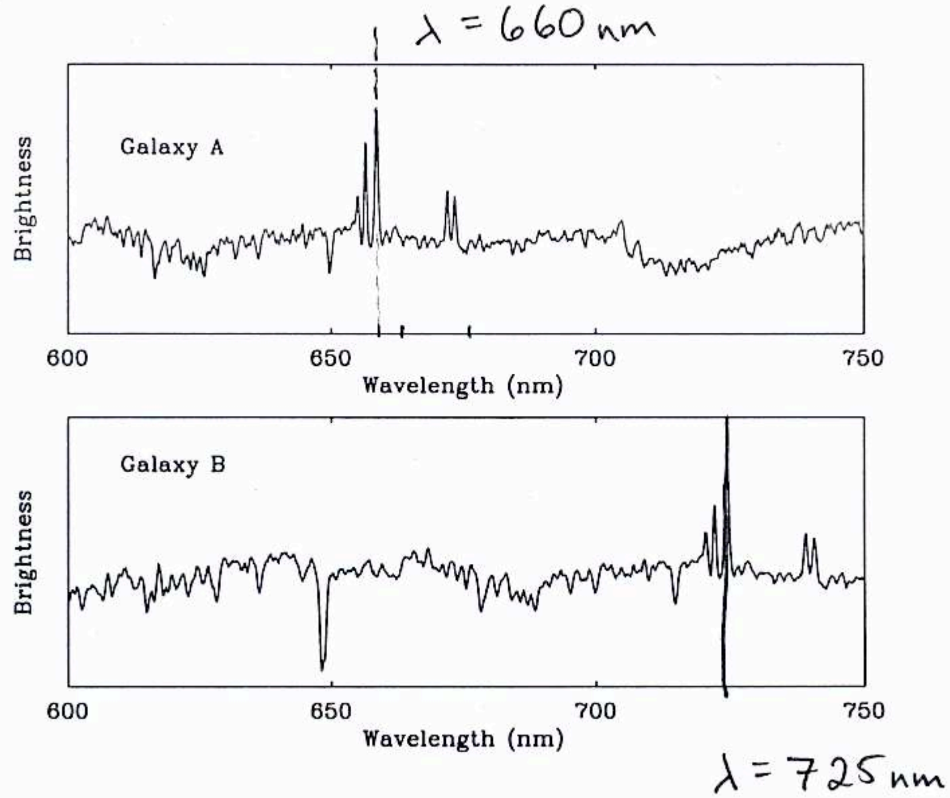
# Astronomy 5, Spring 2007
# Problem Set #3

Due at the beginning of class, May 11.

**Print your name:** Please feel free to ask for hints and/or clarification. Work the problems on this handout. If you need more space, use the back of the pages, or attach extra sheets at the end. Please put circles around final numerical answers. Homework MUST be stapled. Questions marked with a star are especially challenging and may involve kinds of mathematical thinking that are unfamiliar. **Starred problems** will be worked as a group IN SECTION. You can work the problems there and hand in your answers.

1. Measuring the redshifts of galaxies. Look at the figure below. The spectra of two galaxies are shown. Light from each galaxy has been passed through an instrument called a spectrograph. The spectrograph is essentially a prism that spreads the light out into a rainbow, with the bluer light shown at the left end of the figure and the redder light at the right end.



There are lots of little bumps and wiggles in the spectra. These are called "absorption lines" and they tell astronomers about the atoms and molecules in the outer atmospheres of the *stars* in the galaxy. Notice that there are also upward spikes. These are called "emission lines" and are caused by *glowing gas* in the galaxies. All of these features, both absorption lines and emission lines, are emitted at *rest wavelengths* which do not change—they are constant across the Universe. In these galaxy spectra, there are two groups of emission lines: a group of 3 lines (which are from left to right weak, stronger, and strongest), and then a pair of lines to the right of them which are about the same strength. The first group of lines is due to hydrogen and nitrogen in the gas of the galaxy, while the second pair of lines is due to sulfur.

We just said that the wavelengths of these lines are the same everywhere, but looking at these galaxy spectra, the lines are obviously in *different* places. This is an example of *redshift*. Galaxy A is nearby, but Galaxy B is very far away from us. Because the Universe is expanding, Galaxy B is moving away from us quickly. For the sake of this problem, let's pretend that Galaxy A is very nearby, and is hardly moving away from us at all.

