy.

As the gas falls in the atoms collide. The gas heats up due to friction and the release of gravitational potential energy.

The gas emits radiation, though not a thermal spectrum.

Can be incredibly bright, out-shine an entire galaxy.

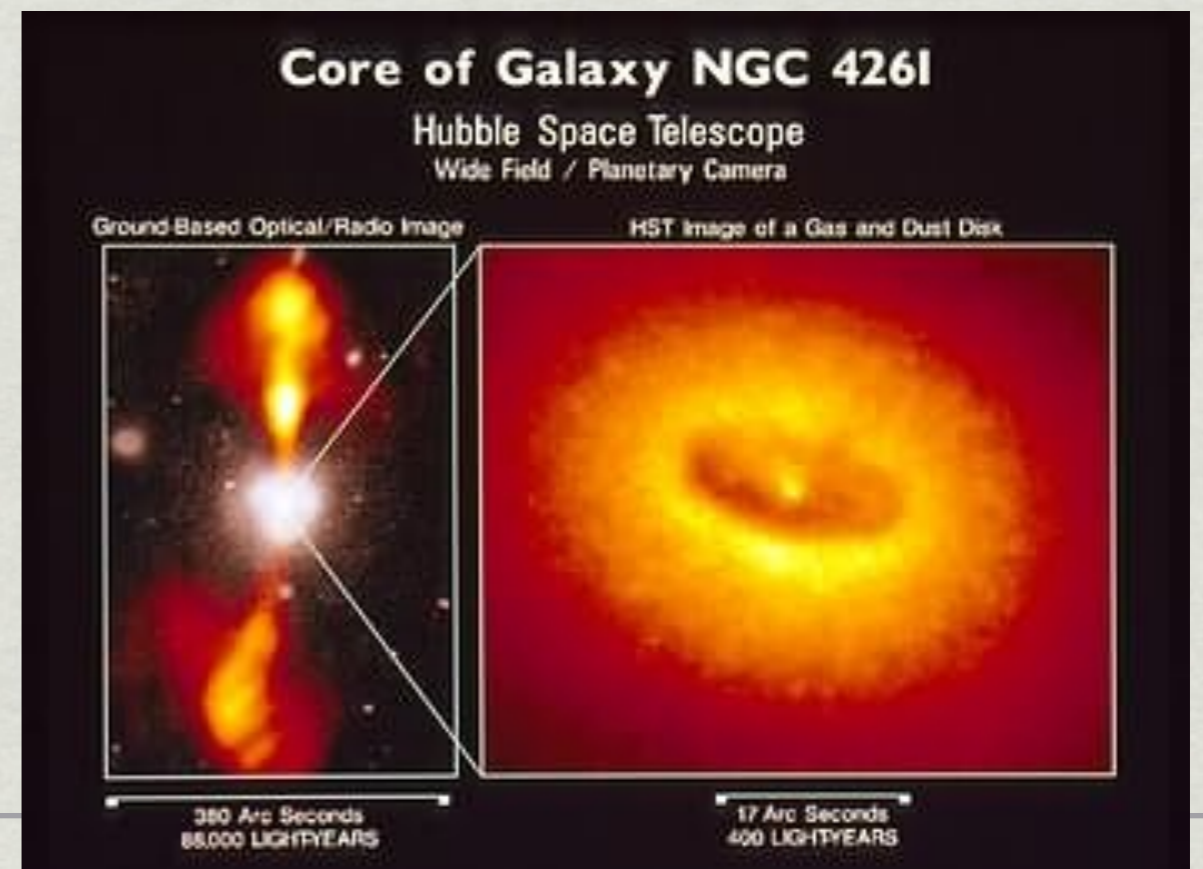
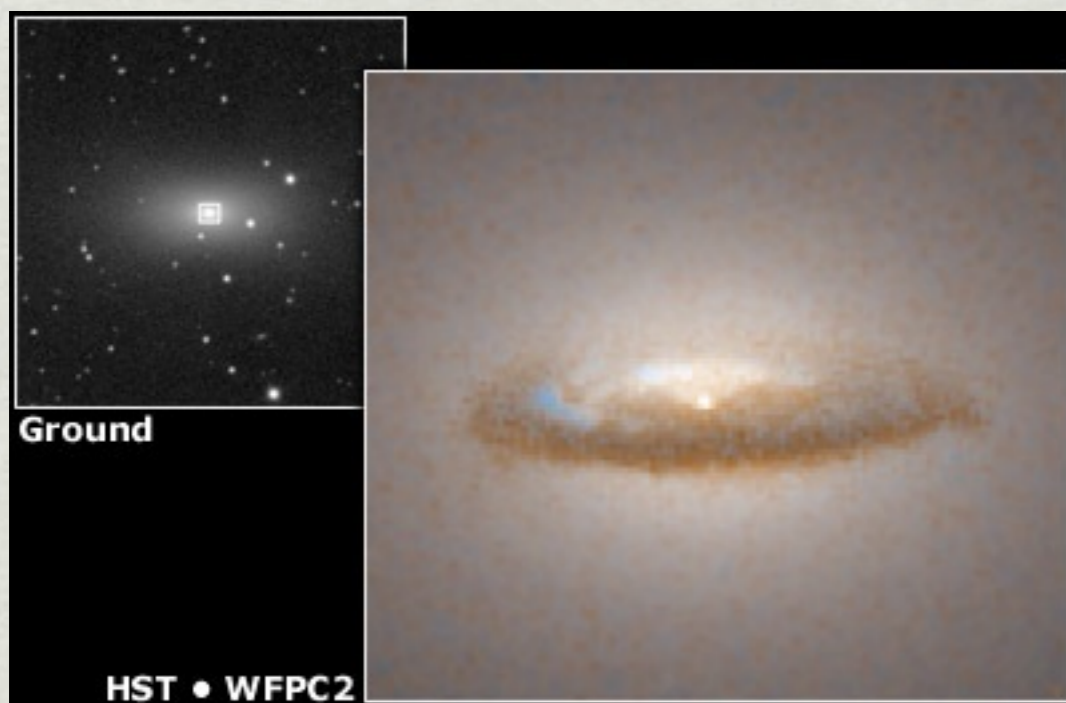


# Quasars

Eventually, realized that quasars are black holes in the centers of galaxies.

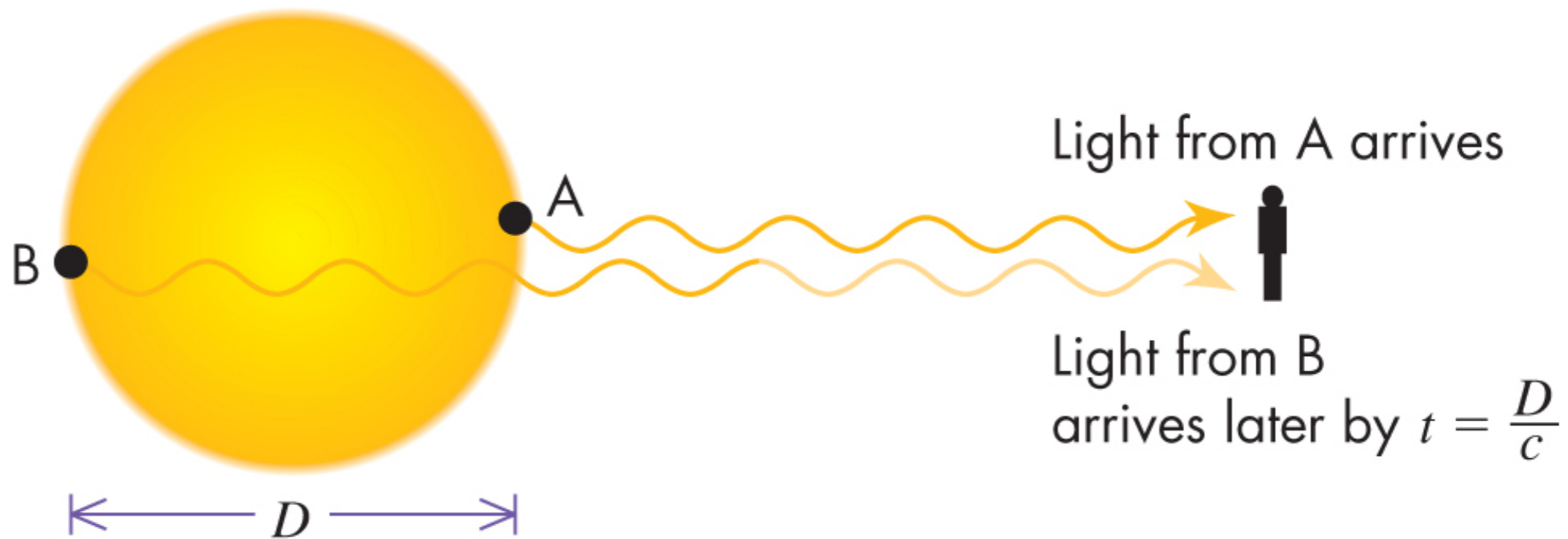
That huge luminosity is the light from gas that is glowing from the energy it gains as it falls in to the black hole.

