Astro 112 – Physics of Stars
Homework #4, Spring 2017
Due on Wednesday, May 17, 2017

1) Let's use some polytropes. Giant planets and brown dwarfs are physical objects with polytrope index around n=1. For the n=1 polytrope, θ=(sin ξ)/ξ, which in our equation for dimensionless density as a function of dimensionless radius. Let's work with Jupiter, which has a radius of 70,000 km (1 R_J) and a mass of 1.9e27 kg (1 M_J). (*20 points*)

a) Using what is given above, solve for K, constant in the polytropic pressure-density relation.
b) Now solve for the central density, ρ_c, in g/cm^3.
c) Write out the equation for the planet's density as a function of r, $\rho(r)$.
d) Using what you know from class, what is the radius of a 50 M_J n=1 polytrope.
e) Let's assume Jupiter, *by number*, is 90% H and 10% He, with the H fully ionized and the He *not* ionized, everywhere. Find mean molecular weight μ of this mixture, for Jupiter.
f) Using these values, and the ideal gas law, estimate the central temperature of Jupiter. The real number is estimated to be about 20,000 K. Do you think Jupiter is an ideal gas?

2) Assume that the pressure and density structure of the Sun can be approximated by an n=3 polytrope. You will need a few of the polytropic constants in Table 5.1 of the book. (*10 points*)

(a) Calculate the radius constant α.
(b) Calculate the central density ρ_c.
(c) Calculate the central pressure P_c. Compare to the real value of 2.48 x 10^{16} pascals.
(d) Calculate the central temperature T_c, assuming that the radiation pressure is negligible, that the ideal gas law holds, and that the interior is fully ionized. Compare to the real inferred value of 1.57 x 10^7 K

3) Explain in words how an ideal gas star behaves, and how a non-relativistic electron-degenerate star behaves, if the rate of energy production in the core due to nuclear reactions, L_{nuc}, is great than L (where L is the rate of emission of radiation). (*10 points*)