A PLANET IN THERMAL BALANCE

FLUX $\left(\frac{W}{m^2} : \frac{ERG}{s \cdot cm^2}\right)$ AT A GIVE DISTANCE, d

$$\boxed{F = \frac{L_*}{4\pi d^2}}$$

CROSS-SECTIONAL AREA OF PLANET: πR_p^2

$\frac{ENERGY}{s}$ ABSORBED: $\frac{L_*}{4\pi d^2}\pi R_p^2 (1-A) = \boxed{\frac{(1-A)L_*}{4}\left(\frac{R_p}{d}\right)^2}$

ENERGY/s IN = $\frac{ENERGY}{s}$ OUT

$$L_p = 4\pi R_p^2 \sigma T_p^4 = \frac{ENERGY}{s}$$

$$4\pi \cancel{R_p^2} \sigma T_p^4 = \frac{(1-A)L_*}{4}\frac{\cancel{R_p^2}}{d^2}$$

- ASSUMES YOU RADIATE AWAY ABSORBED ENERGY OVER FULL SURFACE AREA
- SOMETIMES A BAD ASSUMPTION
  - GOOD IF
    - FAST ROTATOR
    - STRONG ATMOSPHERIC DYNAMICS → "ADVECTION"

$$\boxed{T_p = \left[\frac{(1-A)L_*}{16\pi d^2 \sigma}\right]^{1/4}}$$

IMPORTANT!

$(1-A)^{1/4}$ → ALBEDO NOT AS IMPORTANT AS YOU MIGHT THINK

$T_p = T_{EQ}$

= "EQUILIBRIUM TEMPERATURE"

R_p → NOT IN THE EQUATION

IF $4\pi \to 2\pi$, THEN $T_{EQ} = 2^{1/4}$ LARGER

THERMAL BALANCE II

- ASSUMPTIONS — ATMOSPHERE NOT IMPORTANT
- NO INTERNAL ENERGY SOURCES

EARTH : INTERNAL $\approx \frac{1}{1000}$ ABSORBED

JUPITER : " $\approx$ ABSORBED

HOT JUPITER : " $\approx \frac{1}{10^4}$ ABSORBED

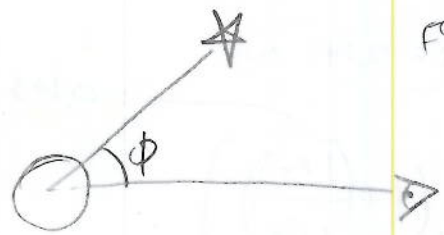
T_{eq}, EARTH $= 255$ K → COLD, OUR SURFACE TEMPERATURE IS HIGHER THAN THIS BECAUSE WE HAVE AN ATMOSPHERE

