Solution Set

Astronomy 5, Spring 2007
Problem Set #2

Due at the beginning of class, April 27.

Print your name here:

Please feel free to ask for hints and/or clarification. Work the problems on this handout. If you need more space, use the back of the pages, or attach extra sheets at the end. Please put circles around final numerical answers. Homework MUST be stapled. Questions marked with a star are especially challenging and may involve kinds of mathematical thinking that are unfamiliar. **Starred problems** will be worked as a group IN SECTION. You can work the problems there and hand in your answers.

1. A galaxy, the Sun, and the Moon all have different intrinsic sizes. On Earth, the Moon appears about the same size as the Sun, but the Sun is, in fact, much larger. If the Sun were closer to the Earth than it is, it would appear larger, and conversely if it was farther away it would appear smaller. If one knows the linear size of an object, the Small-Angle Formula gives a precise relationship between the apparent angular size of an object on the sky and its distance away from us.

$$D = \frac{\alpha d}{3438},$$

Here, D is the intrinsic linear size of an object, α is the angular size of the object in arc-min, and d is the distance to the object. Note that intrinsic linear size is measured in units of length such as light years, and apparent size is measured in angular units such as arc-minutes (There are 60 arc minutes in a degree.). Also, D and d must be in the same units for the above formula to work. For a simple discussion of the Small-Angle Formula, look at pg. 8 to 9 in the text book.

(a) Astronomers believe that morphologically similar galaxies have roughly the same intrinsic size. Thus if spiral galaxy (1) is half the angular size of spiral galaxy (2), then galaxy (1) is approximately twice as far away as (2). Below are two normal spiral galaxies taken in the Sloan Digital Sky Survey, which is an enormous survey which covers about a quarter of the entire celestial sphere. Assume that these galaxies have the same intrinsic size and write them below. Estimate their diameters in arc minutes. (Hint use the arc minutes scale on the picture.)

Galaxy A : diameter ~ 1 arcmin

Galaxy B: diameter ~ 4 arcmin

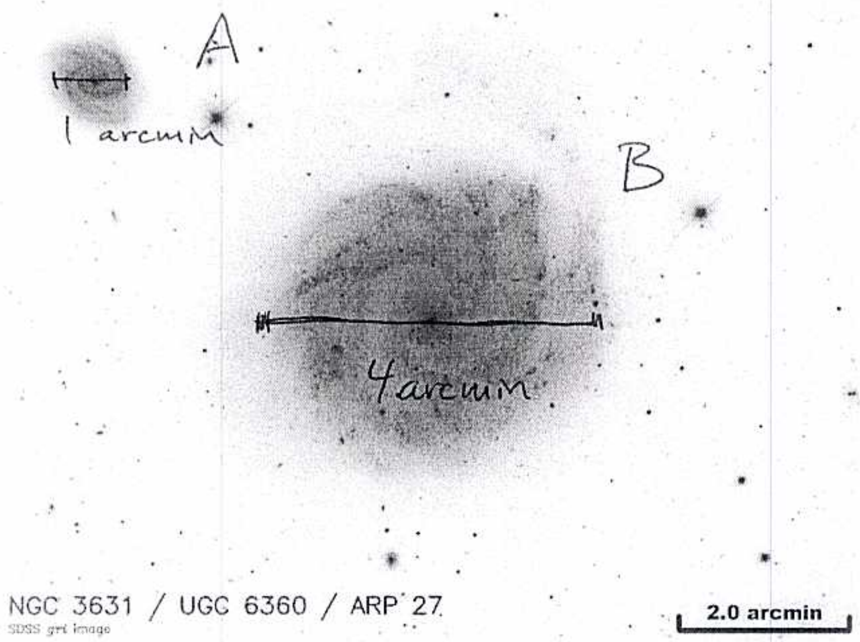


Figure 1: galaxies from Sloan Digital Survey

(b) Galaxies are fuzzy. How did you decide where the edge of each galaxy is?

I chose a region that seemed to include all spiral arms, but that does exclude some of the fainter outer parts of the galaxy.

(c) Which galaxy is the nearer? Farther?

B is nearer. A is farther.

(d) How much farther away from us is the farther galaxy compared to the nearer galaxy, in relative terms, i.e., the farther galaxy might be 5 times farther away?

