GENERAL RELATIVITY
HOMEWORK SET NUMBER 1

1. Two Lorentz tensors have the property in some frame that all of the ratios of each component of tensor #1 to the corresponding component of tensor #2 yield a single constant. For two such second rank tensors, is the ratio of the any arbitrary component of one to the corresponding component of the other a Lorentz invariant? Prove your conclusion.

2. In the rest frame of a conductor, Ohm's law takes the form $\mathbf{J} = \sigma\mathbf{E}$, where $\mathbf{J}$ is the current density and $\mathbf{E}$ is the electric field. Find a covariant generealization of this relation which is valid in any inertial frame.

3. In its own rest frame, a sphere of radius R absorbs all the radiation that hits it. In terms of the energy momentum tensor $T^{\mu\nu}$ of the ambient radiation field, find a covariant expression for $dp^\alpha/d\tau$, the rate at which the sphere absorbs energy and momentum, valid in any inertial frame.

4. Evaluate

$$\varepsilon_{\alpha\beta\gamma\delta}\,\eta^{\alpha\mu}\,\eta^{\beta\nu}\,\eta^{\gamma\sigma}\,\eta^{\lambda\delta}\,\varepsilon_{\mu\nu\sigma\lambda}.$$

What would be the analogous problem (and solution) in five dimensions (four space and one time)?

5. **(a)** Prove that $d^3\mathbf{p}/E$ is a Lorentz invariant. **(b)** Prove that the 6 dimensional volume element in phase space $d^3\mathbf{x}d^3\mathbf{p}$ is a Lorentz invariant. **(c)** If the (invariant) particle distribution function in phase space is $f(\mathbf{x},\mathbf{p})$, find an expression for the energy momentum tensor.