→ EARTH DOES RADIATE AT 255 K, THOUGH

ALBEDOS

A_g, GEOMETRIC ALBEDO → ALBEDO AT A PARTICULAR WAVELENGTH OR BANDPASS AT "FULL PHASE" → $\phi = 0^\circ$

FULL ILLUMINATION



GENERALLY ONLY OBSERVABLE FROM EARTH FOR THE OUTER PLANETS, BEYOND EARTH

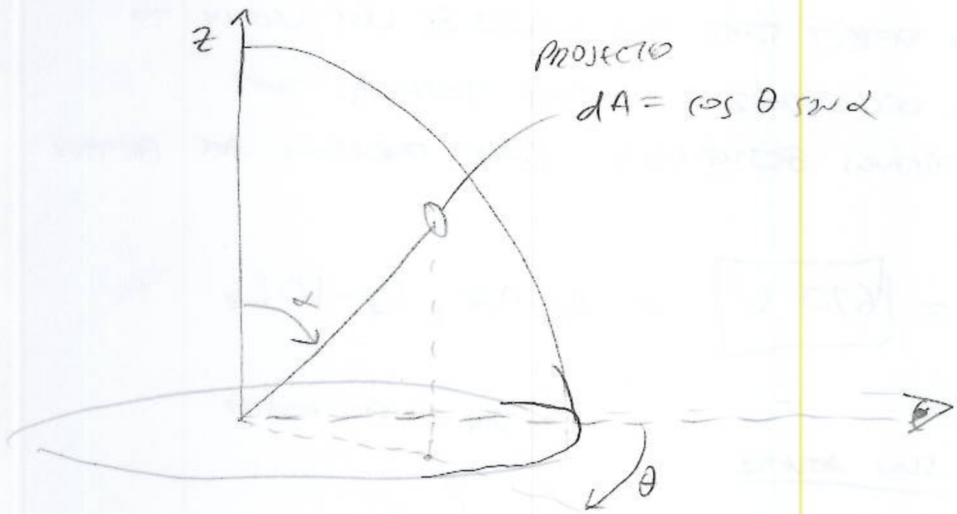
GEOMETRIC ALBEDOS CAN >1, SINCE IT IS RELATIVE TO A LAMBERT DISK

(NOT A LAMBERT SPHERE)

FOR A LAMBERT SPHERE (LIKE A CUE BALL, BUT BETTER),

$$\boxed{A_g = \frac{2}{3}}$$

ALBEDOS II



$$\boxed{A_g = \frac{1}{\pi I_{INC}} \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\alpha=0}^{\pi} I(\theta,\alpha,\phi=0)\cos\theta\sin\alpha \, d\alpha\, d\theta}$$

$$I_{INC} = \frac{\text{ENERGY}}{s \cdot m^2 \cdot \lambda \cdot sr}$$

$$\pi I_{INC} = \text{FLUX} = \frac{\text{ENERGY}}{s \cdot m^2 \cdot \lambda}$$

SPHERICAL ALBEDO, A_s

FRACTION OF INCIDENT RADIATION SCATTERED BY A SPHERE OVER ALL ANGLES
— AT A GIVEN λ OR WAVEBAND

$F(\phi)$ = SCATTERED FLUX AT A GIVEN PHASE ANGLE

$$F(\phi) = \int_{\theta=\phi-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\alpha=0}^{\pi} I(\theta,\alpha,\phi;\theta_0,\alpha_0)\cos\theta\sin\alpha\, d\alpha\, d\theta$$

θ_0 & α_0 → INCIDENT ANGLE OF RADIATION

$$A_s = \frac{1}{\pi I_{INC}} \int_{4\pi} F(\phi)\, d\Omega = \frac{2\cancel{\pi}}{\cancel{\pi} I_{INC}} \int_0^{\pi} F(\phi)\sin\phi\, d\phi$$

$$A_s = A_g \frac{2}{F(\phi=0)} \int_0^{\pi} F(\phi)\sin\phi\, d\phi = (A_g)(q)$$

"PHASE INTEGRAL"

ALBEDOS III

BOND ALBEDO — INTEGRAL OF THE SPHERICAL ALBEDO OVER ALL INCIDENT WAVELENGTHS

$$A_B = \frac{\int_0^\infty A_{S,\lambda} I_{inc,\lambda} \, d\lambda}{\int_0^\infty I_{inc,\lambda} \, d\lambda}$$

$$A_B = \frac{\text{TOTAL SCATTERED POWER}}{\text{TOTAL INCIDENT POWER}}$$ → $1 - A_B$ DICTATES T_{EQ} OF A PLANET

→ DEPENDS EXPLICITLY ON THE SPECTRUM OF THE PARENT STAR → THE INCIDENT LIGHT

→ THE SAME PLANET WILL HAVE A LOWER A_B (AND HIGHER T_{EQ}) AROUND AN M DWARF THAN A G DWARF

→ LESS BLUE LIGHT FROM M DWARF — LESS SCATTERING

IN THE OUTER SOLAR SYSTEM A_S AND A_B CAN ONLY BE OBSERVED FROM SPACECRAFT

TOTAL FLUX

$$F_{PLANET} = F_{EMITTED} + F_{REFLECTED}$$

$F_{inc} = \frac{L_*}{4\pi d^2}$

$$F_{EMITTED} = \frac{L_{EMITTED}}{4\pi D^2} = \frac{4\pi R^2 \sigma T_{EQ}^4}{4\pi D^2}$$

$$F_{REFLECTED} = \frac{1}{4\pi D^2} \left[q(\phi) A_g \pi R_p^2 \right] \frac{L_*}{d^2}$$

