Solution Set

Astronomy 5, Spring 2007
Problem Set #1

Due at the beginning of class, April 18.

Print your name here:

Please feel free to ask for hints and/or clarification. Work the problems on this handout. If you need more space, use the back of the pages, or attach extra sheets at the end. Please put circles around final numerical answers. Homework MUST be stapled. Questions marked with a star are especially challenging and may involve kinds of mathematical thinking that are unfamiliar. Starred problems will be worked as a group IN SECTION. You can work the problems there and hand in your answers.

1. Practice manipulating powers of 10. Fill in the blanks:

   (a) $149.1 \times 10^3 = 14.91 \times 10^?$ 4

   (b) $27695.2 \times 10^7 = 2769.5 \times 10^?$ 8

   (c) $0.0349 \times 10^{-1} = 0.000349 \times 10^?$ 1

   (d) $0.0000842 \times 10^{-4} = 84.2 \times 10^?$ −10

2. **Scientific notation** uses the convention that one significant figure (not 0) is shown to the left of the zero point. Find the answers to the calculations below using scientific notation. Show your work and **circle** your answer. Hint: write all the numbers in scientific notation first, then do the arithmetic on the parts out front and re-express **that** in scientific form. Then combine all the powers of 10 last.

   (a) $(5.2 \times 10^3) \times 3 =$ $15.6 \times 10^3 = \boxed{1.56 \times 10^4}$

   (b) $(2.2 \times 10^6)/40 =$ $\frac{2.2}{40} \times 10^6 = 0.055 \times 10^6 = \boxed{5.5 \times 10^4}$

(c) $\frac{200\times3.1\times10^{25}}{7\times10^3\times2600\times10^{-14}} = \frac{620\times10^{25}}{18200\times10^3\times10^{-14}} = 0.034\times\frac{10^{25}}{10^3\times10^{-14}}$

$= 0.034\times\frac{10^{25}}{10^{-11}} = 0.034\times10^{36}$

$= \boxed{3.4\times10^{34}}$

(d) $1/4600 = \frac{1}{4.6\times10^3} = 0.22\times10^{-3}$

$= \boxed{2.2\times10^{-4}}$

3. Light travels at speed $c = 3\times10^{10}$ cm/s. How long does it take light to travel to Earth from...

$v\text{ (cm/sec)} = \frac{\text{distance (cm)}}{\text{time (sec)}} \Longrightarrow \text{time} = \frac{\text{distance}}{\text{speed}}$

(a) ...a nearby star? (distance = 3 pc) Hint: 1 pc = 3.1×10^{18} cm.

distance = 3pc = $3\times3.1\times10^{18}$ cm = 9.3×10^{18} cm

time = distance/speed = 9.3×10^{18} cm/3×10^{10} cm/s = $\boxed{3.1\times10^8 \text{ s}}$

= $\boxed{9.9 \text{ years}}$

(b) ...the center of the Milky Way Galaxy? (distance = 8 kpc)

$d = 8{,}000\times3.1\times10^{18}$ cm

$= 2.5\times10^{22}$ cm

$t = \frac{d}{v} = \frac{2.5\times10^{22}\text{ cm}}{3\times10^{10}\text{ cm/s}} = \boxed{8.3\times10^{11}\text{ s}}$

$= \boxed{26{,}500 \text{ years}}$

(c) ...our nearest major galaxy neighbor, the Andromeda Galaxy? (distance = 750 kpc)

$d = 750\times10^3\times3.1\times10^{18}$ cm

$= 2.3\times10^{24}$ cm

$t = \frac{d}{v} = \frac{2.3\times10^{24}\text{ cm}}{3\times10^{10}\text{ cm/s}} = \boxed{7.7\times10^{13}\text{ s}}$

$= \boxed{2.4 \text{ million years}}$

(d) How long does it take light to travel from Los Angeles to Santa Cruz? (distance = 550 km)

1 km = 1000m

1 m = 100 cm

$d = 550\times10^3\times10^2$ cm

$= 5.5\times10^7$ cm

$t = \frac{d}{v} = \frac{5.5\times10^7\text{ cm}}{3\times10^{10}\text{ cm/s}} = \boxed{1.8\times10^{-3}\text{ s}}$

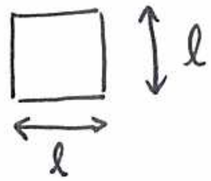
$= \boxed{0.0018 \text{ s}}$

4. Explain in your own words what is meant by the term "cosmic horizon" (this is the term called "horizon" in Lecture 2). Why does it exist?

