MEAN: $\frac{26}{50}$

Name KEY

Astro 118 – Physics of Planetary Systems
Midterm Exam, Winter 2014
Monday, February 10, 2014

The exam is open note, open book. You have 2:00-3:10 p.m. to complete the exam. Partial credit will be given. Don't feel like you need to go on at length. *50 points possible. Each question is worth 10 points. The exam is 20% of the course grade.*

1) Two months ago the European GAIA mission launched. It will perform precise astrometry on 1 billion stars in our galaxy, at the level of 20 micro-arcseconds. With this precision, what is the lowest-mass planet that could be detected around a solar-mass star, at 1 AU, at a distance of 10 parsecs? How about 100 parsecs? Could the stellar radial velocity induced by these planets be detected if 20 cm/s precision is achievable?

EQ. 4.14 : $\Theta = 9.551\times10^{-4} \frac{M_p}{M_*} \frac{r_p}{d}$ ARCSEC

AU

PARSECS

JUPITER MASSES

SOLAR MASSES

$$20\times10^{-6} = 9.551\times10^{-4} \frac{M_p}{1} \frac{1}{10}$$

AT 10 pc $\boxed{M_p = 0.209\ M_J} = 3.97\times10^{26}$ kg

AT 100 pc $\boxed{M_p = 2.09\ M_J} = 3.97\times10^{27}$ kg

$$V_* = \underbrace{\sqrt{\frac{G M_p^2}{r_p M_*}}}_{\text{IN SI UNITS}} \quad (\text{EQ } 4.4) \approx \boxed{\begin{array}{c} 594\ \frac{cm}{s} \text{ AT } 10\ pc \\ \hline 5940\ \frac{cm}{s} \text{ AT } 100\ pc \end{array}}$$

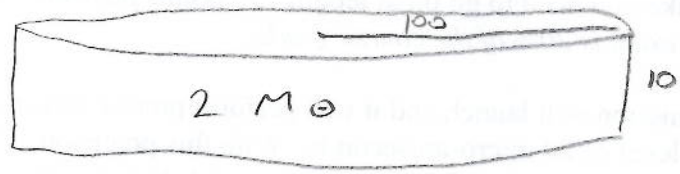
THESE ARE EASILY DETECTABLE

2) Assume that the solar nebula was 100 AU in radius, 10 AU thick, and contained two solar masses of gas and dust, well mixed.

a) What is the average density of the nebula, in g/cm^3 (or kg/m^3)?

b) If the temperature of this gas were 50 K and μ=2.3, what would the Jean's length be?

c) How many lumps (giant gaseous protoplanets) would this nebula fragment into, by a simple comparison of the Jean's length to the diameter of the disk?



a) $\rho = \frac{M}{V} = \frac{2M_\odot}{\pi R^2 H} = \frac{2M_\odot}{\pi(100\,AU)^2(10\,AU)} = \frac{4\times10^{33}\,g}{1.04\times10^{45}\,cm^3} \approx \boxed{3.85\times10^{-12}\ \frac{g}{cm^3}}$

b) $R_{JEAN} = \left(\frac{15kT}{4\pi G\mu m_H \rho}\right)^{1/2} = \left(\frac{15k(50)}{4\pi G(2.3)m_H\rho}\right)^{1/2}$

$$\boxed{R_{JEAN} = 9.14\times10^{13}\ cm} \approx \boxed{6.1\ AU}$$

c) $\# = \frac{\overset{DIAMETER}{200\ AU}}{R_{JEAN}} \approx \boxed{33\ \text{LUMPS}}$

3) Assuming a circular planetary orbit, how close-in could an Earth-mass planet orbit (around the Sun) and still have a moon in the one-month orbit? If an outside observer, at a distant random location, were viewing the system what would be the probability that this Earth-size planet would transit? Through what method could one measure the alignment of the planet's orbital axis relative to the star's rotation axis?

$$R_H = a\left(\frac{M_\oplus}{3M_\odot}\right)^{1/3}$$

30 DAYS

$$P^2 = \frac{4\pi^2}{GM_\oplus} R_H^3$$

$$R_H^3 = \frac{P^2 G M_\oplus}{4\pi^2}$$

$$\frac{P^2 G \cancel{M_\oplus}}{4\pi^2} = a^3 \frac{\cancel{M_\oplus}}{3M_\odot}$$