$D = \frac{\alpha d}{\text{constant}}$ $\Rightarrow$ if D is the same for both (assuming they have the same intrinsic size), α goes as $\frac{1}{d}$

(e) To get a better feel for size and distance, look up the intrinsic diameter of the Moon and its distance from the Earth from the text book. Then, use the Small Angle Formula to

Therefore Galaxy A is 4 times farther away than Galaxy B.

calculate the angular size of the Moon on Earth in arc minutes. (Hint use the same units of length for D and d in the above equation.)

Textbook pg. A-2: Moon distance $d = 384{,}400$ km, size $D = 3476$ k

$$D = \frac{\alpha d}{3438} \Longrightarrow \alpha = \frac{3438\ D}{d} = \frac{3438 \times 3476\ \text{km}}{384{,}000\ \text{km}} = \boxed{31\ \text{arcmi}}$$

(f) It's handy to measure angles using a tool you always have with you, namely your thumb. What is the angular size of your thumb held at arms length? How did you get that angular size?

My thumb is 0.75 inches wide. $\Longrightarrow D = 0.75$ inches
My arm is about 22 inches long $\Longrightarrow d = 22$ inches

$$\alpha = \frac{3438 \times 0.75\ \text{in}}{22\ \text{in}} = 120\ \text{arc min}$$

(g) The next time you see the Moon, hold up your thumb. What fraction of your thumb's width aught to cover the Moon?

1/4 of my thumb should cover the moon.

2. * The apparent brightness of a star depends upon its luminosity and the distance between us and it. If one could move a star farther and farther away, the star's apparent brightness would decrease. Pages 412 to 416 of the text discuss the relationship between luminosity and apparent brightness, which is better known as the Inverse-Square Law:

$$b = \frac{L}{4\pi d^2},$$

where b is the star's apparent brightness measured by a telescope. The star lies at a distance d away with a luminosity L. The problem below helps you use the Inverse-Square Law to find distances to nearby Sun-like stars.

If we want to use the equation above $\left(b = \frac{L}{4\pi d^2}\right)$ we have to do everything in standard physics units: meters, seconds, and kilograms. In this problem, we are comparing other stars to the Sun (i.e., using "Solar" units) so we have to use the equation to derive a scaling relation between two stars. Say we are comparing Star A and the Sun. We can write:

$$b_{sun} = \frac{L_{sun}}{4\pi d_{sun}^2} \quad \text{and} \quad b_A = \frac{L_A}{4\pi d_A^2} \Longrightarrow$$

Now divide one equation by the other:

$$\frac{b_{sun}}{b_A} = \frac{\frac{L_{sun}}{4\pi d_{sun}^2}}{\frac{L_A}{4\pi d_A^2}} \Rightarrow \frac{b_{sun}}{b_A} = \frac{L_{sun}}{\cancel{4\pi} d_{sun}^2} \cdot \frac{\cancel{4\pi} d_A^2}{L_A} \Rightarrow \frac{b_{sun}}{b_A} = \frac{L_{sun}}{L_A} \cdot \frac{d_A^2}{d_{sun}^2}$$

(a) As budding astronomers, we have recorded the apparent brightness b of several stars and have measured their distances from Earth with the help of the Hipparcos satellite [1]. Results are in the table below. The distances are measured in *a.u.* (Astronomical Units)[2] and the apparent brightness b is measured relative to the Sun's apparent brightness. (See Table.) In these units, the Sun has a luminosity of one and a distance of one and the apparent brightness of one. Please calculate the luminosity for Tau Ceti and Beta Canum Venaticorum in solar luminosities. Then enter the values found into the table.

	luminosity	distance	brightness
Sun	1	1	1
Tau Ceti	0.59	7.7×10^5	1.0×10^{-12}
Delta Pavonis	0.94	1.2×10^6	6.5×10^{-13}
Beta Canum Venaticorum	1.2	1.7×10^6	4.2×10^{-13}
Gliese 67	1.5	2.6×10^6	2.1×10^{-13}
51 Pegasi	1.35	2.5×10^6	2.1×10^{-13}

$$\frac{1}{b_{Tau\,Ceti}} = \frac{d_{Tau\,Ceti}^2}{L_{Tau\,Ceti}}$$

$$L_{Tau\,Ceti} = b_{Tau\,Ceti} \times d_{Tau\,Ceti}^2 = (1\times10^{-12})(7.7\times10^5)^2 = \boxed{0.59}$$

Use our "Scaling relation" from above: $\boxed{\frac{b_{sun}}{b_A} = \frac{L_{sun}}{L_A} \cdot \frac{d_A^2}{d_{sun}^2}}$