The cosmic horizon is the edge of the visible universe. The Universe probably continues beyond the horizon, but we cannot see that part. It exists because light takes time to travel and the universe is not infinitely old. For objects beyond the cosmic horizon, light has not had time to travel from there to Earth over the age of the Universe.

5. Practicing with log-log plots. Remember from basic geometry that the area of a rectangle, A, is its width, w, times its length, l. For a square, $l = w$, so $A = l \times w = l^2$. We can make a simple table of the length and the corresponding area for a set of squares.

length	area
1	1
2	4
3	9
4	16
5	25
6	36
8	64
10	100
30	900
50	2500
100	10000



area $A = l^2$

(a) Using one of the sheets of regular graph paper provided, plot the length of the square on the x-axis and the area of the square on the y-axis. Remember to label the axes. (If you make a mistake and need more graph paper, you can print it off the course website at: www.ucolick.org/~faber/ay5/ay5.html.) Plot as many of the squares as you can fit onto the piece of paper. Do they all fit?

No.

(b) You could get all the points to fit by shrinking the scale of the plot, but then what would happen? Try it on the second sheet of regular graph paper.

It becomes almost impossible to plot the data for the small squares because they all get crunched together.

(c) Now use the sheet of log-log graph paper provided. First, look at the number labels on the plot. Each "cycle" of numbering (1–9 and then starting back at 1) covers a factor of 10, but the zeroes have been left off. Thus, the x-axis should really read: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000, etc., and the same for the y-axis. As in part (a), plot the length on the x-axis and the area on the y-axis. Now were you able to fit all of the data on this plot? Which is a better kind of plot for this data set? Can you state a general rule when log-log plots are better than regular ones?

All the data fits. The log-log plot is a more reasonable way to plot this data set. In general, log-log plots are useful for data sets that span many orders of magnitude (i.e., area goes from 1 to 10,000)

(d) When you make the log-log plot in (b), all of the points should fall on a straight line. What is the slope of this line? (Hint: to measure the slope, count in terms of "cycles" of numbering. One cycle on the x-axis corresponds to how many cycles on the y-axis for your data?)

$$\frac{\text{2 powers of 10 on y-axis}}{\text{1 power of 10 on x-axis}} \Rightarrow \text{slope} = 2$$

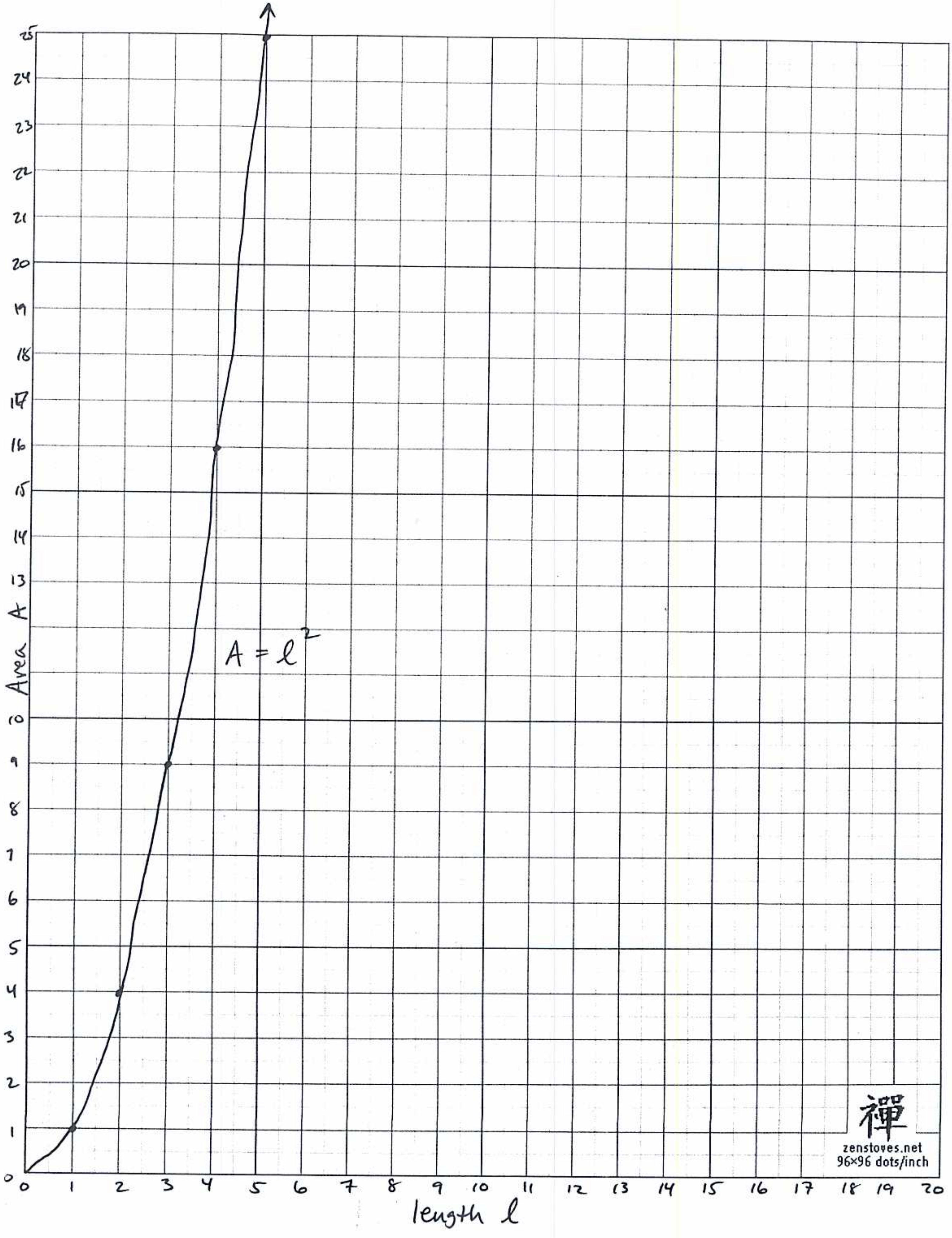
When a data set makes a straight line on a log-log plot, we say the data follows a "power-law" relation. You have just measured the slope of the power-law. We will see many power-law relations throughout this course.