(a) Choose an emission line that you can clearly identify in both galaxy spectra. Mark the line on the spectrum of Galaxy A. Using any method you like, measure the wavelength of your chosen emission line in Galaxy A.

$$\lambda_A = 660 \text{ nm}$$

(b) Now find the same line in Galaxy B and mark it. Using the same method from part (a), measure the wavelength of the emission line in Galaxy B.

$$\lambda_B = 725 \text{ nm}$$

(c) Because Galaxy A is nearby and only moving away from us very slowly, assume that the wavelength you measured in (a) is the *rest wavelength* of the emission line, which we will write as $\lambda_{rest}$. The wavelength you measured in (b) for Galaxy B is the "observed" wavelength of the emission line, which we will write as $\lambda_{obs}$. Calculate the redshift, $z$, of Galaxy B using your measurements from (a) and (b) and the redshift formula from Lecture 8:

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} \tag{1}$$

$$z = \frac{725\,nm - 660\,nm}{660\,nm} = \frac{65\,nm}{660\,nm} = 0.098$$

$$\boxed{z = 0.098}$$

(d) The redshift formula is a version of the low-speed Doppler formula (see discussion in Lecture 8, section 3),

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{v}{c} \tag{2}$$

where $v$ is the speed with which the redshifted galaxy is moving away from us, and $c$ is the speed of light, 300,000 km/s. Calculate the speed of Galaxy B as it moves away from us.

$$z = \frac{v}{c}$$

$$v = z \times c$$

$$v = 0.098 \times 300{,}000\ km/s$$

$$\boxed{v = 29{,}400\ km/s}$$

2. Weighing the black hole at the center of the Milky Way. Objects in orbit under the influence of gravity have to keep moving at an exact speed in order to be in a circular orbit. The speed of the orbit (we'll use the symbol $v$ for velocity) is determined by the enclosed mass, $M(R)$, that the object is orbiting around and the distance (we'll use $R$ for radius) of the object from the center of the orbit. Thus, if we can observe the speed of an orbit, $v$, and the radius of the orbit, $R$, we can calculate the enclosed mass, $M(R)$, that the object is orbiting around. The relevant equation is:

$$M(R) = \frac{v^2 R}{G} \tag{3}$$

where $G$ is a constant, called the gravitational constant, and $M(R)$, $R$, and $v$ are the enclosed mass, the orbital radius, and the velocity of the orbiting object, as described above.

(a) When the gravitating central body is a star, argue that the enclosed mass of a planetary orbit is just the star's mass. For example, argue that the enclosed mass for Earth's orbit around the Sun is about equal to the mass of the Sun. (Hint: in all solar systems that we know about, the mass of the planets is negligible compared to the mass of the star.)

The enclosed mass of a planetary orbit is everything that the planet orbits around. This includes the central star, but also any other interior planets (like Mercury and Venus) or asteroids or other space junk. In practice, the star is so much more massive than rocky planets or asteroids that the "enclosed mass" is essentially that of the star.

(b) Suppose you had measured the speeds and radii of two different orbiting objects, planet A which orbits around star A, and planet B which orbits around star B. This means that you know the speeds, $v_A$ and $v_B$, and the orbital radii, $R_A$ and $R_B$, for each orbiting planet. You would like to compare the masses of the two stars, $M_A$ and $M_B$. Use the equation above to write down a ratio relationship of the form:

$$\frac{M_A}{M_B} = ? \tag{4}$$

using the information you have measured: $v_A$, $v_B$, $R_A$, and $R_B$.

$$\frac{M_A}{M_B} = \frac{v_A^2 R_A / \cancel{G}}{v_B^2 R_B / \cancel{G}}$$

$$\boxed{\frac{M_A}{M_B} = \frac{v_A^2 R_A}{v_B^2 R_B}}$$

The ratio you wrote in part (b) can be used to compare the central masses of any orbits. For example, the Sun is orbiting the center of the Milky Way. It orbits at 26,000 ly from the center of the galaxy at a speed of 220 km/s. Astronomers have used this information to measure the total mass of the Milky Way inside the Sun's orbit: it's about 90 billion Solar masses. We can write this as $9 \times 10^{10} M_\odot$, where $M_\odot$ is the mass of the Sun.