The Sun has: $b_{sun} = 1$, $L_{sun} = 1$, $d_{sun} = 1$

in the units for this problem. Therefore: $\boxed{\frac{1}{b_A} = \frac{1}{L_A} \cdot d_A^2}$

(b) In the case of Delta Pavonis and 51 Pegasi, the situation is different. We know their intrinsic luminosities by placing them on the main sequence. Their placement is determined by their surface temperature, but we don't know their distances a priori. So these values in the table have been left blank. However, we do know their apparent brightnesses. Now, use the Inverse Square Law to calculate their distances in *a.u.*.

$$\frac{1}{b_A} = \frac{d_A^2}{L_A}$$

Delta Pavonis

$$\frac{1}{b_{DP}} = \frac{d_{DP}^2}{L_{DP}} \Rightarrow d_{DP}^2 = \frac{L_{DP}}{b_{DP}}$$

$$d_{DP} = \sqrt{\frac{L_{DP}}{b_{DP}}} = \sqrt{\frac{0.94}{6.5\times10^{-13}}} = \boxed{1.2\times10^6 \text{ A.U.}}$$

[1] Hipparcos was a space mission funded by the European Space Agency. It carefully measured the distance to over 120,000 stars using parallax. Hipparcos was an ancient Greek astronomer who discovered the use of parallax. You don't need to understand parallax to do this problem. You only need to know that it can be used to measure distances to nearby stars. If you want to know more see pp.410-411 in the text.

[2] One Astronomical Unit (*a.u.*) is the distance from the Sun to the Earth

51 Pegasi

$$\frac{1}{b_{51P}} = \frac{d_{51P}^2}{L_{51P}}$$

$$d_{51P}^2 = \frac{L_{51P}}{b_{51P}}$$

$$d_{51P} = \sqrt{\frac{L_{51P}}{b_{51P}}} = \sqrt{\frac{1.35}{2.1\times10^{-13}}} = \boxed{2.5\times10^6}$$

Beta Canum Venaticorum

$$\frac{1}{b_{BCV}} = \frac{d_{BCV}^2}{L_{BCV}}$$

$$L_{BCV} = b_{BCV} \times d_{BCV}^2 = (4.2\times10^{-13})\times(1.7\times10^6)^2$$

$$\boxed{L_{BCV} = 1.2}$$

Fake Star A: $d_A = 10^5$ a.u. $L_A = 1$

Fake Star B: $d_B = 10^7$ a.u. $L_B = 1$

$\frac{1}{b_A} = \frac{d_A^2}{L_A}$ $b_A = \frac{L_A}{d_A^2} = \frac{1}{(10^5)^2} = \boxed{10^{-10}}$ $b_B = \frac{L_B}{d_B^2} = \frac{1}{(10^7)^2} = \boxed{10^{-14}}$

(c) Practicing with log-log plots: now plot the apparent brightness b vs. distance d of the stars in the above table on a log-log plot **omit** the Sun. To expand the range of the table add two more "fake" stars each with a solar luminosity but one at a distance of $10^5 a.u.$ and the other fake star at a distance of $10^7 a.u.$. Get the "fake" star's apparent brightnesses by using the Inverse Square Law. (Use the graphing paper at the end of this problem set to make this plot.)

$b_A = 10^{-10}$ $b_B = 10^{-14}$

(d) Draw a straight line through the two fake Suns at $10^5 a.u.$ and at $10^7 a.u.$. This is the exact Inverse-Square Law for stars with a luminosity exactly equal to the Sun. What is the slope of this line on the log-log plot?

$\frac{-4 \text{ orders of magnitude in } y}{2 \text{ orders of magnitude in } x} \Rightarrow \text{slope} = -2$

(e) Now consider stars close but not exactly equal to the Sun in luminosity L. Where should they lie relative to this line? (Hint: look at the locations of all the stars brighter than the Sun versus the stars fainter than the Sun.)

Stars more luminous than the Sun should lie above the line (more luminous at the same distance → brighter). Stars less luminous than the Sun should lie below the line (because they are fainter when seen from the same distance).

3. Properties of galaxies. For a given property, identify that it is a property of either an elliptical galaxy, a spiral galaxy, or both, by circling either spiral galaxy, elliptical galaxy or both.