(e) Let's write again the length-area relation for a square: $A = l^2$. Is the slope of your line in the log-log plot the same as the value of the exponent of l? Based on this, write down a general rule for the slope of a power-law relation.

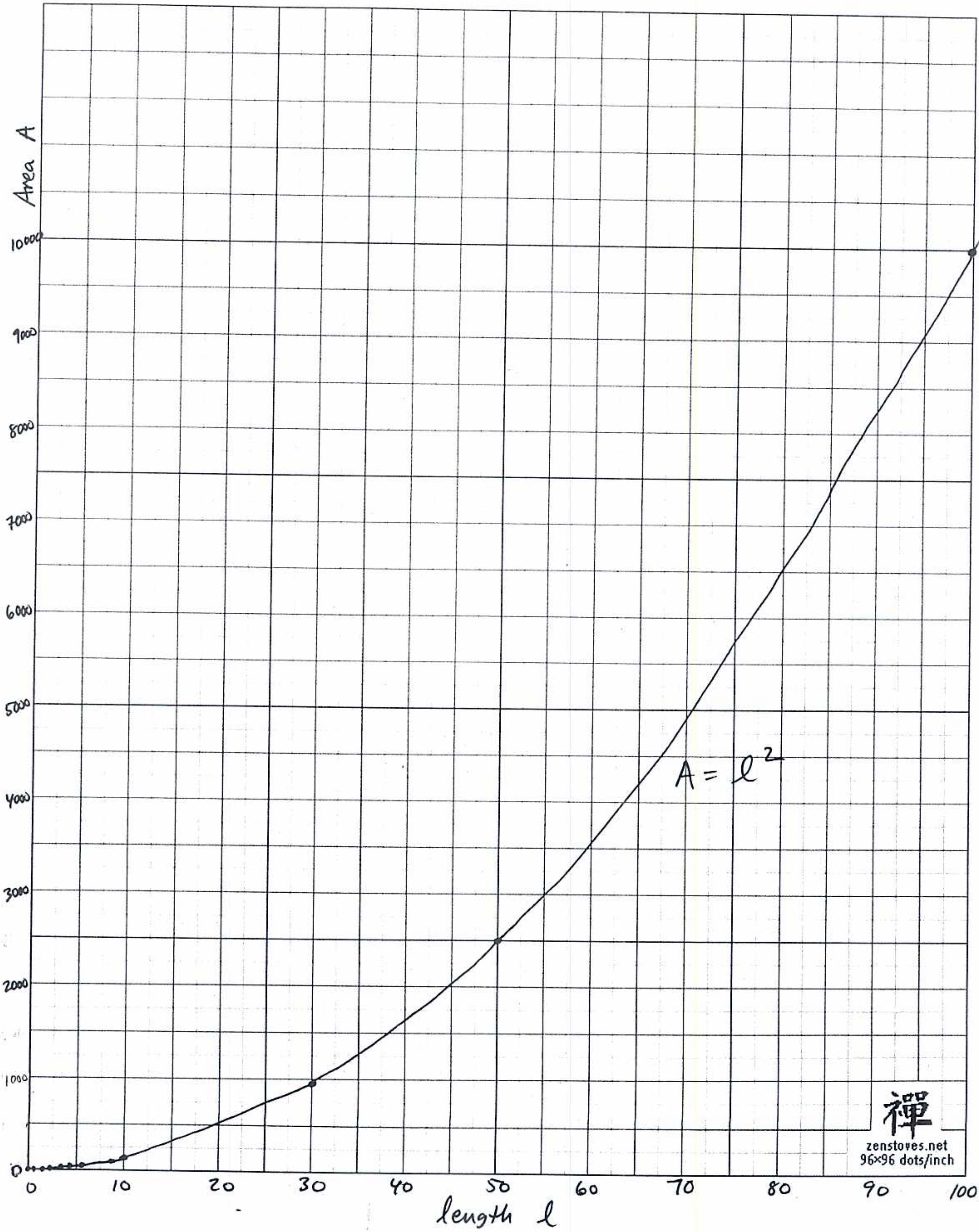
$A = l^2$ exponent $= 2$

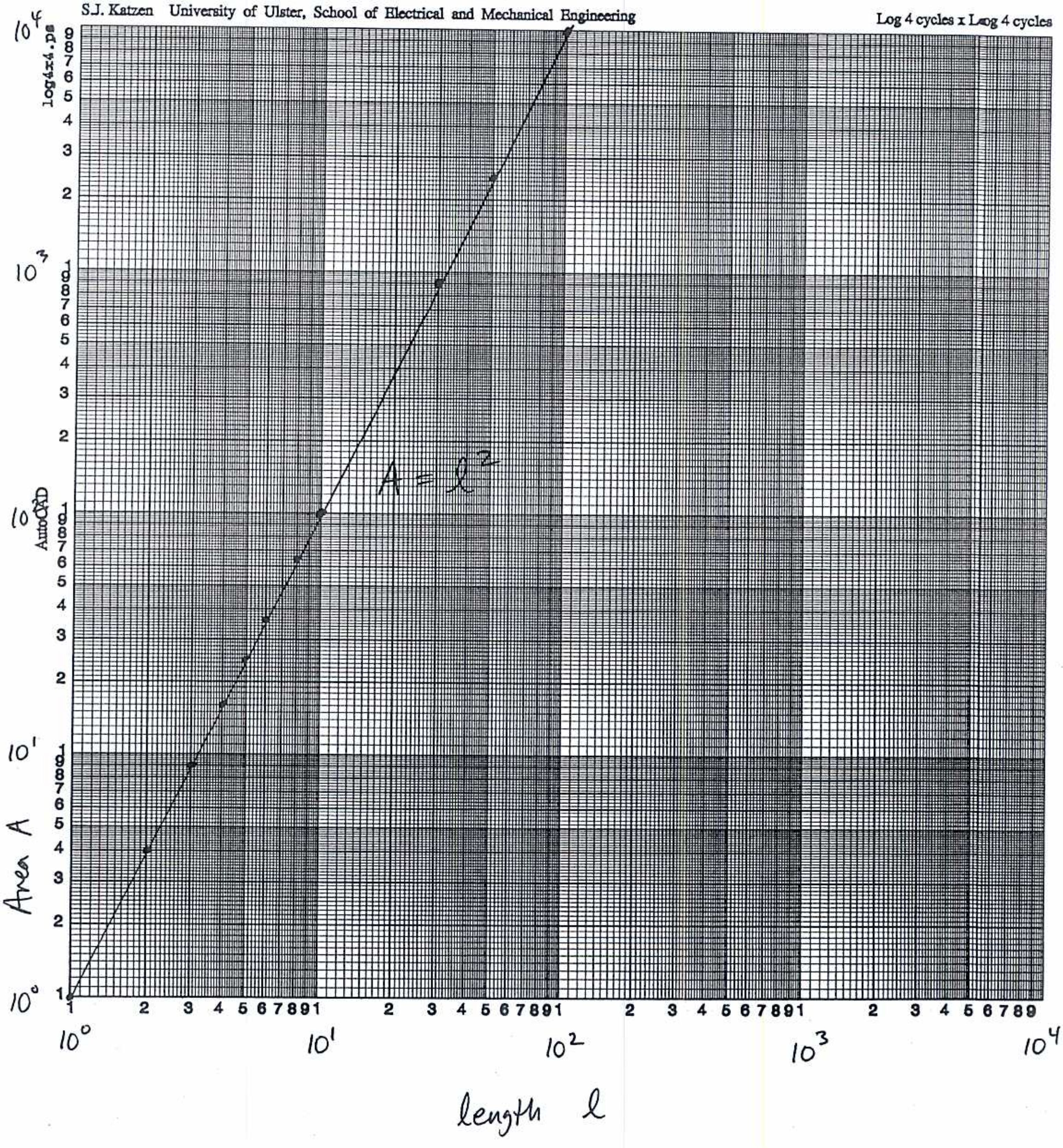
slope of powerlaw $= 2$

The slope of a power-law relation shows the exponential relationship between x and y.

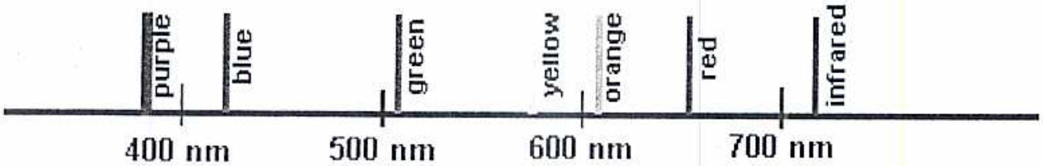


Area A
length l
A = l²
禪
zenstoves.net
96×96 dots/inch


Area A
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
10
20
30
40
50
60
70
80
90
100
length l
A = l²
zenstoves.net
96×96 dots/inch


S.J. Katzen University of Ulster, School of Electrical and Mechanical Engineering
Log 4 cycles x Log 4 cycles
log4x4.ps
AutoCAD
$A = l^2$
Area A
length l
10^0
10^1
10^2
10^3
10^4

*6. The term "error" signifies a deviation of a measurement from some "true" value. Often we cannot know what the true value is, and we can determine only estimates of the errors inherent in the experiment. If we repeat the experiment, the result may differ from that on the first attempt. We can express this difference as a discrepancy between the two results. The