Infall converts gravitational potential energy to other kinds of energy. Stored in the energy levels of atoms, released as emission line spectra and other interactions



# Measuring the Size of Quasars

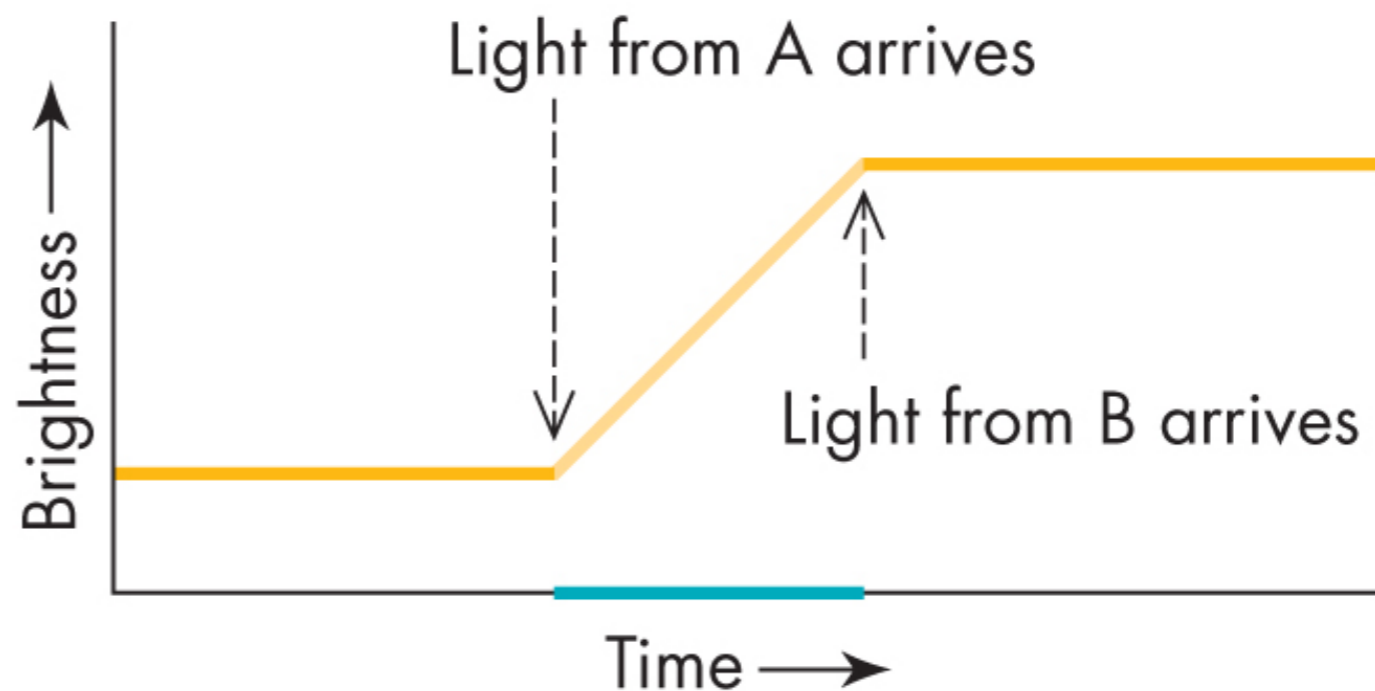
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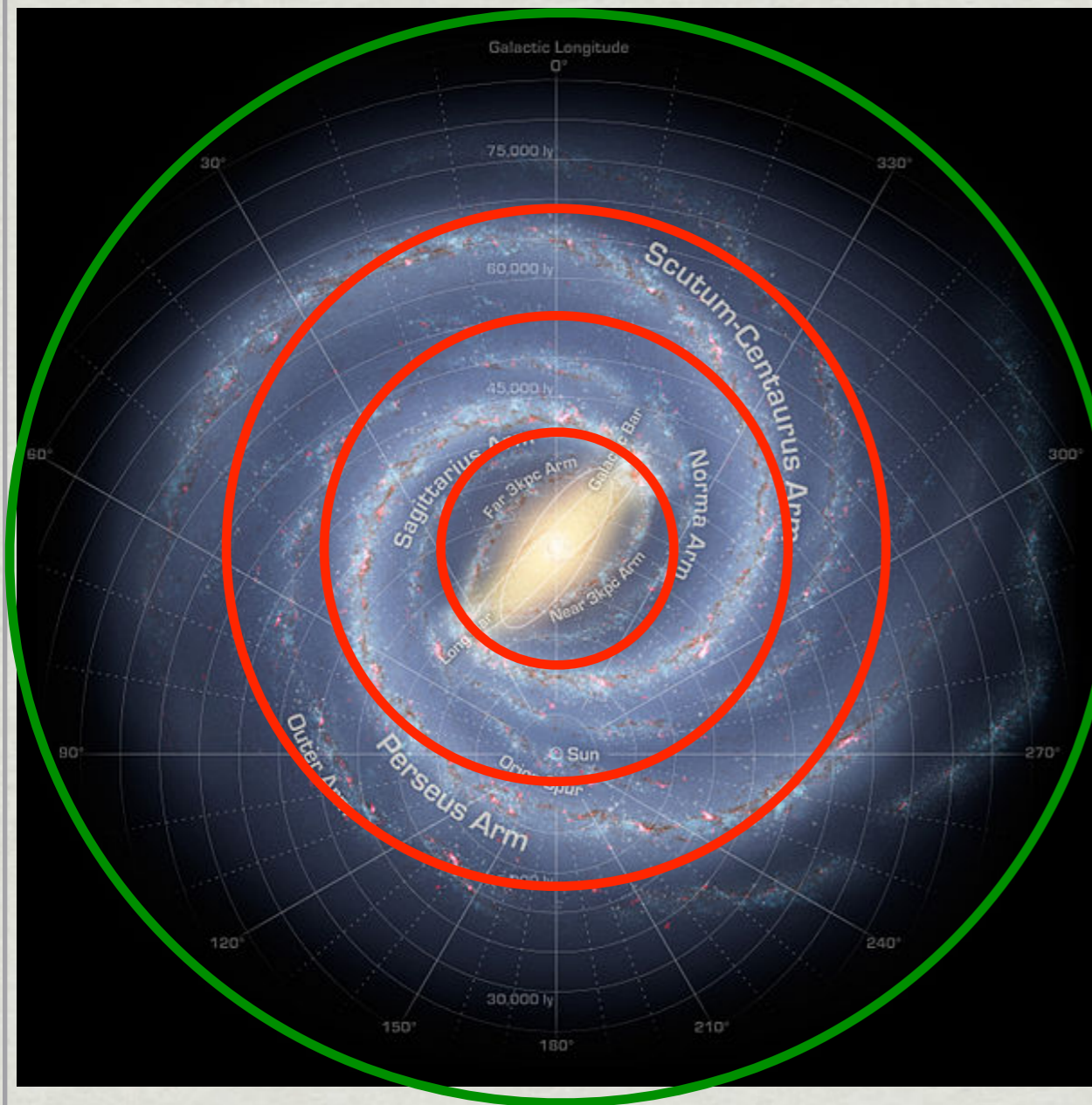
Distance = velocity  $\times$  time

$$D = ct$$

$$\frac{D}{c} = t$$



# Milky Way: Mass



What do you expect will happen to the velocity once the **circles get big enough** to enclose all the stars?