When astronomers look into the very center of the Milky Way galaxy, the stars closest to the center of the galaxy are moving very fast. Stars that are only 15 ly from the galaxy center are orbiting at speeds near 1,000 km/s—almost 5 times as fast as the Sun's orbit. And yet, when we look at the very center of the Galaxy to see what they are orbiting around, there is nothing there! However, the fact that the stars are orbiting fast means that something massive must be lurking in the center of the Galaxy, even though it is invisible. In fact, there is a massive black hole hiding there. But how massive is the black hole?

(c) Use the ratio relationship you wrote down in (b) to calculate the mass of the black hole at the center of the Galaxy, by comparing the mass of the black hole to the total mass of the Milky Way inside the Sun's orbit.

System A : Black Hole + orbiting stars

System B : the Sun orbiting the galaxy

$$\frac{M_A}{M_B} = \frac{v_A^2 R_A}{v_B^2 R_B} = \frac{(1000\ \text{km/s})^2 (15\ \text{ly})}{(220\ \text{km/s})^2 (26{,}000\ \text{ly})} = \frac{1000^2 \times 15}{220^2 \times 26{,}000} = \frac{15\times 10^6}{1.3\times 10^9}$$

$$\frac{M_A}{M_B} = \frac{15\times 10^6}{1.3\times 10^9} = 0.012 \qquad M_{\text{Black Hole}} = 0.012 \times M_{\text{Galaxy}}$$

$$M_{BH} = 0.012 \times 9\times 10^{10}\ M_\odot$$

$$\boxed{M_{BH} = 1\times 10^9\ M_\odot = 1 \text{ billion } M_\odot}$$

3. Type Ia supernovae are very bright and can be seen to large distances. They also all have approximately the same intrinsic luminosity. This makes them excellent "standard candles" for measuring large distances in the Universe. Astronomer Adam Reiss and his research team have used the known intrinsic luminosity and the observed brightness of several hundred supernovae to measure the distances to the supernovae. Fifteen of these are given in the table below, with distances measured in Megaparsecs (Mpc). It is also possible to measure the redshift of the supernovae using their spectra, similar to what you did in problem 1. Once the redshift is measured, we can calculate the speed at which the supernova is moving away from us, using the low-speed Doppler formula from problem 1d:

$$z = \frac{v}{c} \tag{5}$$

where $z$ is the redshift, $v$ is the speed at which the galaxy is moving away, and $c$ is the speed of light. These "recession velocities" are also given in the table.

| Name | Distance (Mpc) | Recession velocity $cz$ (km/s) |
|---|---|---|
| SN01V | 57.8 | 4800 |
| SN92ae | 309.0 | 22500 |
| SN92aq | 474.2 | 30300 |
| SN99aa | 71.1 | 4500 |
| SN97cn | 75.5 | 5100 |
| SN92al | 57.5 | 4200 |
| SN95ac | 176.2 | 14700 |
| SN98dx | 211.8 | 15900 |
| SN92bg | 147.9 | 10800 |
| SN98eg | 101.4 | 6900 |
| SN92bc | 84.7 | 5400 |
| SN98co | 72.4 | 5100 |
| SN93ah | 110.2 | 8400 |
| SN91U | 110.2 | 9900 |
| SN92bs | 295.1 | 18900 |

Data from Reiss et al. 2007

(a) Using the attached sheet of linear graph paper (no log-log plots this time!), plot the distance to each supernova on the x-axis and the recession velocity on the y-axis.

See plot

(b) Use a straight-edge to draw a straight line that gives a good fit to the data *and goes through the origin at (0,0)*. What is the slope of your line? (Hint: it will be in units of km/s/Mpc.) This is the "Hubble constant", which describes the rate of expansion of the Universe.