The Milky Way is this kind of galaxy? **Elliptical Spiral**

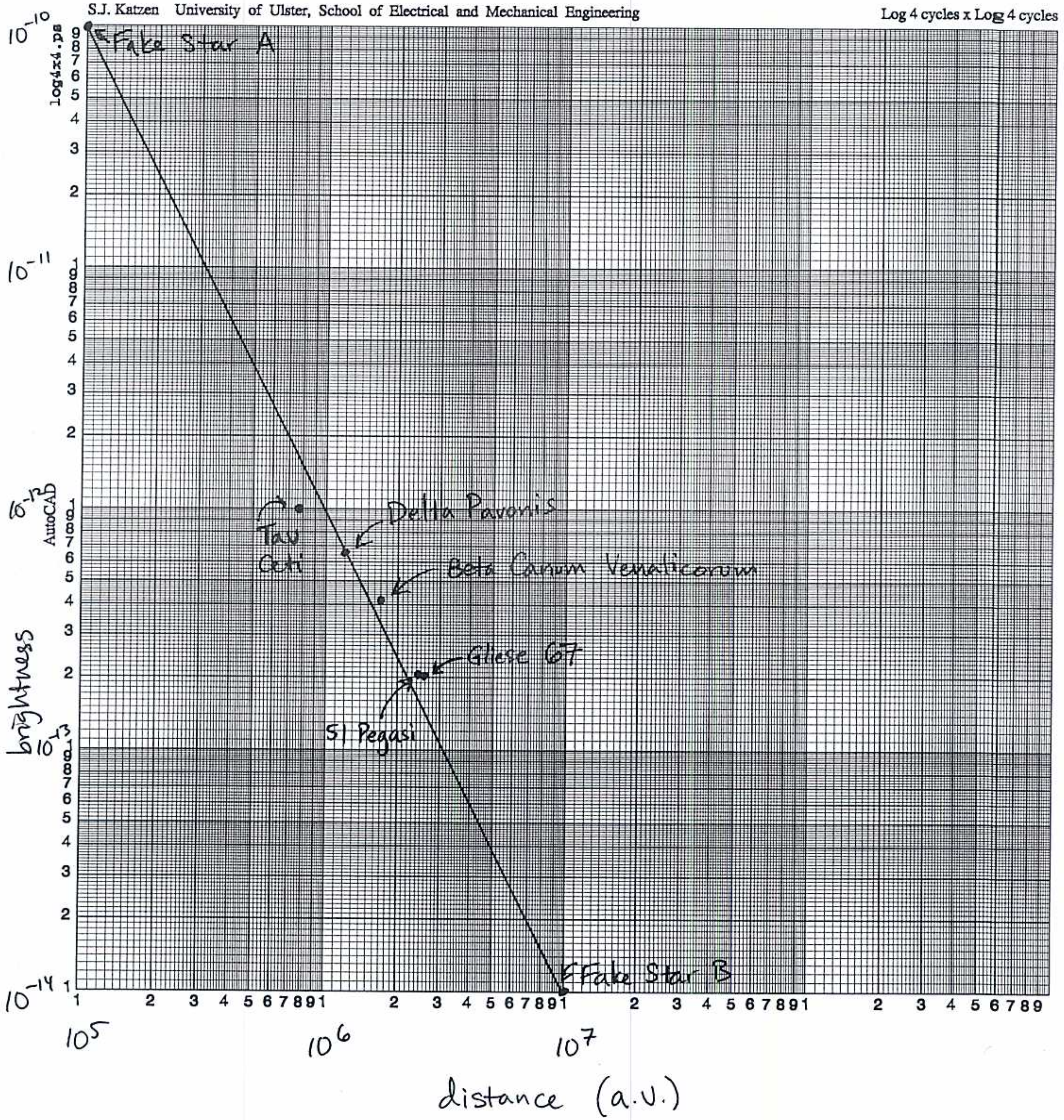
What kind of galaxy has a large spheroid? **Elliptical Spiral**

What kind has a large disk? **Elliptical Spiral**

What kind has orderly circular orbits? **Elliptical Spiral**

What kind has randomly scrambled elliptical orbits? **Elliptical Spiral**

Which kind is still making stars? **Elliptical Spiral**



S.J. Katzen University of Ulster, School of Electrical and Mechanical Engineering
Log 4 cycles x Log 4 cycles
log4x4.ps
AutoCAD
Fake Star A
Fake Star B
Tau Ceti
Delta Pavonis
Beta Canum Venaticorum
Gliese 67
51 Pegasi
brightness
distance (a.u.)