PROBLEM SET 1 – Astronomy 113

1. Draw a spacetime diagram of the earth, the moon, and of an Apollo spacecraft which travels to the moon, lands, waits a while, and then returns. Also show the worldline of a radio signal from earth telling the spacecraft to lift off the moon as well as the spacecraft's acknowlegement signal back to earth.

2. Can you think of another homogeneous universe that is not isotropic (besides the surface of an infinite cylinder)?

3. Redo Olber's paradox on an infinite two dimensional plane.

4. If we observe an object with $(\lambda_{obs} - \lambda_{em})/\lambda_{em} = 2$, what radial velocity must the object have?

5. Find a formula which relates the angular size of a galaxy whose diameter is 50 kpc to the wavelength we would observe for Hα emitted by the galaxy. The rest wavelength of $H\alpha$ is 6562 Å.

6. Draw spacetime diagrams in proper distance and in co-ordinate distance for two galaxies and the light received by one from the other.

7. How are proper density and co-ordinate density (mass per unit co-ordinate volume) related? How do they depend on time or on scale factor?

8. Find $R(t)$ for $\Omega_o < 1$. Hint: Instead of using $R \propto \sin^2\theta$, try using $R \propto \sinh^2\theta$, where $\sinh x = \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})$.

9. In the Einstein–deSitter universe, what is the density as a function of Hubble constant and time?