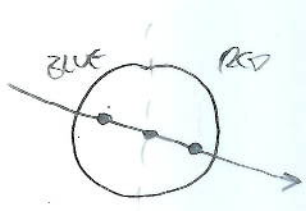
$$\frac{3(2.59\times10^6\,s)^2 G M_\odot}{4\pi^2} = a^3$$

$$a = 4.1\times10^{12}\ cm$$

$$\boxed{a = 0.27\ AU}$$

- TRANSIT PROBABILITY $= \frac{R_*}{a} = \frac{1\,R_\odot}{0.27\,AU} = \frac{6.95\times10^{10}\,cm}{4.1\times10^{12}\,cm} = \boxed{0.017}$

- ORIENTATION OF THE ORBIT FROM THE ROSSITER McLAUGHLIN EFFECT, OR SOMETHING SIMILAR — LOTS OF GOOD ANSWERS



4) In the limit of very strong focusing, and assuming constant v, ρ_s, and ρ_E, derive an expression for the time (t) it takes for an embryo to grow from a starting mass M_A, up to mass M_B, if the embryo starts with mass M_A at t=0.

$$\frac{dM_e}{dt} = \pi R_e^2 \left(1 + \frac{2GM_e}{R_e v^2}\right) \rho_s v \quad \text{(eq 36)}$$

STRONG FOCUSING: $\frac{2GM_e}{R_e v^2} \gg 1$

$$\frac{dM_e}{dt} = \pi R_e^{\cancel{2}} \frac{2GM_e}{\cancel{R_e} v^{\cancel{2}}} \rho_s \cancel{v}$$

$$\rho_e = \frac{M_e}{\frac{4}{3}\pi R_e^3}$$

$$R_e = \left(\frac{M_e}{\frac{4}{3}\pi \rho_e}\right)^{1/3}$$

$$\frac{dM_e}{dt} = \pi \left(\frac{M_e}{\frac{4}{3}\pi \rho_e}\right)^{1/3} \frac{2GM_e \rho_s}{v}$$

$$\frac{dM_e}{dt} = [\quad] M_e^{4/3}$$

$$M_e^{-4/3} dM_e = \frac{2\pi G \rho_s}{v} \left(\frac{1}{\frac{4}{3}\pi \rho_e}\right)^{1/3} dt$$

$$\left.\frac{-3}{M_e^{1/3}}\right|_{M_A}^{M_B} = (\quad)\Big|_0^t$$

$$-3\left(\frac{1}{M_B^{1/3}} - \frac{1}{M_A^{1/3}}\right) = \frac{2\pi G \rho_s}{v}\left(\frac{1}{\frac{4}{3}\pi \rho_e}\right)^{1/3}(t)$$

$$\boxed{\frac{3v\left(\frac{4}{3}\pi \rho_e\right)^{1/3}}{2\pi G \rho_s}\left(\frac{1}{M_A^{1/3}} - \frac{1}{M_B^{1/3}}\right) = t}$$

5) A synchronous satellite is one that orbits a planet in a circular orbit with an orbital period that is the same as the planet's rotational period. (Think of "stationary" communication satellites in the sky.) How slow of a rotator could the Earth be, and still be able to have stable synchronous satellites? Ignore any gravitational effects due to our Moon or the other planets.

(1) $$P^2 = \frac{4\pi^2}{G(M+m)} a^3$$

AT WHAT P DOES a FROM (1) EQUAL r_{HILL} FROM (2)?

(2) $$r_{HILL} = a\left(\frac{m}{3M}\right)^{1/3}$$

— ORBIT OF SATELLITE WOULD BECOME UNSTABLE IF IT ORBITED BEYOND THE EARTH'S HILL RADIUS

$$r_{HILL} \approx 0.001 \text{ AU}$$

$$P^2 = \frac{4\pi^2}{GM_\oplus} a_{SAT}^3$$

$$\boxed{\text{LET } a_{SAT} = r_{HILL}}$$

$$r_{HILL}^3 = (1 \text{ AU})^3\left(\frac{M_\oplus}{3M_\odot}\right)$$

$$P^2 = \frac{4\pi^2}{GM_\oplus}(1 \text{ AU})^3\left(\frac{M_\oplus}{3M_\odot}\right)$$

$$P = 1.81 \times 10^7 \text{ s} = \boxed{0.57 \text{ YEARS}}$$ ROTATIONAL PERIOD OF THE EARTH

$$\omega = \frac{2\pi}{P} = \boxed{3.47 \times 10^{-7} \ \frac{\text{RAD}}{\text{s}}}$$