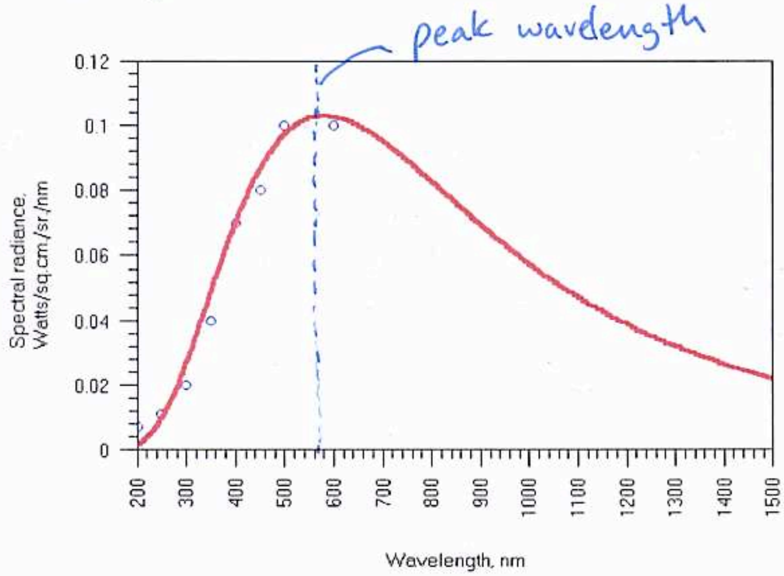
Astronomy 5, Spring 2007
Problem Set #4

Due at the beginning of class, May 21.

Print your name:

Please feel free to ask for hints and/or clarification. Work the problems on this handout. If you need more space, use the back of the pages, or attach extra sheets at the end. Please put circles around final numerical answers. Homework MUST be stapled. Questions marked with a star are especially challenging and may involve kinds of mathematical thinking that are unfamiliar. **Starred problems** will be worked as a group IN SECTION. You can work the problems there and hand in your answers.

1. Thermal Radiation: Below is a spectrum of a thermal body. This type of radiation is called "thermal radiation", or "blackbody radiation". The brightness at each wavelength is shown in the plot.



$\lambda_{peak} = 570$ nm

(a) In your own words, what is thermal or blackbody radiation?

Blackbody radiation is radiation produces by an object with non-zero temperature. The radiation is produced by the motion of charged particles (electrons and protons) in the object which are moving because of the heat (kinetic energy) of the object.

(b) Give three examples of objects that produce thermal radiation:

the Sun
a lightbulb filament
a human

(c) For the thermal radiation spectrum shown in the plot above, what is the peak wavelength, λ_{peak}?

$$\lambda_{peak} = 570\ nm$$

(d) Calculate the temperature of the object producing the radiation.

$$\lambda_{peak} = \frac{0.29\ cm\text{-}K}{T}$$

$$T = \frac{0.29\ cm\, K}{\lambda_{peak}}$$

$$570\ nm = 570\times10^{-9}\ m = 570\times10^{-7}\ cm$$

$$T = \frac{0.29\ \cancel{cm}\ K}{570\times10^{-7}\ \cancel{cm}}$$

$$\boxed{T = 5100\ K}$$

2. Falling into a black hole: Imagine that you are out in space, very near to a black hole which is 3 times more massive than the sun. You have brought two clocks with you: Clock 1 and Clock 2. Clock 1 is sitting in a spaceship which is hovering in place outside of the Schwarzschild radius of the black hole. Clock 2 has been tossed out of the spaceship and is falling into the black hole.

(a) If you are sitting in the spaceship with Clock 1, watching Clock 2 fall into the black hole, does the time measured by Clock 2 seem to be moving faster, slower, or the same as the time measured by Clock 1 on the spaceship?

The ~~[illegible]~~ time measured by clock 2 seems to be moving slower than clock 1 because it takes light time to climb out of the gravitational field of the black hole.

(b) If you are falling into the black hole with Clock 2, looking back toward the spaceship, does the time measured by Clock 1 on the spaceship seem to be moving faster, slower, or the same as the time measured by Clock 2?

The time measured by clock 1 appears to be moving faster than clock 2 — this is the opposite scenario of what happens in part (a).

3. Precision Cosmology and distances: You are an astronomer and you have just come back from the telescope with a spectrum of a galaxy. When you look at the emission and absorption lines in the galaxy spectrum (as you did in Problem Set #3), you are able to measure the redshift of the galaxy, and discover that the galaxy lies at very high redshift, $z = 3$!

(a) Use the low-speed Doppler formula to calculate the speed at which the galaxy is moving away from you. Then use the Hubble Law to calculate the distance to the galaxy. (The low-speed Doppler formula is written as $z = v/c$, where z is the redshift of the object, v is its speed along the line of sight, and c is the speed of light. Hubble's Law is $v = H_0 \times r$, where v is the speed at which the galaxy is moving away from us, H_0 is the Hubble constant, and r is the distance to the galaxy. Use a Hubble constant $H_0 = 70$ km / sec / Mpc.)

