GENERAL RELATIVITY
HOMEWORK SET NUMBER 2

1. In 1+1 dimensions, to go from an inertial frame (t, x) to a uniformly accelerated frame in which the acceleration is constant (t', x'), one can use the transformations

$$t = t' \quad \text{and} \quad x = x' + \frac{1}{a'}\left[\sqrt{1 + (a't')^2} - 1\right]$$

where a' is the constant acceleration "felt" in the primed frame. **(a)** Find the metric tensor in the primed frame. **(b)** A twin goes on a journey in which she experiences a radial acceleration of $g = 980$ cm/s/s for 15 years, a deceleration of $-g$ for 30 years (not counting a brief stop at a distant outpost), and another radial acceleration of g for 15 years, bringing her back to rest at earth. How many years have passed for the twin that was left behind?

2. Consider the following transformation from a 1+1 dimensional inertial frame $(t_{\text{inert}}, x_{\text{inert}})$ to a new (t, x) frame:

$$t_{\text{inert}} = t \qquad \text{and} \qquad x_{\text{inert}} = tx.$$

Find all the affine connections in the new frame.

3. Consider the metric

$$-d\tau^2 = -dt^2 + F(t)G(r)\left[dr^2 + r^2 d\theta^2\right]$$

in 2+1 dimensions, where F and G are arbitrary functions of one variable. Find the Christoffel symbols.

4. Find the metric and the affine connections in a rotating co-ordinate system (2+1 dimensions).

5. The Schwartzchild metric is

$$d\tau^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2.$$

What is the gravitational redshift $z_{grav} \equiv -1 + \nu_{emitted}/\nu_{observed}$ for light emitted at rest at $r = 4GM$ and observed at $r = \zeta GM$, where $\zeta \gg 1$?

6. For neutron stars, degeneracy implies that mass and radius are related by MR^3 =constant. How does the gravitational redshift from the surface of a neutron star depend upon M?