ALBEDOS IV

$$f_{PLANET} = f_{EMIT} + f_{REFLEC}$$

$$f_{EMIT} = \frac{L_{EMIT}}{4\pi D^2} = \boxed{\frac{R_p^2 \sigma T_p^4}{D^2}} = R_p^2 \sigma \frac{(1-A_B)L_*}{16\pi d^2 \sigma D^2}$$

$$f_{REFLEC} = \frac{\frac{L_*}{4\pi d^2} \pi R_p^2 A_g q}{4\pi D^2} = \frac{L_*}{4\pi d^2} \frac{\pi R_p^2 A_g q}{4\pi D^2}$$

$$D = R_P$$

$$f_{REFLECTED} = \frac{L_* R_p^2 A_g q}{16\pi d^2 D^2}$$

$$L_{EMIT} = \frac{\cancel{R_p^2} \cancel{\sigma} (1-A_B) L_*}{16\pi d^2 \cancel{\sigma} \cancel{R_p^2}} \quad 4\pi$$

$$L_{EMIT} = \frac{(1-A_B) L_*}{16\pi d^2}$$

$$= \frac{L_*}{4\pi d^2} \frac{R_p^2 A_g q}{4 D^2}$$

$$\cancel{L_{REFL} = \frac{L_* R_p^2 A_g}{16\pi d^2 R_p^2}}$$

$$= \frac{\cancel{4\pi} R_*^2 f_*}{\cancel{4\pi} d^2} \frac{R_p^2 A_g}{4 D^2} q$$

$$=$$

$$\boxed{f_{REFLEC} = \frac{A_g}{4} \left(\frac{R_*}{D}\right)^2 \left(\frac{R_p}{d}\right)^2 f_* q}$$

PLANETARY ATMOSPHERES

$$H = \text{SCALE HEIGHT} = e\text{-FOLDING DISTANCE} = \frac{kT}{\mu m_H g} = \frac{RT}{\mu g}$$

μ : DIMENSIONLESS

	V	E	M	J	S	U	N	TITAN	HD209458b
H(km)	16	8	18	18	35	20	19	20	400

$$\rho(z) = \rho_0 e^{-z/H} \rightarrow \rho_0 e^{-z/H(z)}$$

$$\boxed{H(z) = \frac{kT(z)}{\mu(z) m_H g(z)}}$$

① $\frac{dP}{dz} = -\rho g$ ② $P = \frac{\rho}{\mu m_H} kT$

$$\frac{dP}{dz} = \frac{P \mu m_H}{kT}(-g)$$

$$\int \frac{dP}{P} = -dz \frac{\mu m_H}{kT} g = \int \frac{-dz}{H}$$

$$\boxed{P = P_0 \exp(-z/H)}$$

$H = \frac{RT}{\mu g}$

$\frac{dp}{dr} = -\rho g$

$= -\rho g$

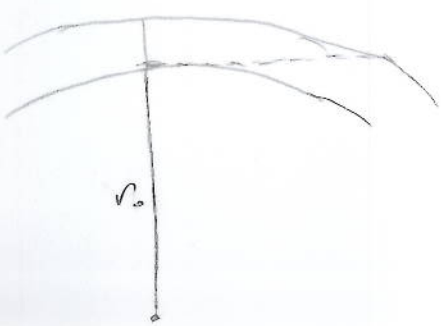
$\frac{dp}{dr} = -\rho g$

$dp = -\rho g \, dr$

$dr = -\frac{1}{\rho g} dp$

VERTICAL COLUMN DENSITY, N, ASSUME T = CONST, g = CONST, μ = CONST, IDEAL GAS

$$N(r_0) = \int_{r_0}^{\infty} n(r)\, dr = \int_{r_0}^{(\infty)} \frac{\rho(r)}{\mu m_H} dr = \int_0^{p(r_0)} \frac{1}{m_H \mu g(r)} dp = \frac{P(r_0)}{g(r_0)\mu m_H} = \frac{n_0 kT}{g(r_0)\mu m_H} = \boxed{n_0 H}$$



