

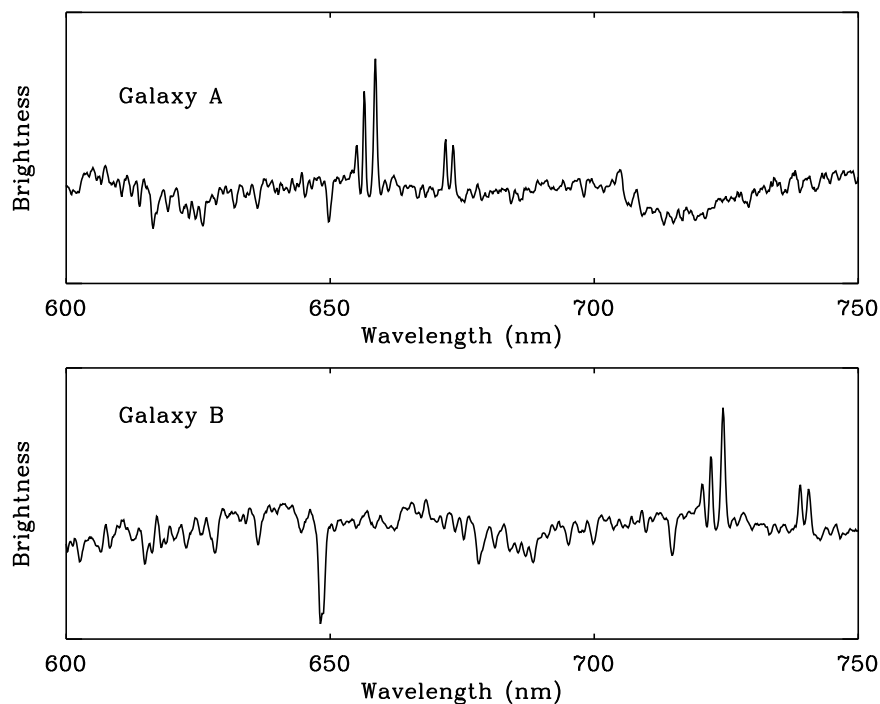
Astronomy 5, Spring 2007

Problem Set #3

Due at the beginning of class, May 11.

Print your name: Please feel free to ask for hints and/or clarification. Work the problems on this handout. If you need more space, use the back of the pages, or attach extra sheets at the end. Please put circles around final numerical answers. Homework **MUST** be stapled. Questions marked with a star are especially challenging and may involve kinds of mathematical thinking that are unfamiliar. **Starred problems** will be worked as a group IN SECTION. You can work the problems there and hand in your answers.

1. Measuring the redshifts of galaxies. Look at the figure below. The spectra of two galaxies are shown. Light from each galaxy has been passed through an instrument called a spectrograph. The spectrograph is essentially a prism that spreads the light out into a rainbow, with the bluer light shown at the left end of the figure and the redder light at the right end.



There are lots of little bumps and wiggles in the spectra. These are called “absorption lines” and they tell astronomers about the atoms and molecules in the outer atmospheres of the *stars* in the galaxy. Notice that there are also upward spikes. These are called “emission lines” and are caused by *glowing gas* in the galaxies. All of these features, both absorption lines and emission lines, are emitted at *rest wavelengths* which do not change—they are constant across the Universe. In these galaxy spectra, there are two groups of emission lines: a group of 3 lines (which are from left to right weak, stronger, and strongest), and then a pair of lines to the right of them which are about the same strength. The first group of lines is due to hydrogen and nitrogen in the gas of the galaxy, while the second pair of lines is due to sulfur.

We just said that the wavelengths of these lines are the same everywhere, but looking at these galaxy spectra, the lines are obviously in *different* places. This is an example of *redshift*. Galaxy A is nearby, but Galaxy B is very far away from us. Because the Universe is expanding, Galaxy B is moving away from us quickly. For the sake of this problem, let’s pretend that Galaxy A is very nearby, and is hardly moving away from us at all.

(a) Choose an emission line that you can clearly identify in both galaxy spectra. Mark the line on the spectrum of Galaxy A. Using any method you like, measure the wavelength of your chosen emission line in Galaxy A.

(b) Now find the same line in Galaxy B and mark it. Using the same method from part (a), measure the wavelength of the emission line in Galaxy B.

(c) Because Galaxy A is nearby and only moving away from us very slowly, assume that the wavelength you measured in (a) is the *rest wavelength* of the emission line, which we will write as λ_{rest} . The wavelength you measured in (b) for Galaxy B is the “observed” wavelength of the emission line, which we will write as λ_{obs} . Calculate the redshift, z , of Galaxy B using your measurements from (a) and (b) and the redshift formula from Lecture 8:

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} \quad (1)$$

(d) The redshift formula is a version of the low-speed Doppler formula (see discussion in Lecture 8, section 3),

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{v}{c} \quad (2)$$

where v is the speed with which the redshifted galaxy is moving away from us, and c is the speed of light, 300,000 km/s. Calculate the speed of Galaxy B as it moves away from us.

2. Weighing the black hole at the center of the Milky Way. Objects in orbit under the influence of gravity have to keep moving at an exact speed in order to be in a circular orbit. The speed of the orbit (we'll use the symbol v for velocity) is determined by the enclosed mass, $M(R)$, that the object is orbiting around and the distance (we'll use R for radius) of the object from the center of the orbit. Thus, if we can observe the speed of an orbit, v , and the radius of the orbit, R , we can calculate the enclosed mass, $M(R)$, that the object is orbiting around. The relevant equation is:

$$M(R) = \frac{v^2 R}{G} \quad (3)$$

where G is a constant, called the gravitational constant, and $M(R)$, R , and v are the enclosed mass, the orbital radius, and the velocity of the orbiting object, as described above.