A) velocity increase as radius gets larger (green circle and beyond)

B) velocity should decrease as radius gets larger (green circle and beyond)

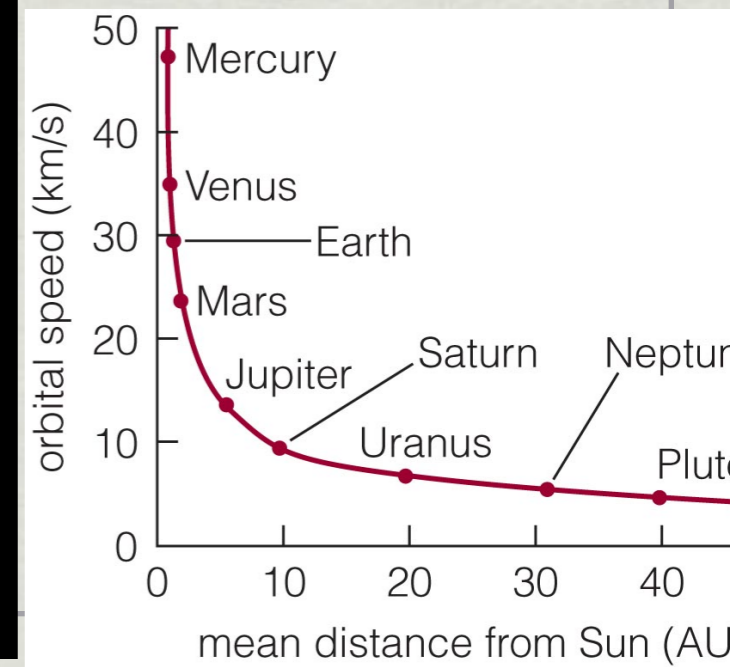
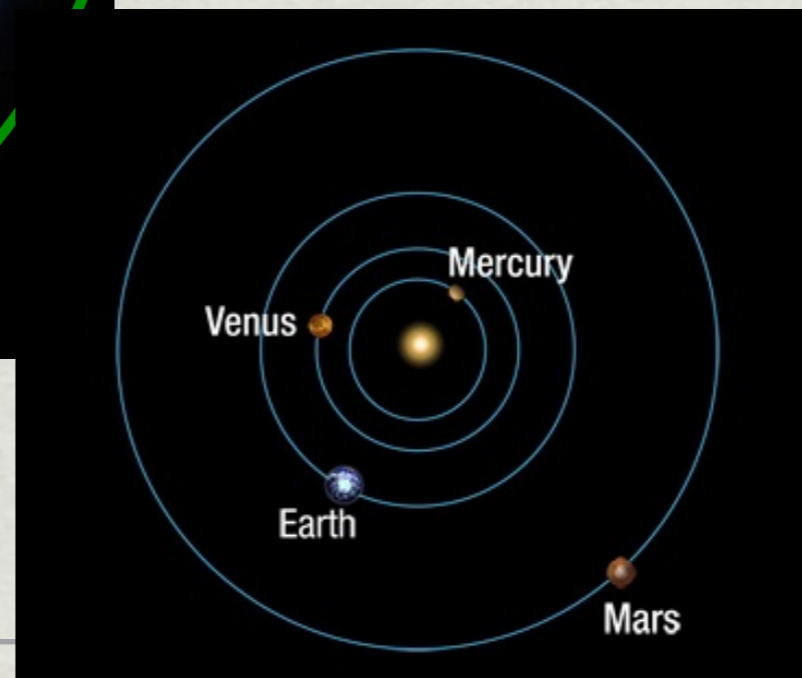
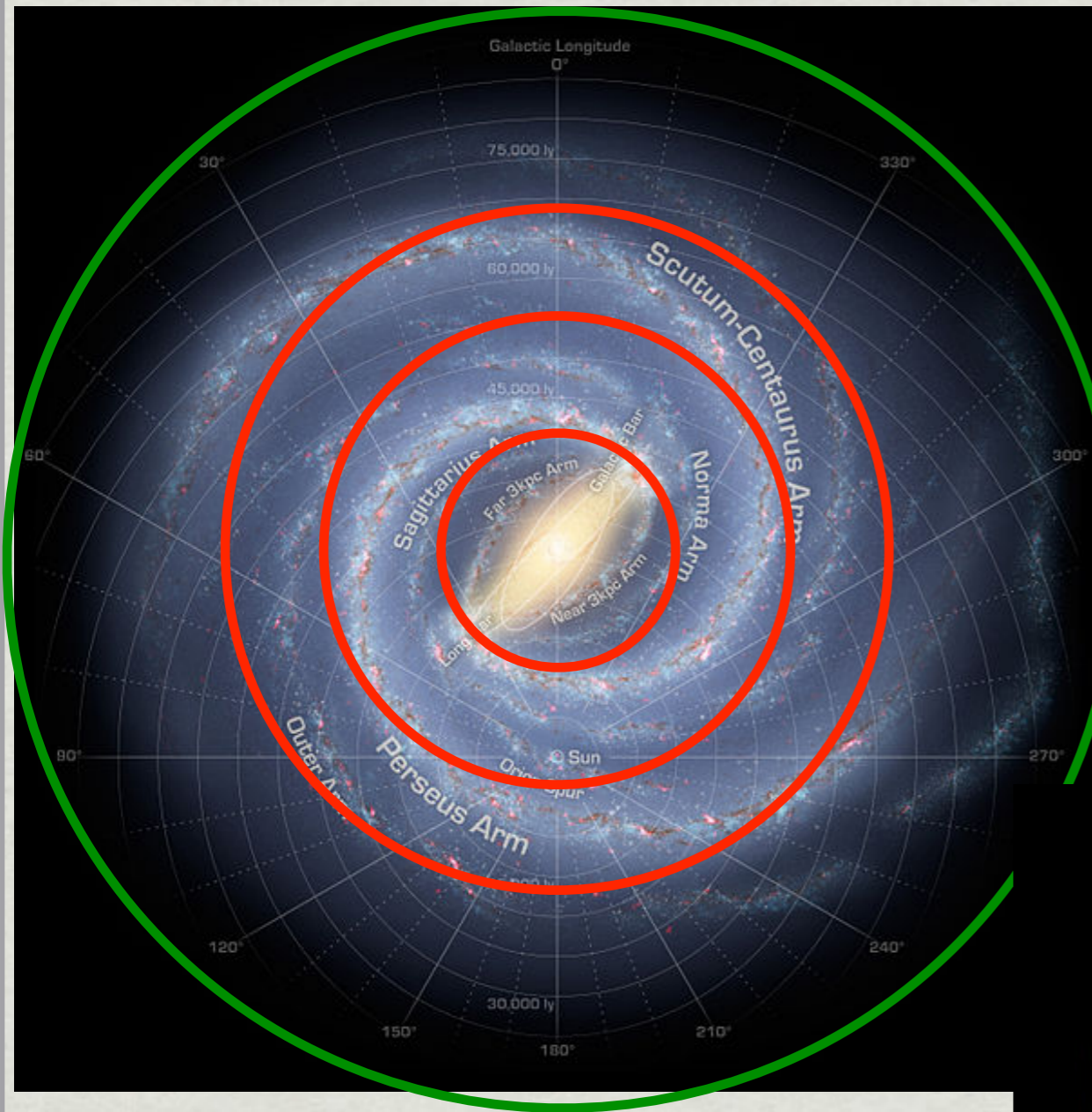
Remember, speed of an orbit around mass M:

$$v^2 = \frac{GM}{r}$$

# Milky Way: Mass

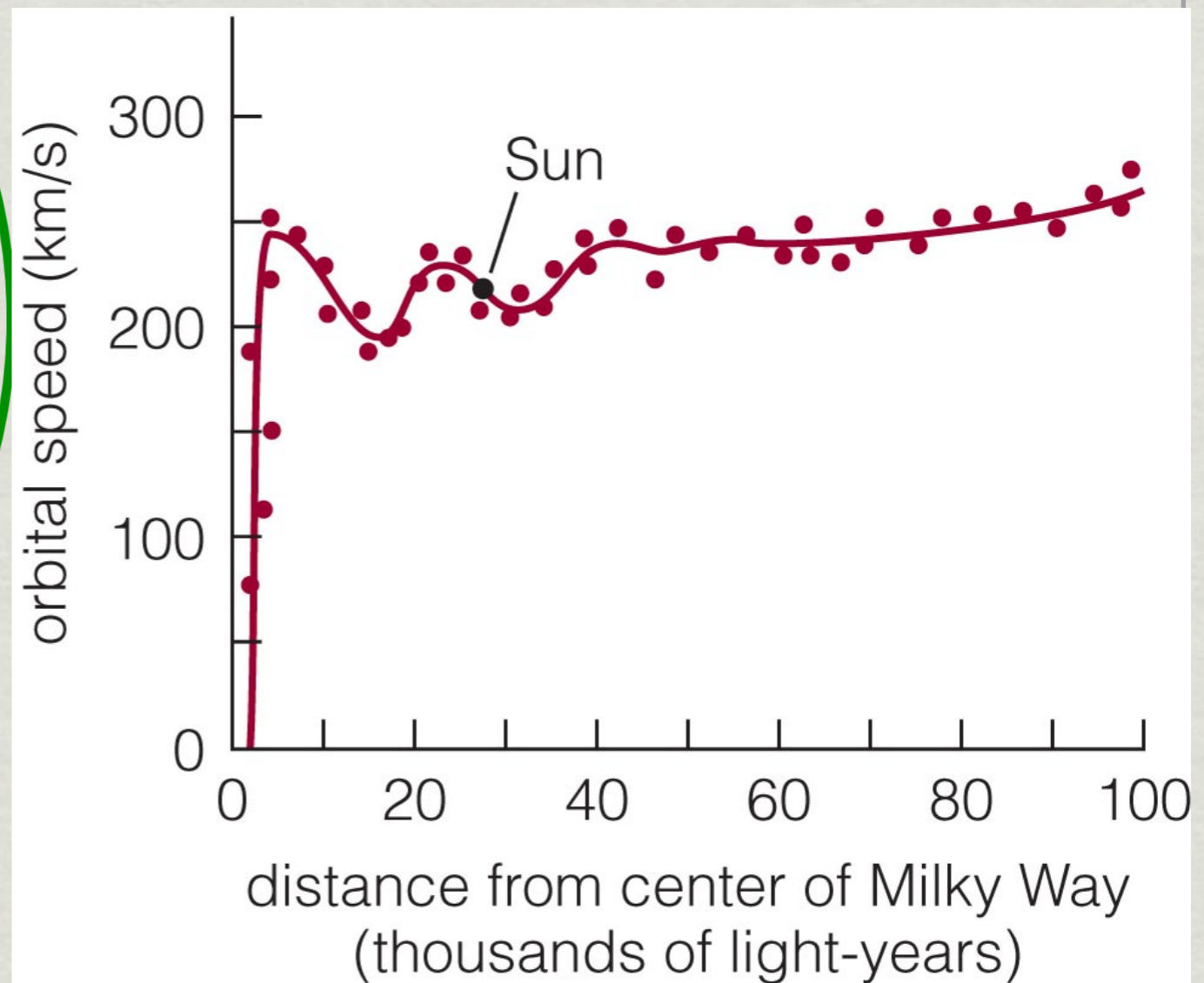
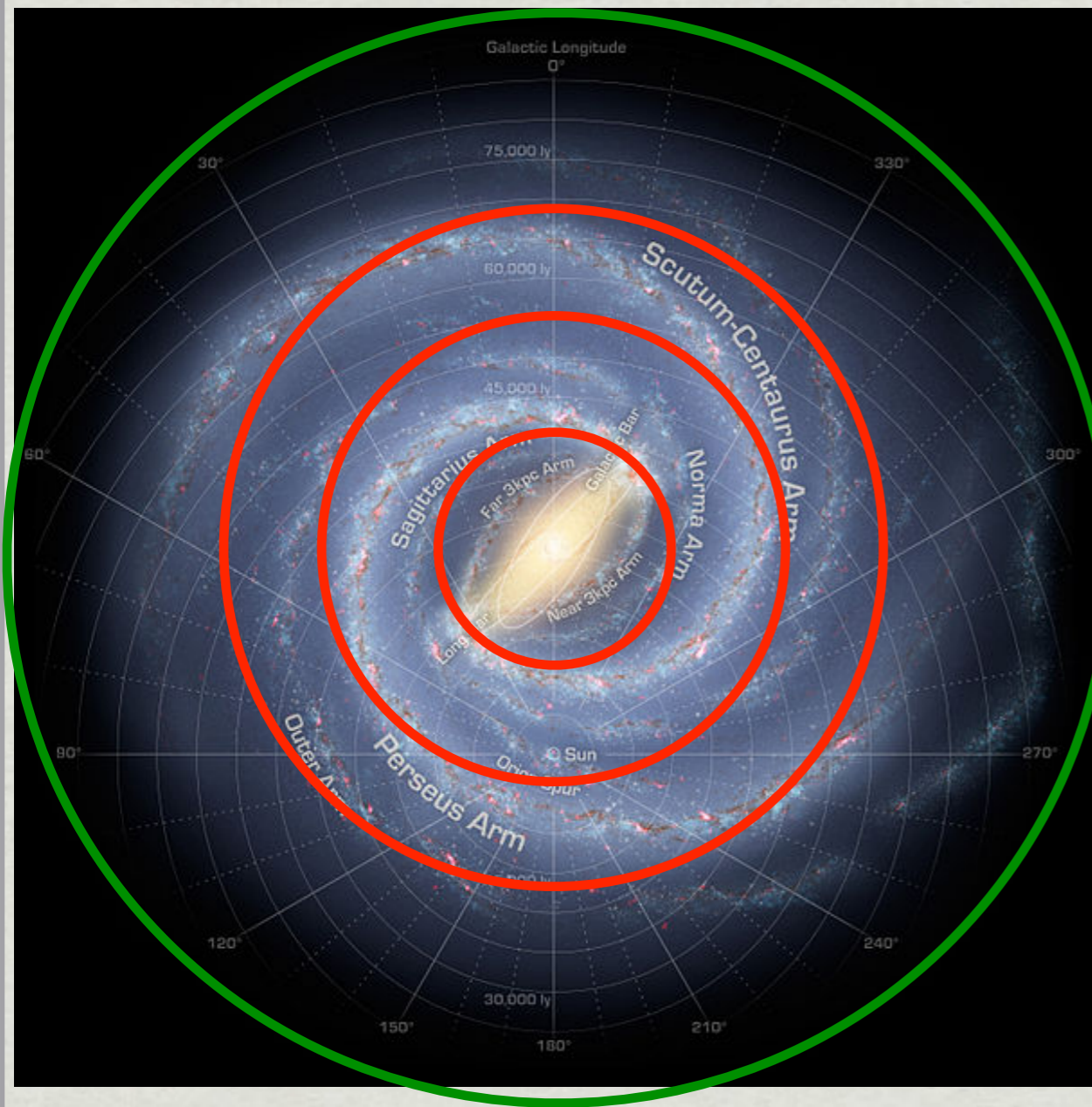
What do you expect will happen to the velocity once the **circles get big enough** to enclose all the stars?

Velocity should decrease at larger radii, just like for the solar system. All the mass is inside the large circles, right?



