Name KEY

Astro 112 – Physics of Stars
Exam #2, Fall 2011
Wednesday, November 9, 2011

The exam is open note, open book. You have 9:30-10:40 a.m. to complete the exam. Partial credit will be given. If you can answer a question with a few lines of math or text, as opposed to filling up the whole page, great! Don't feel like you need to go on at length.

1) If a 120 M_{Sun} star forms with $\log_{10}$ (Teff)=4.73 and $\log_{10}(L/L_{Sun})$=6.25: (*5 points*)

a) Estimate its Eddington luminosity, assuming the opacity is due to electron scattering only. Ratio your answer to the actual luminosity of the star.

b) Compare the star's radius and mean density to that of the Sun (again as a ratio).

A) $L_{EDD} = \dfrac{4\pi c G M}{\kappa} = 3.2\times10^4\left(\dfrac{M}{M_\odot}\right)L_\odot$ (EQ. 5.37)

$L_{EDD} = 3.2\times10^4(120)L_\odot$

$L_{ACTUAL} = 1.79\times10^6\ L_\odot$

$\dfrac{L_{ACT}}{L_{EDD}} = \dfrac{1.79\times10^6\ L_\odot}{3.84\times10^6\ L_\odot} = \boxed{0.47}$

B) $L = 4\pi R^2\sigma T_{EFF}^4$

$R = \sqrt{\dfrac{L}{4\pi\sigma T_{EFF}^4}} = \left(\dfrac{1.79\times10^6(3.85\times10^{26})}{4\pi\sigma(53333\,K)}\right)^{1/2} = 1.09\times10^{10}\ m$

$= \boxed{15\ R_\odot}$

$\bar{\rho} = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{120\ M_\odot}{\frac{4}{3}\pi(1.09\times10^{10}\,m)^3} \approx 44\ \frac{kg}{m^3}$

$\approx \boxed{0.03\ \rho_\odot}$

$\rho_\odot = 1409\ \frac{kg}{m^3}$

2) What source of opacity (*9 points*)

a) Is independent of wavelength?

e^- SCATTERING

FREE-FREE IS ALSO OK, DEPENDENT ON YOUR DEFINITION OF "INDEPENDENT"...

b) Is strongly peaked around a specific wavelength?

BOUND-BOUND

c) Is non-zero only at wavelengths shorter than a specific wavelength?

BOUND-FREE

3) Under the assumption of $\boxed{\text{complete ionization,}}$ find: *(9 points)*

a) μ_e for Pure ^{16}O

$$\frac{1}{\mu_e} = \text{\# OF FREE } e^- \text{ PER NUCLEON}$$

$$\frac{1}{\mu_e} = \frac{8}{16}$$

$$\boxed{\mu_e = 2}$$

b) μ_I for a mixture of X=0.7, Y=0.3

$$\frac{1}{\mu_I} = X + \frac{Y}{4} + \frac{\cancel{Z}^{\,0}}{\langle A \rangle}$$

$$\frac{1}{\mu_I} = 0.7 + \frac{0.3}{4}$$

$$\boxed{\mu_I = 1.29}$$

c) μ for Pure ^{16}O

$$\mu = \frac{\text{\# OF MASSES}}{\text{\# OF PARTICLES}}$$

$$= \frac{16}{1 \text{ NUCLEUS} + 8e^-}$$

^{16}O HAS $8e^-$ BECAUSE IT HAS 8 PROTONS

$$\boxed{\mu = \frac{16}{9}}$$

4) A star emits light (at a particular wavelength) from a gas region within its envelope, below the visible surface of the star. At this particular depth, the gas number density is $n_o=2\times10^{12}$ atoms/m^3. Above this region, the gas density falls off exponentially as $n= n_o\exp(-z/\lambda)$ where $\lambda=10^8$ m. The scattering cross section for photons is $\sigma= 10^{-19}$ m^2/atom.

a) Calculate the optical depth for the photon to travel upwards along the straight path (+z) until it escapes at $z=\infty$. Assume the optical path is strictly in the +z direction.

b) If the star has a temperature T=7,400 K at an optical depth of τ=2/3, estimate the temperature down at this gas region from which the light is coming from. You can assume that the plane parallel gray atmosphere Eddington approximation holds. (*12 points*)

$n=n_o e^{-z/\lambda}$, $\lambda=10^8$ m

A) $d\tau = n\sigma ds$

$$\tau=\int_0^\infty n\sigma dz = n_o\sigma\int_0^\infty e^{-z/\lambda}dz = -n_o\sigma\lambda e^{-z/\lambda}\Big|_0^\infty$$

$$\tau=-n_o\sigma\lambda(0-1) = \sigma n_o\lambda$$

$$\tau=(10^{-19})(2\cdot10^{12})(10^{8})$$

$$\boxed{\tau=20}$$

B) $T^4=\frac{3}{4}T_{EFF}^4\left(\tau+\frac{2}{3}\right)$

$T = 7400$ K AT $\tau=2/3$,

SO THAT $T = T_{EFF}$ THERE

$$T^4=\frac{3}{4}7400^4\left(20+\frac{2}{3}\right)$$

$$\boxed{T=14{,}683\ \text{K}}$$

5) Let's calculate the minimum mass of a true star (or rather, the maximum mass of a brown dwarf.) We've discussed that giant planets and brown dwarfs can be approximated by an n=1 polytrope, with radius R=7x10^7 m. Let us set a core temperature, T_c=4x10^6 K, as the threshold temperature for p-p hydrogen burning. Assume: 1) full ionization, 2) an ideal gas, and 3) a composition X=0.73 and Y=0.27. Find the mass of the brown dwarf at the hydrogen-burning limit. Express your answer in Jupiter masses (M_J), where 1 M_J=1.9x10^{27} kg. *(15 points)*

$$P = \frac{\rho}{\mu} \mathcal{R} T$$

IF $X = 0.73$ AND $Y = 0.27$

THEN $\mu = 0.61$

$n=1$ POLYTROPE

$$\rho_c = D_n \bar{\rho} \qquad P_c = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

$$P_c = \frac{\rho_c}{\mu} \mathcal{R} T_c$$

$$\frac{\rho_c \mathcal{R} T_c}{\mu} = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{4/3}$$

$$\frac{\mathcal{R} T_c}{\mu} = (4\pi)^{1/3} B_n G M^{2/3} \rho_c^{1/3}$$

$$= (4\pi)^{1/3} B_n G M^{2/3} (D_n \bar{\rho})^{1/3}$$

$$\frac{\mathcal{R} T_c}{\mu} = (4\pi)^{1/3} B_n G M^{2/3} D_n^{1/3} \frac{M^{1/3}}{\left(\frac{4}{3}\pi R^3\right)^{1/3}} = \frac{(4\pi)^{1/3} B_n G D_n^{1/3}}{\left(\frac{4}{3}\pi\right)^{1/3} R} M$$

ALL QUANTITIES KNOWN, EXCEPT FOR M

$\mu = 0.61$

$\mathcal{R} = 8.31 \times 10^3$

$T_c = 4 \times 10^6$

$B_n = B_1 = 0.233$

$D_n = D_1 = 3.290$

$R = 7 \times 10^7$ m

$$M = \frac{\left(\frac{4}{3}\pi\right)^{1/3} R \mathcal{R} T_c}{\mu (4\pi)^{1/3} B_n G D_n^{1/3}} = \frac{3.75 \times 10^{15}}{3.27 \times 10^{-11}}$$

$M = 1.14 \times 10^{29}$ kg

$$\boxed{M = 60 \ M_J}$$

REAL ANSWER IS 75 M_J. NOT A BAD APPROXIMATION