$z = 3, \quad c = 300{,}000$ km/s

$$z = \frac{v}{c}$$

$$3 = \frac{v}{300{,}000 \text{ km/s}}$$

$$v = 900{,}000 \text{ km/s}$$

(faster than the speed of light!)

$$v = H_0 r$$

$$r = \frac{v}{H_0}$$

$$r = \frac{900{,}000 \cancel{\text{km/s}}}{70 \cancel{\text{km/s}}/\text{Mpc}}$$

$$\boxed{r = 12{,}860 \text{ Mpc}}$$

(b) Now we'll use Ned Wright's cosmology calculator to calculate the distance to the galaxy. This Javascript tool calculates distances and light travel times to cosmological sources, accounting for the effects of cosmology. Most astronomers I know use this tool all the time to calculate distances to objects based on their redshifts!

The cosmology calculator can be found as a link on the course website in the section for "Online Tools". When you go there, notice that on the left-hand side of the tool, you can fill in the cosmological parameters for the Universe: H_0, the redshift of the galaxy, Omega$_M$ (Omega matter or Ω_M in the lecture notes), and Omega$_{vac}$ (Omega of the vaccuum or Ω_Λ in the lecture notes, ie. the cosmological constant). We saw in Lecture 14 that the results from the Supernova Cosmology Project indicate that Omega$_M$ is about 0.3 and Omega$_{vac}$ is about 0.7. Put in these values, then hit the button that says "General" to perform the calculation. There are lots of output numbers on the right. The one you want is the "luminosity distance D_L", which is the distance to the galaxy you would get by measuring is brightness.

$$D_L = ? \tag{1}$$

$D_L = 25{,}000$ Mpc

(c) This distance in part (b) is different from the one you measured in (a). Why are they different? (Hint: What assumptions do you have to make to use the formulas you used in part (a)? Why are each of those equations "wrong" for this situation?) If you are confused, there is a discussion of this topic in the textbook in box 26-2, on page 590.

Both equations are <u>wrong</u> for this situation. To use the low-speed Doppler formula ($z = \frac{v}{c}$), the galaxy has to be moving no more than 10% of the speed of light (0.1c). It is obviously moving faster than that, which is why the low-speed Doppler formula gives us a ~~s~~velocity that is <u>more</u> than the speed of light. The correct special-relativistic version of the Doppler formula has to be used for objects at high redshift. To use the Hubble law, we have assumed that the Hubble constant, H_0, is always the same. However, H_0 is actually changing with time and cannot be used for high redshift objects.

4. *The scale-factor of the Universe, a: Let's figure out how quickly the Universe is growing. If the Universe expands at a constant rate, it grows the same amount every year so its size is proportional to the amount of time it has been growing. This means that if the Universe get 10% older, it gets 10% bigger as well. If you were comparing the scale-factor of the Universe at two different ages, you could write:

$a_{now} = 1$

$$\frac{a_{now} - a_{then}}{a_{now}} = \frac{age_{now} - age_{then}}{age_{now}} \tag{2}$$

(a) What was the scale factor of the Universe 1 billion years ago?

$$\frac{1 - a_{then}}{1} = \frac{14 \times 10^9 - 1 \times 10^9}{14 \times 10^9} = \frac{1 \times 10^9}{14 \times 10^9} = \frac{1}{14}$$

$$1 - a_{then} = 0.07 \quad \text{so} \quad \boxed{a_{then} = 0.93}$$

(b) What will be the scale factor of the Universe 1 million years in the future?

$1 \text{ million} = 10^6 \longrightarrow age_{now} - age_{then} = -1 \text{ million years}$

$$\frac{1 - a_{then}}{1} = \frac{-1 \text{ million years}}{14 \text{ billion years}} = \frac{-1 \times 10^6}{14 \times 10^9} = -\frac{1}{14{,}000}$$

$$1 - a_{then} = -7.1 \times 10^{-5} \quad \text{so} \quad \boxed{a_{then} = 1.000071}$$

(c) We first discussed the scale factor of the Univers in lecture about 2 weeks ago. How much has the scale factor of the Universe changed since we first talked about it?

$age_{now} - age_{then} = 2 \text{ weeks}$

$1 \text{ week} = \frac{1}{52} \text{ years} \longrightarrow 2 \text{ weeks} = \frac{2}{52} = 0.038 \text{ years}$

$$\frac{1 - a_{then}}{1} = \frac{0.038 \text{ years}}{14 \times 10^9 \text{ years}} = 2.7 \times 10^{-12}$$

$$1 - a_{then} = 2.7 \times 10^{-12} \quad \text{so} \quad \boxed{a_{then} = 1 - 2.7 \times 10^{-12}}$$