# Milky Way: Mass

What we measure is wonderfully weird:  
*the velocity stays constant.*

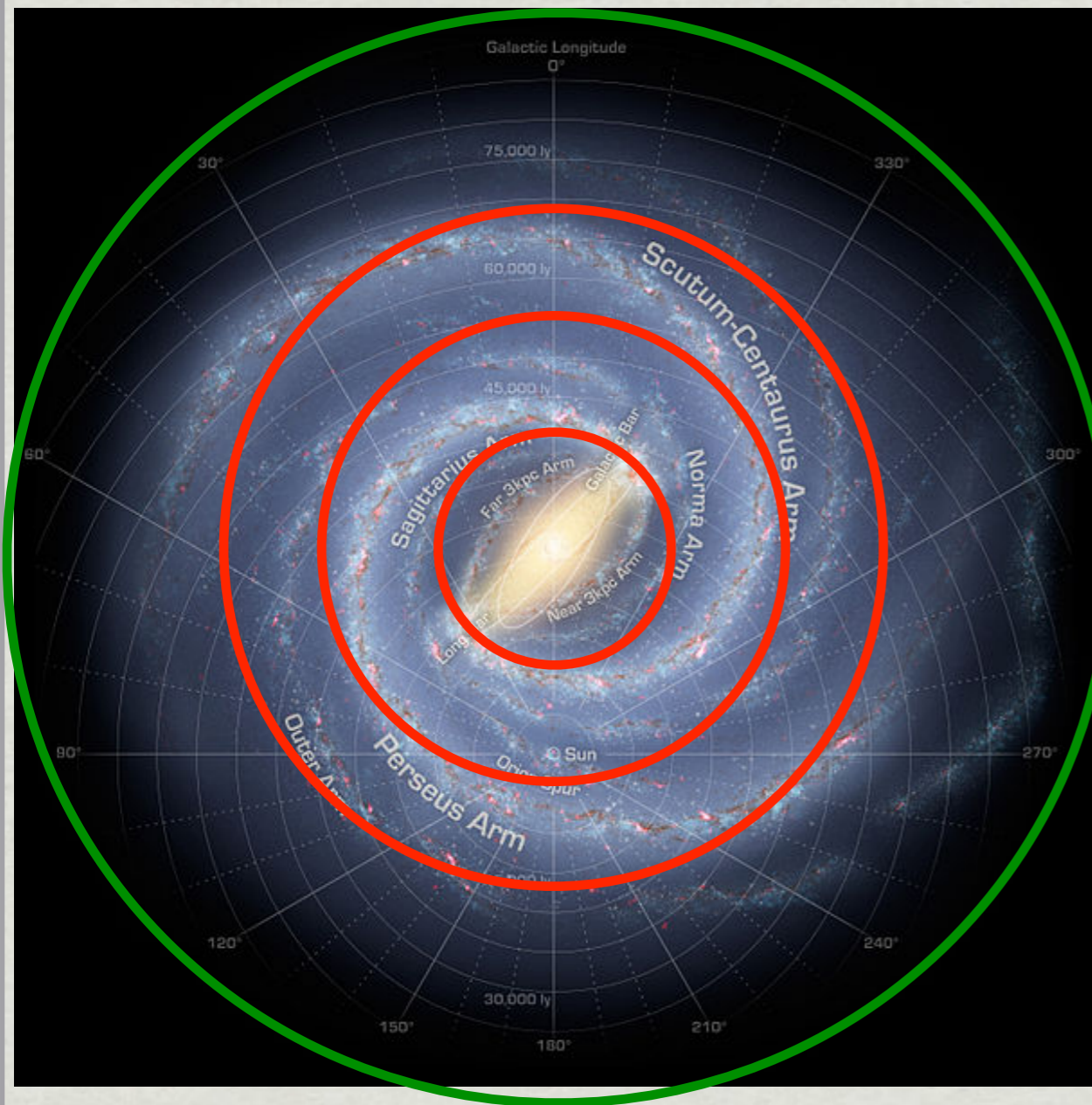


So mass must still be increasing with radius!