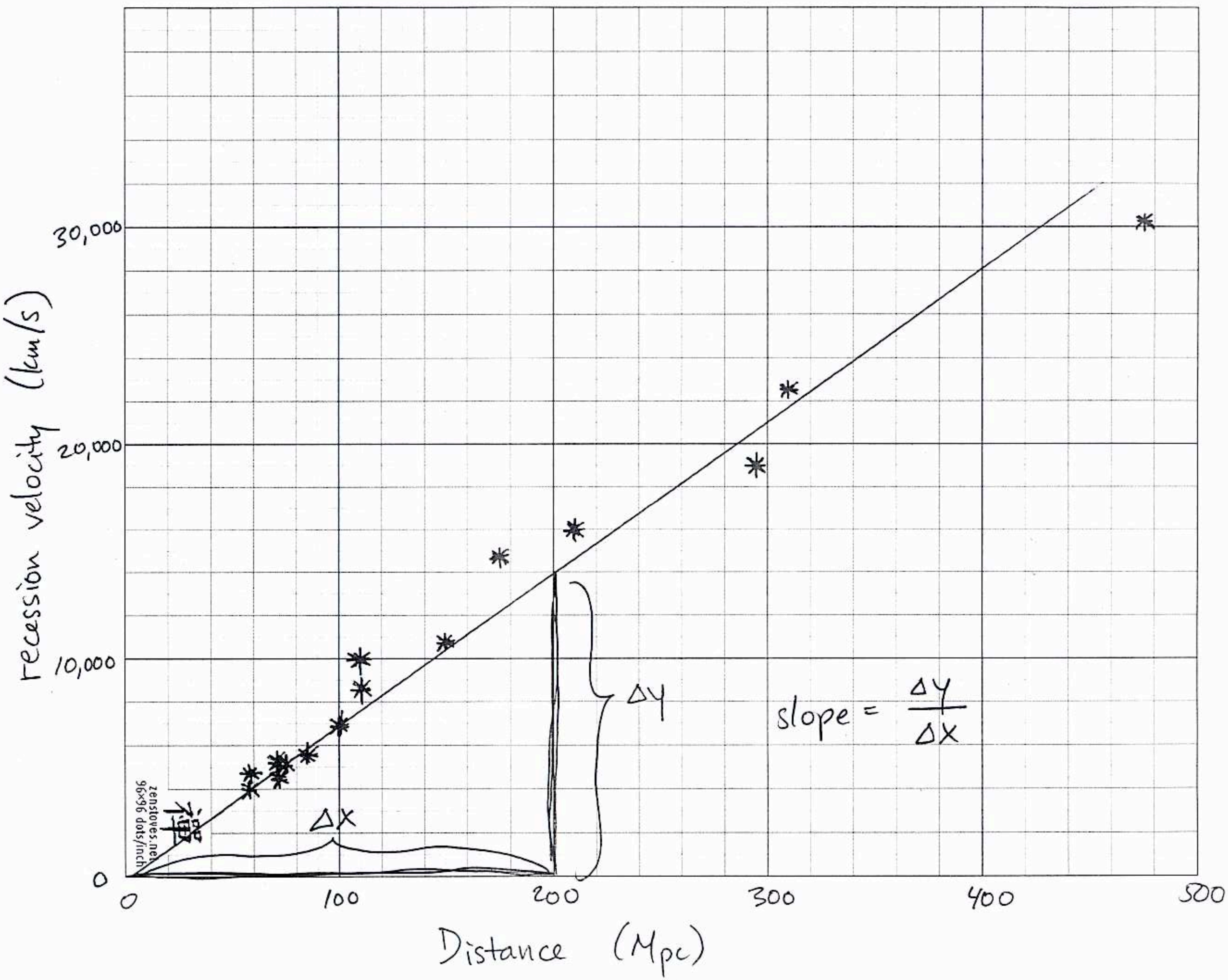
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - 0}{x_2 - 0} = \frac{y_2}{x_2}$$

$$\text{slope} = H_0 = \frac{14{,}000 \text{ km/s}}{200 \text{ Mpc}} \Rightarrow \boxed{H_0 = 70 \text{ km/s/Mpc}}$$

(c) Clearly not all the supernovae lie exactly on the line. Give at least 2 reasons that this might happen.

1. Errors in the measurement of the SN brightness would lead to errors in the distance measurement, which will scatter SN off the line.

