Solutions

Astronomy 5, Spring 2007
Problem Set #5

Due at the beginning of class, June 6.

Print your name:

Please feel free to ask for hints and/or clarification. Work the problems on this handout.

1. The Hubble Constant in the Future: Assume that there is no matter in the Universe. In this case, we are living in a very open universe [1]. Therefore, galaxies always recede from us with the **same speed** (i.e. there is no gravitational deceleration and no slowing down). Four galaxies today at distances of 200, 400, 600, and 800 Mpc away from the Milky Way, all follow the Hubble Law perfectly. Thus if we plotted their speed on the y-axis in units of km/sec and their location on the x-axis in units of Mpc, they would all lie on a straight line whose slope would be $H_0 =$ ~~74~~ 70 km/sec per Mpc.

$v = H_0 r$ $r_1 = 200$ Mpc $\longrightarrow v_1 = 14{,}000$ km/s $r_3 = 600$ Mpc $\rightarrow v_3 = 42{,}000$ km/s

$r_2 = 400$ Mpc $\longrightarrow v_2 = 28{,}000$ km/s $r_4 = 800$ Mpc $\rightarrow v_4 = 56{,}000$ km/s

(a) (6pts) You can think of the recession velocity as the speed at which each galaxy had to move in order to travel to its present distance from the Milky Way over the lifetime of the Universe. Over the course of 14 billion years, the closest galaxy in the plot has moved a total of 200 Mpc. Now imagine that you can come back and observe the same four galaxies another 14 billion years in the future. How far away would they be, if they continue to move with the same speed? Using the linear-linear graph paper attached to this problem set, plot the data given above. Now, also plot the position of each galaxy as it will be 14 billion year from now. (Hint: leave room on the x-axis for the larger distance of galaxies at the later time.)

If they continue to move with the same speed, their speeds v_1, v_2, v_3, v_4 will be the same but they will have gone twice as far (in twice as much time). → see plot

(b) (5pts) Next draw a line through the four data points that also goes through the origin at (0,0). The slope of this line is the Hubble constant measured 14 billion years in the future. What is the Hubble constant 14 billion years from now? How does it compare to the Hubble constant today?

[1] Obviously, saying there are galaxies but no matter is a contradiction. However for the purpose of this problem, lets consider galaxies to be massless "test particles" that follow the expansion but do not themselves generate gravity.

after 14 billion more years:

$r_1 = 400$ Mpc $\rightarrow v_1 = 14{,}000$ km/s
$r_2 = 800$ Mpc $\rightarrow v_2 = 28{,}000$ km/s
$r_3 = 1200$ Mpc $\rightarrow v_3 = 42{,}000$ km/s
$r_4 = 1600$ Mpc $\rightarrow v_4 = 56{,}000$ km/s

same speeds as before

they've all gone as far again in the next 14 billion years as they went in the first 14 billion years.

(c) (3pts) What does this tell you about the Hubble constant? Will it really stay *constant* over the entire age of the Universe?

New Hubble constant is $35\ \frac{\text{km/s}}{\text{Mpc}}$. This is ½ the current Hubble constant.

The Hubble constant can't stay the same over the entire age of the Universe — it is not constant over time.

2. The Critical Mass Density of the Universe

(a) (3pts) You are trying to explain to a friend (who hasn't taken this course) what critical density means in cosmology. Give an intuitive, physical explanation. Assume in this question that the Universe has only matter generation normal, decelerating gravity.

In a Universe with only matter (no dark energy!), that matter will attract the other matter and tend to pull things back toward one another. This acts against the expansion of the Universe and tends to slow down the expansion. If you only have a little bit of matter, it won't be enough to slow down the Universe very much and it will expand forever. If you have a lot of matter, eventually the expansion will slow to a halt and the Universe will collapse back in on itself. The critical mass density of the Universe is the amount of mass needed to just exactly balance the outward →

expansion so that the Universe stops expanding as it reaches infinity. Any less mass than the critical density, and the Universe expands forever. Any more mass than the critical density, and the Universe will eventually collapse back in on itself.