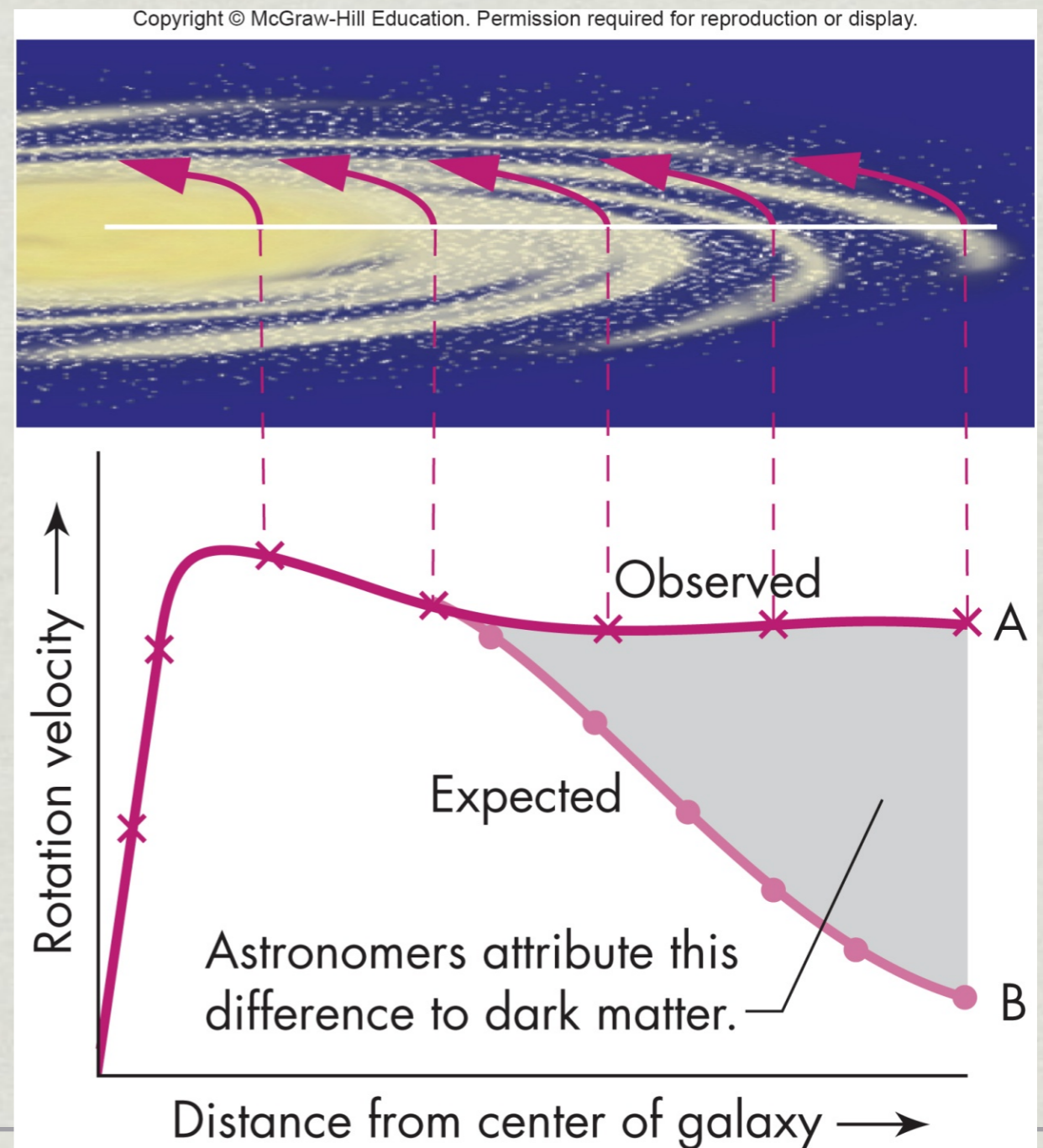


# Milky Way: Mass

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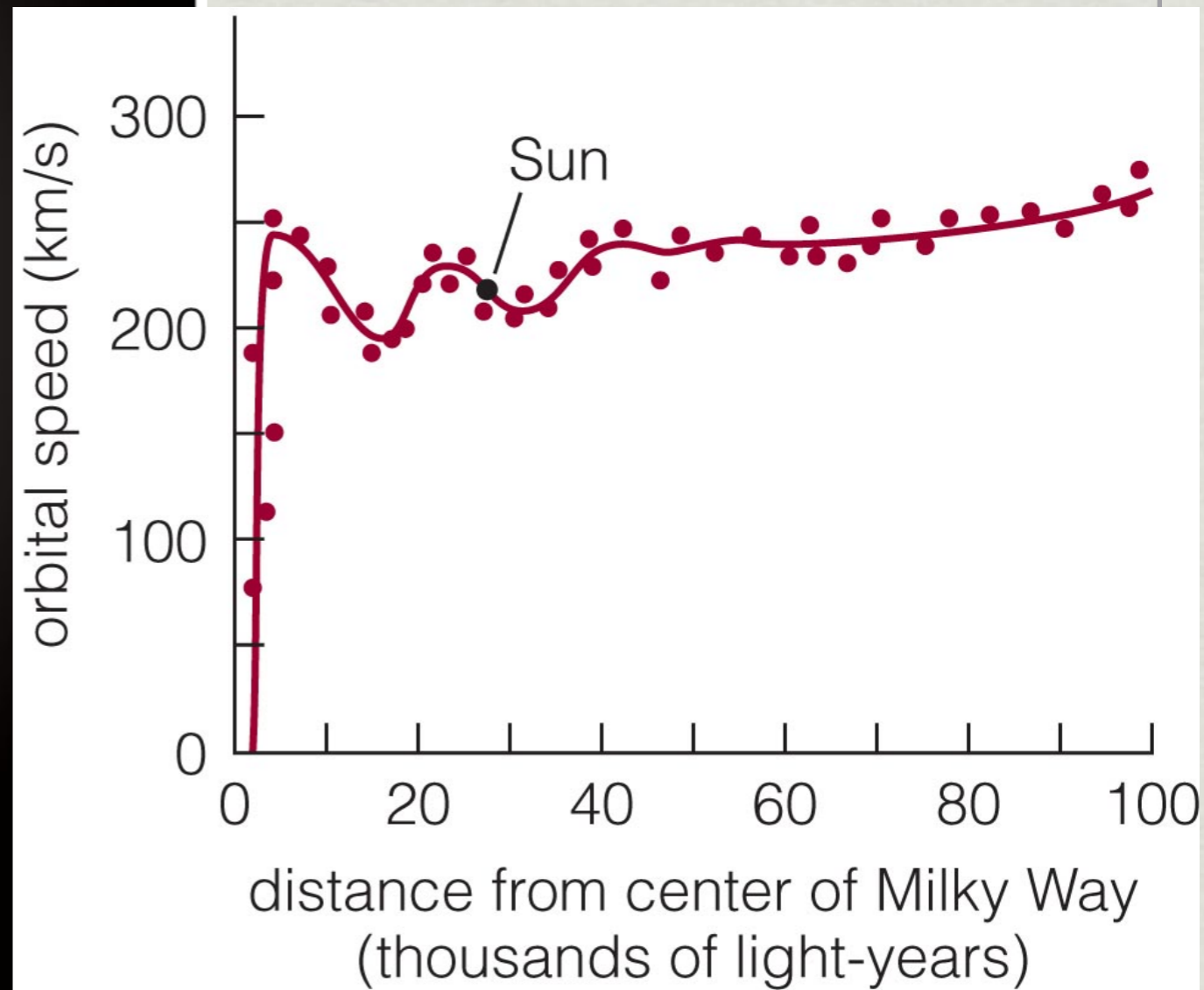
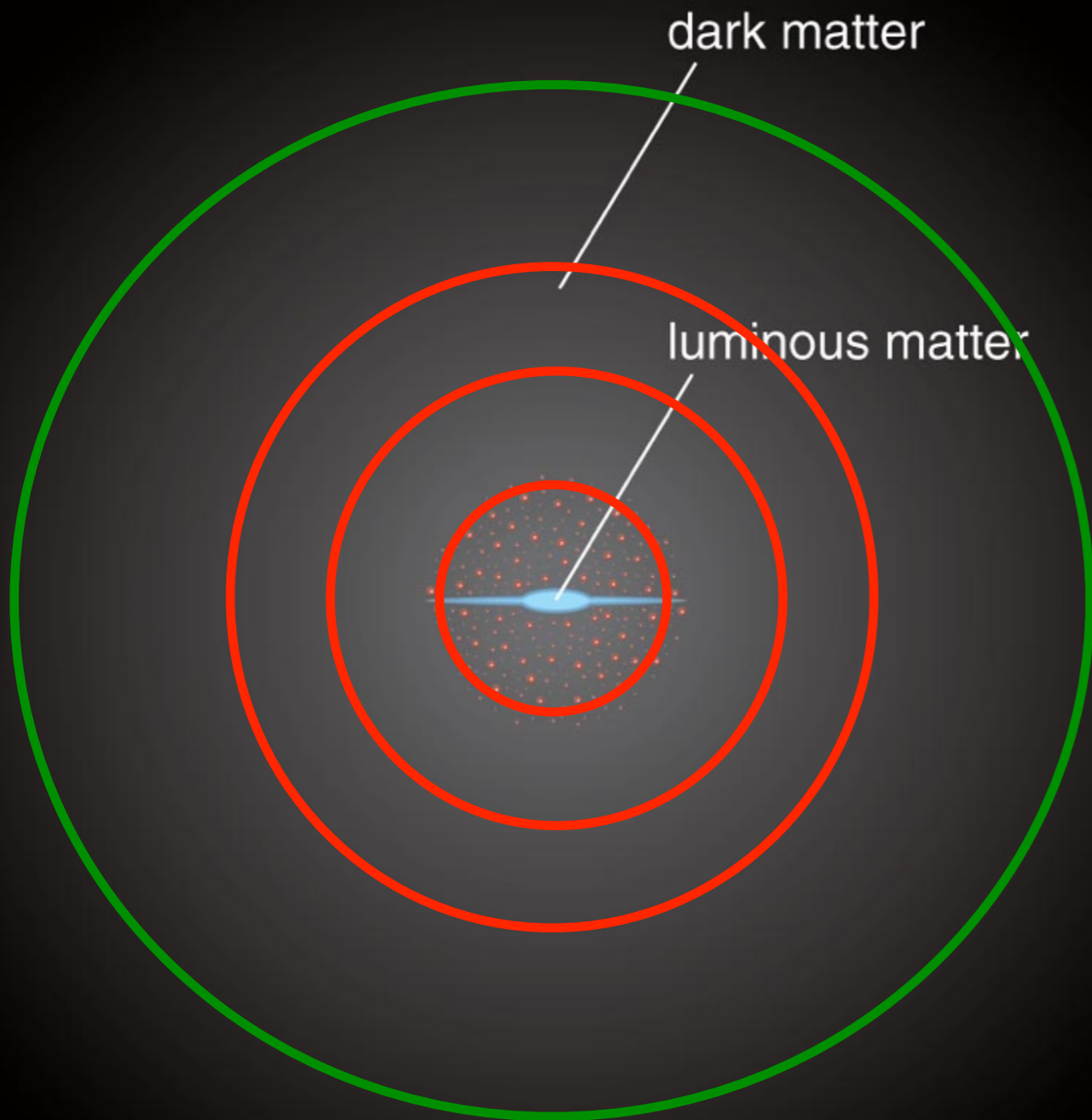


So mass must still be increasing with radius!



# Milky Way: Mass

We can't see it, but the velocity measurements tell us there is mass at larger distances than all the mass we can see



# Galaxies in the Universe

- ❖ Light travels at a finite speed (300,000 km/s).
- ❖ Galaxies at large distances → light left those galaxies a long time ago.
- ❖ Large distance = large “look-back time”

The distance to Andromeda, the nearest galaxy to the Milky Way (our galaxy), is 2.5 million light-years.

Stars in the Andromeda galaxy emitted the light we see when we look at Andromeda today 2.5 million years ago.