2. Some galaxies, in addition to the fact that they are moving away with the expanding Universe, are orbiting other galaxies while they do. This makes their velocity along the line of sight slightly different from the prediction of the Hubble law.



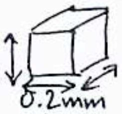
30,000
20,000
10,000
0
recession velocity (km/s)
0
100
200
300
400
500
Distance (Mpc)
ΔY
ΔX
slope = ΔY / ΔX
zenstoves.net
96×96 dots/inch

4. *An example of order-of-magnitude estimates. Order of magnitude estimates are a good way to do a quick mental calculation. They give you reasonable ball-park estimates, which can enable you to make decisions on the fly. This can help you address real-world problems, like estimating how much you will owe in taxes, or deciding whether or not it is worth purchasing an insurance policy. One order of magnitude is a power of 10, so an order-of-magnitude estimate is one that is good to a factor of ten or so. Here's a problem that gives practice in making order-of-magnitude estimates, with a cosmological application:

In chapter 2, figure 2.15, the textbook claims that there are more stars in the visible Universe than grains of sand on the beaches of the Earth. Let's see if this is reasonable:

(a) First, we'll estimate the number of grains of sand, $N_{gr}$. That is equal to the volume of all beaches, $V_b$, divided by the volume of a grain of sand, $V_{gr}$, so $N_{gr} = V_b/V_{gr}$.

Let's start by estimating $V_{gr}$. About how long on a side is a grain of sand in mm? Assuming a sand grain is a cube, what is $V_{gr}$? $\longrightarrow$ convert all units to meters




$V_{gr} = (0.2 \text{ mm})^3$

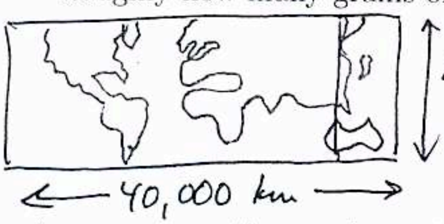
$0.2 \text{ mm} = 0.2\times10^{-3} \text{ m} = 2\times10^{-4} \text{ m}$

$V_{gr} = (2\times10^{-4} \text{ m})^3 = 8\times10^{-12} \text{ m}^3$

$\boxed{V_{gr} = 8\times10^{-12} \text{ m}^3}$

Now let's estimate $V_b$. Estimate the total length of coastline, $L_b$ on the Earth. (Hint: the circumference of the Earth is about 40,000 km.) Now estimate the typical width, $W_b$, and thicknes, $T_b$, of a typical beach. Explain why your values for $L_b$, $W_b$, and $T_b$ are reasonable. Find $V_b$.

Roughly how many grains of sand are there on all the worlds' beaches?




$L_b = 100{,}000 \text{ km} = 10^5 \text{ km} = 10^8 \text{ m}$

$W_b \sim 100 \text{ m}$ $\quad$ $T_b \sim 3 \text{ m}$

Americas: ~40,000 km
Europe + Africa: ~20,000 km
India, Asia + Australia = 40,000 km

$V_b = L_b \cdot W_b \cdot T_b = 10^8 \text{ m} \times 100 \text{ m} \times 3 \text{ m}$

$\boxed{V_b = 3\times10^{10} \text{ m}^3}$

(b) Second, estimate the number of stars. Assume that the visible Universe contains 100 billion galaxies, each one containing 100 billion stars. Was the textbook right in its claim?

$N_{gr} = \frac{V_b}{V_{gr}} = \frac{3\times10^{10} \text{ m}^3}{8\times10^{-12} \text{ m}^3}$

$\boxed{N_{gr} \approx 4\times10^{21} \text{ grains}}$

$100 \text{ billion} = 100\times10^9 = 10^{11}$

$N_{stars} = 10^{11} \frac{\text{stars}}{\text{galaxy}} \times 10^{11} \text{ galaxies}$

(c) Which of the steps of this problem do you think is the most uncertain?

More stars than grains of sand! $\boxed{N_{stars} = 10^{22}}$

I think the estimate of $L_b$ is most uncertain because the coastline wiggles in and out a lot, and only some coast has sandy beaches. But $W_b$ is also hard, b/c you could stop at the water's edge, or go farther out underwater...