

Astro 112 – Physics of Stars
 Homework #5, Fall 2011
 Due in class on Wednesday, November 30, 2011

1) Crack open your book to Section 7.4 and show that Equations a) 7.36, b) 7.37, and c) 7.38 are true.

$$7.36) F_{\star} = \int_0^{\rho_{\star}} T_{\star}^n M = \frac{ac}{k} \left(\frac{\mu G}{R} \right)^4 M^3$$

$$\int_0^{\left(\frac{M}{R_{\star}^3}\right)} \left(\frac{\mu GM}{R_{\star} R} \right)^n M = \frac{ac}{k} \left(\frac{\mu G}{R} \right)^4 M^3$$

$$T_{\star} = \frac{\mu R_{\star}}{R \rho_{\star}} \quad \rho_{\star} = \frac{GM^2}{R_{\star}^3} \quad \left. \vphantom{\int_0^{\left(\frac{M}{R_{\star}^3}\right)}} \right\} T_{\star} = \frac{\mu GM}{R_{\star} R}$$

$$\int_0^{\left(\frac{\mu G}{R}\right)^{n-4}} \left(\frac{M^n}{R_{\star}^{n+3}} \right) = \frac{ac}{k} M$$

$$\Downarrow$$

$$\frac{M^n}{R_{\star}^{n+3}} \propto M \Rightarrow R_{\star} \propto \left(\frac{M^n}{M} \right)^{\frac{1}{n+3}} \propto \frac{M^{\frac{n}{n+3}}}{M^{\frac{1}{n+3}}}$$

$$R_{\star} \propto M^{\frac{n}{n+3} - \frac{1}{n+3}}$$

$$7.37) \rho_{\star} = \frac{M}{R_{\star}^3} \propto \frac{M}{M^{\frac{3(n-1)}{n+3}}}$$

$$\rho_{\star} \propto M^1 M^{-3\left(\frac{n-1}{n+3}\right)}$$

$$\boxed{\rho_{\star} \propto M^{2\left(\frac{3-n}{n+3}\right)}}$$

$$\boxed{R_{\star} \propto M^{\frac{n-1}{n+3}}}$$

$$7.38) L \propto M^3 \quad R_{\star} \propto M^{\frac{n-1}{n+3}} \quad L^{\frac{1}{3}} \propto M$$

$$L = 4\pi\sigma R^2 T_{\text{eff}}^4$$

$$L = 4\pi\sigma M^{2\left(\frac{n-1}{n+3}\right)} T_{\text{eff}}^4$$

$$L = 4\pi\sigma L^{\frac{2}{3}\left(\frac{n-1}{n+3}\right)} T_{\text{eff}}^4$$

$$\boxed{L^{1 - \frac{2}{3}\left(\frac{n-1}{n+3}\right)} \propto T_{\text{eff}}^4}$$

2) Consider an inhomogeneous fully ionized star that obeys the ideal gas law. It is all hydrogen exterior to some mass shell $m(r)$, and all helium interior of $m(r)$. Find the ratio of interior density divided by exterior density at the density discontinuity $m(r)$.

T AND P MUST BE CONTINUOUS IN A STAR.

FOR IDEAL GAS, $P = \frac{\rho}{\mu} RT$

$$\left. \frac{P}{T} \right|_{\text{ABOVE (H)}} = \left. \frac{P}{T} \right|_{\text{BELOW (He)}}$$

$$\frac{\rho_{\text{He}}}{\rho_{\text{H}}} = \frac{\mu_{\text{He}}}{\mu_{\text{H}}} = \frac{4/3}{1/2} = \boxed{\frac{8}{3}}$$

$$\mu = \frac{\# \text{ OF MASSSES}}{\# \text{ OF FREE PARTICLES}}$$

$$\mu_{\text{He}} = \frac{4}{1 \text{ nucleus} + 2e^-} = \frac{4}{3}$$

$$\mu_{\text{H}} = \frac{1}{1 \text{ nuc.} + 2e^-} = \frac{1}{2}$$

3) Why does a star less than $0.3 M_{\odot}$ star live longer than you would expect from a simple approximation that τ_{MS} is proportional to M^{-2} (eq. 7.41) ?

Nearly all main sequence stars burn through just the hydrogen in their core, which is around 10% of their mass (give or take). Of course low mass stars burn through this slower, which is already taken into account in 7.41. However, stars below $0.3 M_{\text{Sun}}$ are fully convective, so they are well mixed everywhere. Therefore, fresh hydrogen is constantly cycled into the core, and these low-mass stars can burned 100% of their hydrogen, not just 10%.

4) The age of the Universe is 13.7 Gyr. Compare this value so the main sequence lifetime of a $0.8 M_{\odot}$ star. Why isn't it useful to compute detailed models of the post main sequence evolution of stars much less massive than the Sun?

The Sun's MS lifetime is ~ 10 Gyr, so, using eq. 7.41, for an $0.8 M_{\text{Sun}}$ star, the lifetime is $10/0.8^2$, for about 15.6 Gyr. This is longer than the age of the Universe, so no $0.8 M_{\text{Sun}}$ stars have left the main sequence. Hence, there is no data to compare to.

5) For a $5 M_{\odot}$ star, briefly describe how the star's main sequence lifetime would change in duration if mass loss at the surface were enhanced during its time on the main sequence.

Low mass stars have lower central temperatures, so they burn through their hydrogen more slowly. A star with enhanced (faster) mass-loss gradually becomes a lower mass star through time, so that it will live longer on the main sequence than a star of the same initial mass that has slower mass-loss.