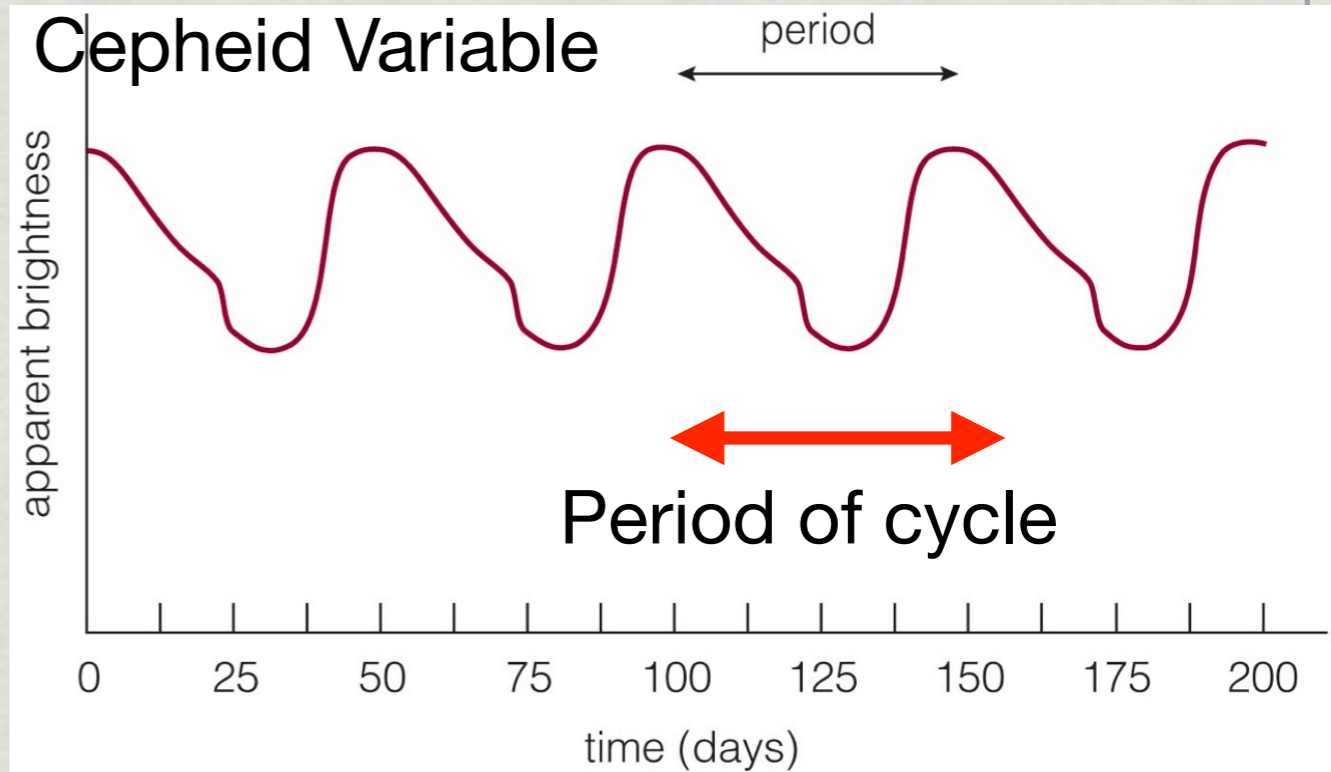
# Distances to Galaxies

**More luminous Cepheids have longer periods**

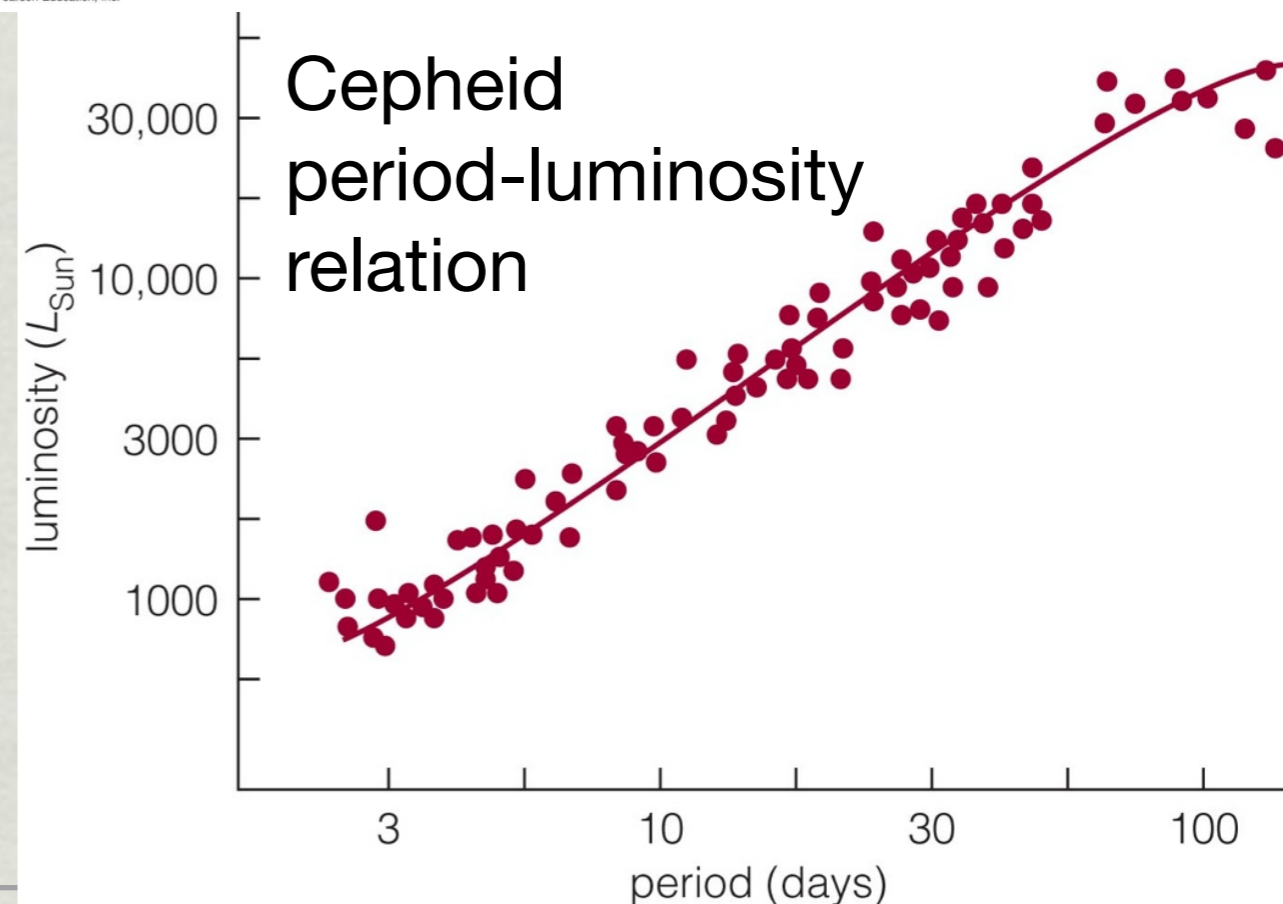
This is *incredibly useful*: if you can measure period, you can learn luminosity of a Cepheid without knowing its distance!