fact that a discrepancy arises is due to the fact that we can determine our results only within a given **uncertainty** or error. The more precise our measuring tool, the more precise our answer will be; the more accurate our input data, the more accurate the results. But, **we will always have some uncertainty**. As a simple example, consider the following figure, which shows the wavelengths of some well known emission lines from astronomical objects:



(a) Estimate the wavelengths in nm (nanometers are 10^{-9} meters) of the blue, yellow, and red lines in the figure. Just use your eyes to estimate in this first step.

blue: 425 nm   red: 655 nm

yellow: 575 nm

(b) What is your estimated uncertainty in your answers? That is, how far off (in nm) could your determinations be? Imagine yourself changing the wavelength slightly. How big a change could you allow?

± 5 nm

(c) If you compared your wavelength measurements to those of your friend, would her/his results differ from yours? Why or why not?

Results might differ slightly because they are estimates. They should be within the ± 5 nm error.

(d) A finely graded ruler with a mm scale is printed at the end of your homework set. Cut this out. Devise a method to use the ruler in order to get **more accurate** wavelength estimates. Compare your new answers with the old ones by filling in the following table:

on my ruler: 100 nm $\rightarrow$ 27 mm $\Rightarrow$ 1 mm = 3.7 nm

blue line is 6 mm away from 400nm. 6mm = nm

$\therefore$ blue line is at 400 + = $\boxed{422 \text{ nm}}$

yellow line is 20mm away from 500nm. 20mm = 74 nm

$\therefore$ yellow line is at 500 + 74 = $\boxed{574 \text{ nm}}$

red line is 12.5mm <u>below</u> 700 nm   12.5 mm = 46 nm

$\therefore$ red line is at 700 − 46 = $\boxed{654 \text{ nm}}$

Color	1^{st} Wavelength from part (a)	Estimated Error from part (b)	2^{nd} Wavelength from part (d)	Wavelength Difference (1^{st} - 2^{nd})
Blue	425 nm	± 5 nm	422 nm	3 nm
Yellow	575 nm	± 5 nm	574 nm	1 nm
Red	655 nm	± 5 nm	654 nm	1 nm

(e) Presumably your ruler-estimated numbers are more accurate than your eyeball estimates from part (a). Let's take the ruler numbers as "truth." Compare your original error estimates from part (b) with the "real" errors found by taking the difference between the eye and ruler estimates in part (d). How good were your original error estimates?

My original error estimates were a little high. My actual errors were only 1-3 nm, less than the ±5 nm I had estimated.