

# Astronomy 5, Spring 2007

## Problem Set #5

Due at the beginning of class, June 6.

**Print your name:**

Please feel free to ask for hints and/or clarification. Work the problems on this handout.

1. The Hubble Constant in the Future: Assume that there is no matter in the Universe. In this case, we are living in a very open universe <sup>1</sup>. Therefore, galaxies always recede from us with the **same speed** (i.e. there is no gravitational deceleration and no slowing down). Four galaxies today at distances of 200, 400, 600, and 800 Mpc away from the Milky Way, all follow the Hubble Law perfectly. Thus if we plotted their speed on the y-axis in units of km/sec and their location on the x-axis in units of Mpc, they would all lie on a straight line whose slope would be  $H_0 = 71$  km/sec per Mpc.

(a) (6pts) You can think of the recession velocity as the speed at which each galaxy had to move in order to travel to its present distance from the Milky Way over the lifetime of the Universe. Over the course of 14 billion years, the closest galaxy in the plot has moved a total of 200 Mpc. Now imagine that you can come back and observe the same four galaxies another 14 billion years in the future. How far away would they be, if they continue to move with the same speed? Using the linear-linear graph paper attached to this problem set, plot the data given above. Now, also plot the position of each galaxy as it will be 14 billion year from now. (Hint: leave room on the x-axis for the larger distance of galaxies at the later time.)

(b) (5pts) Next draw a line through the four data points that also goes through the origin at (0,0). The slope of this line is the Hubble constant measured 14 billion years in the future. What is the Hubble constant 14 billion years from now? How does it compare to the Hubble constant today?

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<sup>1</sup>Obviously, saying there are galaxies but no matter is a contradiction. However for the purpose of this problem, lets consider galaxies to be massless “test particles” that follow the expansion but do not themselves generate gravity.

(c) (3pts) What does this tell you about the Hubble constant? Will it really stay *constant* over the entire age of the Universe?

2. The Critical Mass Density of the Universe

(a) (3pts) You are trying to explain to a friend (who hasn't taken this course) what critical density means in cosmology. Give an intuitive, physical explanation. Assume in this question that the Universe has only matter generation normal, decelerating gravity.

(b) (7pts) What is  $\Omega_M$  and what does it have to do with critical density? Fill in the following table by briefly describing the geometry and ultimate fate of a universe with the given value of  $\Omega_M$ .

$\Omega_M$	Geometry	Fate
$\Omega_M < 1$		
$\Omega_M = 1$		
$\Omega_M > 1$		

3. (8pts) Galaxy Formation: In the table below, we have outlined the major steps in the formation and evolution of a galaxy. In each one, a physical process plays an important role. Describe what that role is. Note: in some cases, there may be more than one role. Think of all you can. For example, star formation creates heavy elements.

Era	Physical Process	Role in Formation of the Galaxy
Inflation	Uncertainty principle	
Post-inflation after inflation	Gravitational growth	
Collapse phase	Cooling of ordinary matter	
Post-collapse phase	Star formation	
Post-post-collapse phase	Galaxy collisions	

4. (10pts) **Extra Credit Question: What was the size of the Universe, compared to today, when electron-positron pairs ceased to exist?**

This problem is complicated, and will take a number of steps to solve. Recall the basic process: two photons combine to make two equal-mass particles, the electron and the positron. Thus the energy of one photon goes into the electron and the other photon into the positron. First, determine how much energy it takes to produce an electron and this is the same needed for the positron. (Hint:  $E = mc^2$ , and you can look up the mass of an electron in the back of your textbook.) Next, what is the wavelength of a photon which has enough energy to do this? It may be helpful to know that a photon with a wavelength of 1 meter has an energy of  $2 \times 10^{-25}$  joules. Subsequently, what temperature did the Universe have when it was filled with radiation at this wavelength? (Hint: photons in the universe today at 3 K have a wavelength near 1 mm.) And finally, how much has the Universe has expanded between the time when it was at that temperature and today, when the Cosmic Microwave Background is about 3 K.

This is the most complicated problem we have given you so far. Remember that we will give you partial credit for partial solutions, so it's worth going as far as you can. If you get lost, you can come to office hours or section for extra help. Good luck!