If you also measure the apparent brightness, you can learn the distance to the Cepheid, and therefore to the star cluster or galaxy it lives in:

$$L = B \times 4\pi d^2$$



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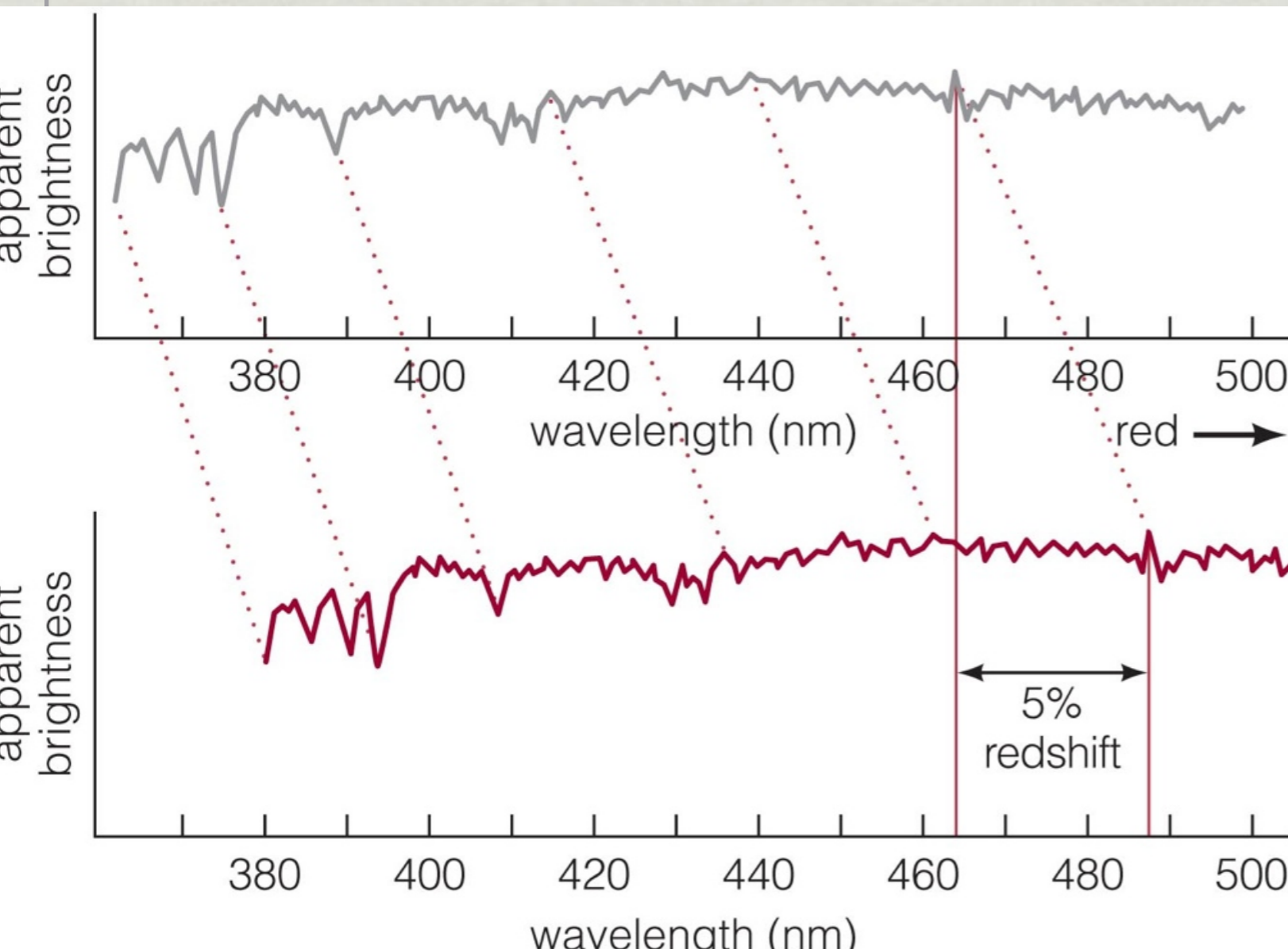


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# Distances to Galaxies

Edwin Hubble (him again) used the inverse square law for light again to measure the distances to other galaxies.

Astronomers already knew that the spectra of almost all other galaxies is red-shifted.



← normal galaxy spectrum. Looks like the sum of many star spectra. More light from bright stars, less from faint stars.

← This is what we observe.

Redshift ( $z$ ):

$$z = \frac{v}{c} = \frac{\lambda_{shift} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda}$$

This is just the Doppler formula for light (Chapter 5)

# Hubble's Law and the Expanding Universe

Hubble's Law  $v = H_0 \times D$

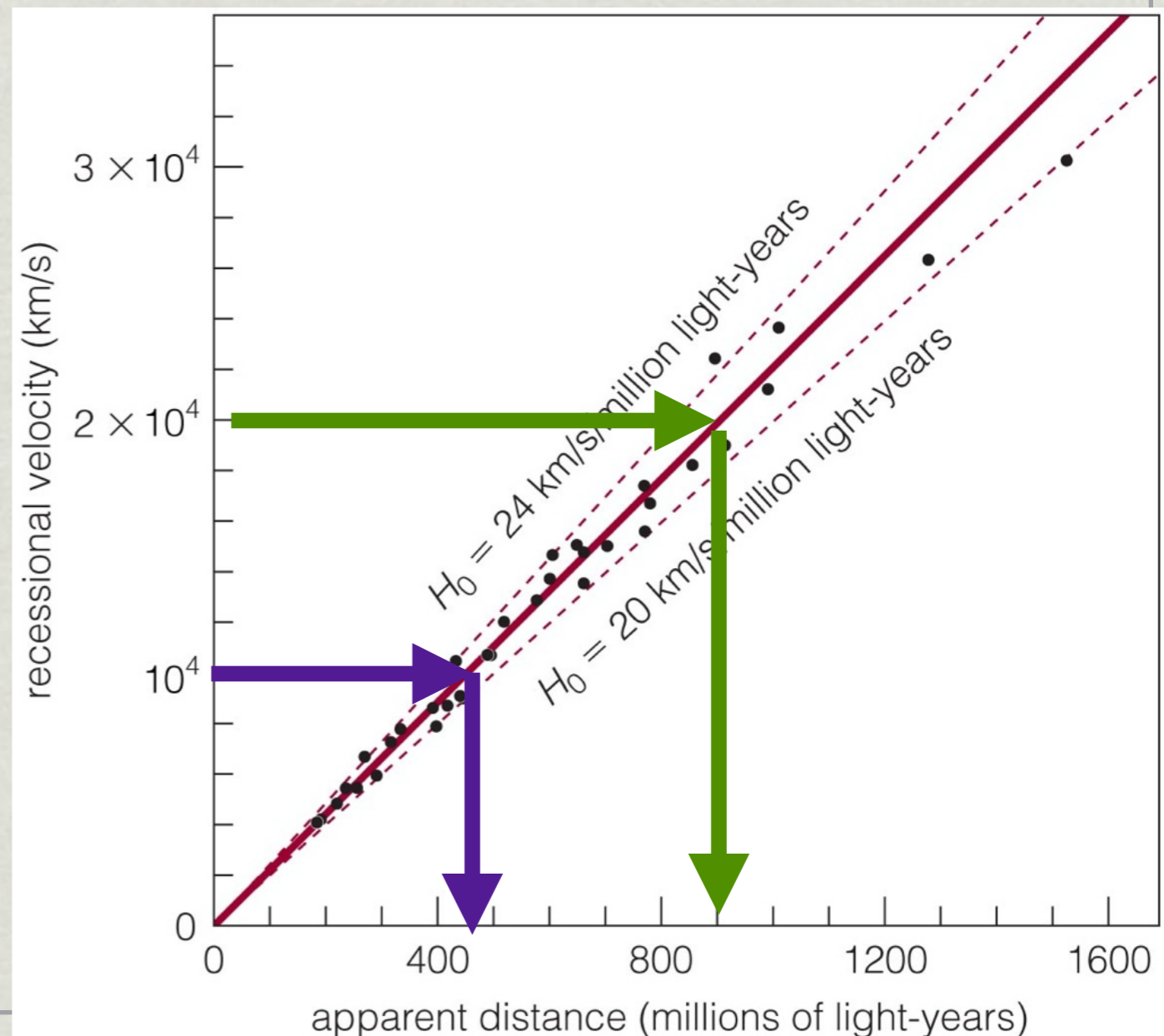
$H_0 = 21 \text{ (km/s)/Mly}$

$H_0$  = Hubble's Constant, relates distance and velocity

Looks like all the galaxies are moving away from us  
More distant galaxies are moving faster.

Space is expanding, so the distance between galaxies is growing.

Measure a galaxy's Doppler velocity (redshift), know  $H_0$ , find its distance from us.



# Hubble's Law and the Expanding Universe

Hubble's Law:  $v = H_0 \times D$

$\frac{1}{H_0}$  has units of seconds = time!

What time is that? Time since the expansion started.  
That's the age of the universe.

Galaxies that are farther away are going faster.

All galaxies take the same amount of time to get where they are now if they all started very close together and the universe expands at a constant rate.