(a) When the gravitating central body is a star, argue that the enclosed mass of a planetary orbit is just the star's mass. For example, argue that the enclosed mass for Earth's orbit around the Sun is about equal to the mass of the Sun. (Hint: in all solar systems that we know about, the mass of the planets is negligible compared to the mass of the star.)

(b) Suppose you had measured the speeds and radii of two different orbiting objects, planet A which orbits around star A, and planet B which orbits around star B. This means that you know the speeds, v_A and v_B , and the orbital radii, R_A and R_B , for each orbiting planet. You would like to compare the masses of the two stars, M_A and M_B . Use the equation above to write down a ratio relationship of the form:

$$\frac{M_A}{M_B} = ? \quad (4)$$

using the information you have measured: v_A , v_B , R_A , and R_B .

The ratio you wrote in part (b) can be used to compare the central masses of any orbits. For example, the Sun is orbiting the center of the Milky Way. It orbits at 26,000 ly from the center of the galaxy at a speed of 220 km/s. Astronomers have used this information to

measure the total mass of the Milky Way inside the Sun’s orbit: it’s about 90 billion Solar masses. We can write this as $9 \times 10^{10} M_{\odot}$, where $M_{\odot}$ is the mass of the Sun.

When astronomers look into the very center of the Milky Way galaxy, the stars closest to the center of the galaxy are moving very fast. Stars that are only 15 ly from the galaxy center are orbiting at speeds near 1,000 km/s—almost 5 times as fast as the Sun’s orbit. And yet, when we look at the very center of the Galaxy to see what they are orbiting around, there is nothing there! However, the fact that the stars are orbiting fast means that something massive must be lurking in the center of the Galaxy, even though it is invisible. In fact, there is a massive black hole hiding there. But how massive is the black hole?

(c) Use the ratio relationship you wrote down in (b) to calculate the mass of the black hole at the center of the Galaxy, by comparing the mass of the black hole to the total mass of the Milky Way inside the Sun’s orbit.

3. Type Ia supernovae are very bright and can be seen to large distances. They also all have approximately the same intrinsic luminosity. This makes them excellent “standard candles” for measuring large distances in the Universe. Astronomer Adam Reiss and his research team have used the known intrinsic luminosity and the observed brightness of several hundred supernovae to measure the distances to the supernovae. Fifteen of these are given in the table below, with distances measured in Megaparsecs (Mpc). It is also possible to measure the redshift of the supernovae using their spectra, similar to what you did in problem 1. Once the redshift is measured, we can calculate the speed at which the supernova is moving away from us, using the low-speed Doppler formula from problem 1d:

$$z = \frac{v}{c} \tag{5}$$

where z is the redshift, v is the speed at which the galaxy is moving away, and c is the speed of light. These “recession velocities” are also given in the table.

Name	Distance (Mpc)	Recession velocity cz (km/s)
SN01V	57.8	4800
SN92ae	309.0	22500
SN92aq	474.2	30300
SN99aa	71.1	4500
SN97cn	75.5	5100
SN92al	57.5	4200
SN95ac	176.2	14700
SN98dx	211.8	15900
SN92bg	147.9	10800
SN98eg	101.4	6900
SN92bc	84.7	5400
SN98co	72.4	5100
SN93ah	110.2	8400
SN91U	110.2	9900
SN92bs	295.1	18900

Data from Reiss et al. 2007

(a) Using the attached sheet of linear graph paper (no log-log plots this time!), plot the distance to each supernova on the x-axis and the recession velocity on the y-axis.

(b) Use a straight-edge to draw a straight line that gives a good fit to the data *and goes through the origin at $(0,0)$* . What is the slope of your line? (Hint: it will be in units of km/s/Mpc.) This is the “Hubble constant”, which describes the rate of expansion of the Universe.

(c) Clearly not all the supernovae lie exactly on the line. Give at least 2 reasons that this might happen.

4. *An example of order-of-magnitude estimates. Order of magnitude estimates are a good way to do a quick mental calculation. They give you reasonable ball-park estimates, which can enable you to make decisions on the fly. This can help you address real-world problems, like estimating how much you will owe in taxes, or deciding whether or not it is worth purchasing an insurance policy. One order of magnitude is a power of 10, so an order-of-magnitude estimate is one that is good to a factor of ten or so. Here's a problem that gives practice in making order-of-magnitude estimates, with a cosmological application:

In chapter 2, figure 2.15, the textbook claims that there are more stars in the visible Universe than grains of sand on the beaches of the Earth. Let's see if this is reasonable:

(a) First, we'll estimate the number of grains of sand, N_{gr} . That is equal to the volume of all beaches, V_b , divided by the volume of a grain of sand, V_{gr} , so $N_{gr} = V_b/V_{gr}$.

Let's start by estimating V_{gr} . About how long on a side is a grain of sand in mm? Assuming a sand grain is a cube, what is V_{gr} ?

Now let's estimate V_b . Estimate the total length of coastline, L_b on the Earth. (Hint: the circumference of the Earth is about 40,000 km.) Now estimate the typical width, W_b , and thickness, T_b , of a typical beach. Explain why your values for L_b , W_b , and T_b are reasonable. Find V_b .

Roughly how many grains of sand are there on all the worlds' beaches?

(b) Second, estimate the number of stars. Assume that the visible Universe contains 100 billion galaxies, each one containing 100 billion stars. Was the textbook right in its claim?

(c) Which of the steps of this problem do you think is the most uncertain?