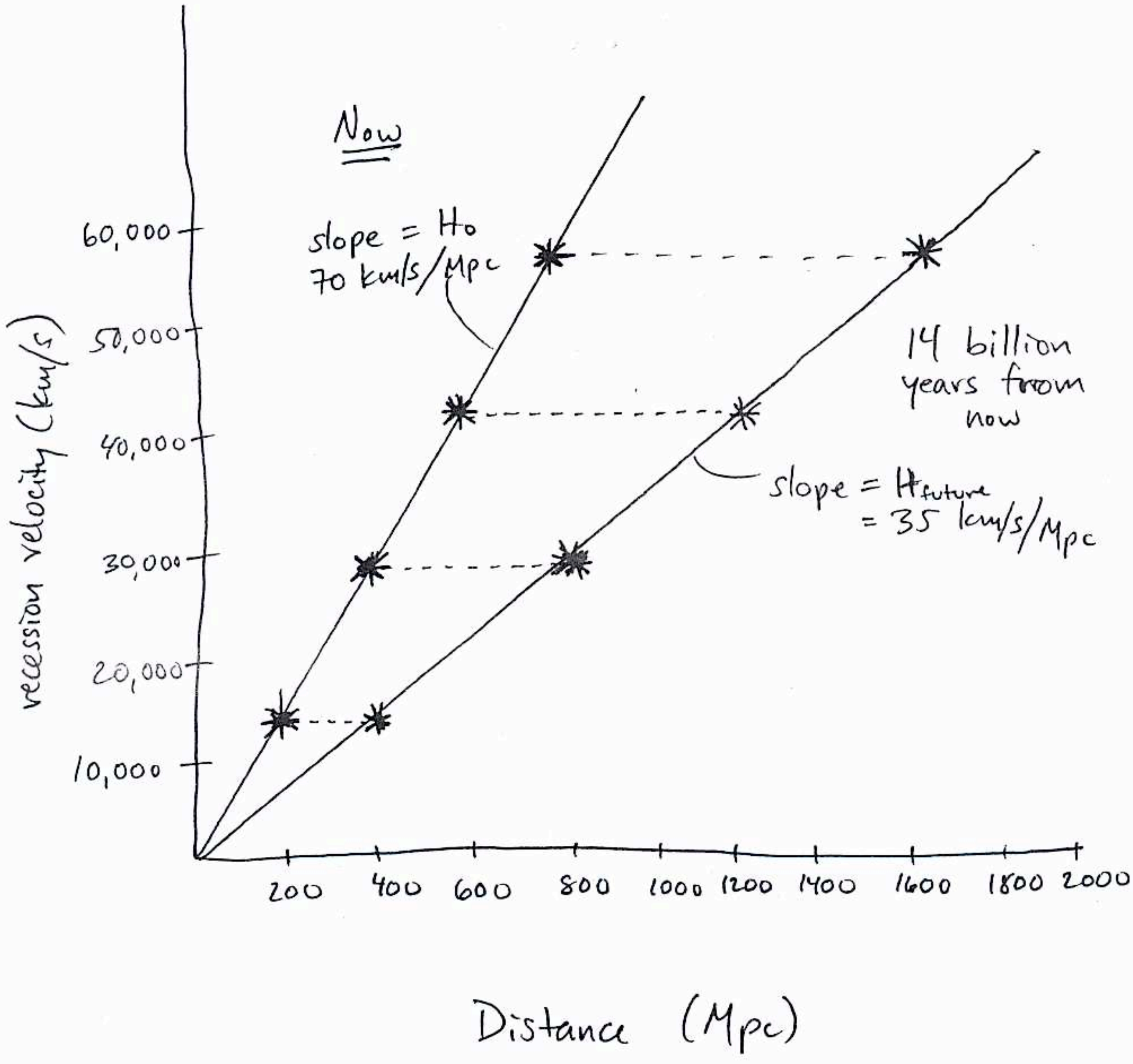
(b) (7pts) What is Ω_M and what does it have to do with critical density? Fill in the following table by briefly describing the geometry and ultimate fate of a universe with the given value of Ω_M.

Ω_M	Geometry	Fate
$\Omega_M < 1$	Open	Universe expands forever
$\Omega_M = 1$	Flat	Expansion of the Universe slows to zero as it reaches infinity, never recollapses.
$\Omega_M > 1$	Closed	Universe collapses back in on itself

3. (8pts) Galaxy Formation: In the table below, we have outlined the major steps in the formation and evolution of a galaxy. In each one, a physical process plays an important role. Describe what that role is. Note: in some cases, there may be more than one role. Think of all you can. For example, star formation creates heavy elements.

see table ⟶

Era	Physical Process	Role in Formation of the Galaxy
Inflation	Uncertainty principle	Produces particle-antiparticle pairs (quantum fluctuations) in the vacuum which are separated as the Universe inflates. These produce initial overdense and underdense regions of the Universe.
Post-inflation after inflation	Gravitational growth	Overdense regions tend to collapse under their own gravity and pull in matter from the underdense regions. The initial quantum fluctuations grow into large structures this way.
Collapse phase	Cooling of ordinary matter	Ordinary matter can emit photons and lose energy by colliding against other matter particles, heating up, and radiating. Losing energy this way allows it to sink to the center of the dense clump to make *very* dense gas clouds. ← Note: dark matter only interacts through gravity, so it can't lose energy this way and does not get nearly as dense.
Post-collapse phase	Star formation	Dense gas clouds collapse in on themselves and form stars. These stars are the parts of the galaxies we can see!
Post-post-collapse phase	Galaxy collisions	Galaxies in dense regions (clusters) collide with one another. The frequency of collisions determines the final shape (elliptical or spiral) and the fate of the galaxy.


Now
slope = Ho
70 km/s/Mpc
14 billion
years from
now
slope = Hfuture
= 35 km/s/Mpc
recession velocity (km/s)
60,000
50,000
40,000
30,000
20,000
10,000
200
400
600
800
1000
1200
1400
1600
1800
2000
Distance (Mpc)