UNDERSTANDING PLANETARY SPECTRA

UNDERSTANDING ATMOSPHERIC COMPOSITION — DE PATER & LISSAUER, FIG. 4.4 - 4.6

HABITABILITY

WATER — APPEARS ESSENTIAL TO LIFE

TRIPLE POINT — 0.006 BAR (600 Pa), 273.16 K

AT $P < 0.006$ BAR, LIQUID WATER NOT POSSIBLE $\longleftrightarrow$ VAPOR TO SOLID PHASE

CRITICAL POINT — 220 BAR, 646 K — BEYOND THERE IS NO DISTINCTION BETWEEN LIQUID AND VAPOR — "SUPERCRITICAL FLUID"

ORIGIN OF WATER ON ROCKY WORLDS

EARTH: 70% OF SURFACE AT 4 KM DEEP

MASS WATER $\approx 1\text{-}2 \times 10^{24}$ g

WATER FROM ROCKY BUILDING BLOCKS + OUTGASSING

"LATE VENEER" OF COMETS

- D/H OF COMETS = $2\times$ $\frac{D}{H}$ OF EARTH'S OCEANS
- ONLY ~10% OF EARTH'S WATER COULD COME FROM COLD COMETS

CLASSICAL HABITABLE ZONE

T_{surf} = 273–373 K, FOR LIQUID WATER, FOR 1 BAR SURFACE PRESSURE

$$a = \left[\frac{(1-A_B)L_*}{16\pi\sigma T_{eq}^4}\right]^{1/2} \qquad A_B = 0.3, \quad L_* = L_\odot$$

LEADS TO $\boxed{a = 0.47\text{–}0.87 \text{ AU}}$

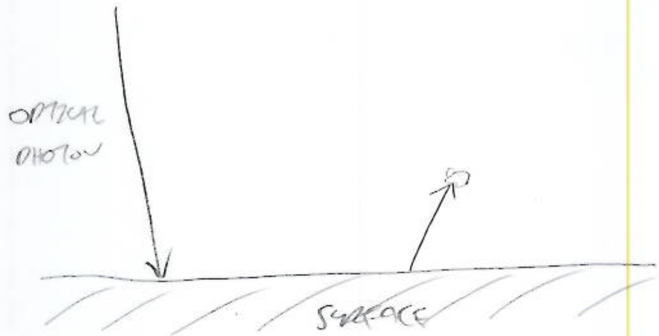
- EARTH IS NOT IN THE HABITABLE ZONE
- BUT WE HAVE IGNORED THE GREENHOUSE EFFECT

HABITABLE ZONE II

$\lambda_{max} T = 0.29$ cm K $\quad T_{surf} = 287$ K, $\underline{\lambda_{max} = 10\,\mu m}$

$\overbrace{H_2O, CO_2, CH_4}$ ALL STRONG ABSORBERS AT THESE WAVELENGTHS

(SHOW PLOTS FROM SHABRAM PAPER)



$$\sigma = \frac{ac}{4}$$

$$\frac{dT}{dr} = \frac{-L}{4\pi r^2} \frac{3\kappa\rho}{16\sigma T^3}$$

$$F = \frac{-4acT^3}{3\kappa\rho} \frac{dT}{dr} \qquad \frac{dP}{dr} = -\rho g$$

$$-\frac{3\kappa\rho F}{4acT^3} = \frac{dT}{dr} \qquad \frac{dP}{dr} = -\rho g$$

$$\frac{dT}{dP} = \frac{3\kappa\cancel{\rho}F}{4acT^3\cancel{\rho}g} \frac{4}{4}$$

$$= \frac{3\kappa F}{16\sigma T^3 g}$$

$$= \frac{3\kappa\cancel{\sigma}T_{eff}^4}{16\cancel{\sigma}T^3 g}$$

$$T^3 dT = \frac{3\kappa T_{eff}^4}{16g} dP$$

$$\frac{T^4}{4}\Big|_0^T = \frac{3\kappa T_{eff}^4}{16g}\Big|_0^{P_0}$$

$$T^4 = \frac{3\kappa T_{eff}^4}{16g} P_0$$

$$T = T_{eff}\left(\frac{3\kappa}{16g} P\right)^{1/4}$$

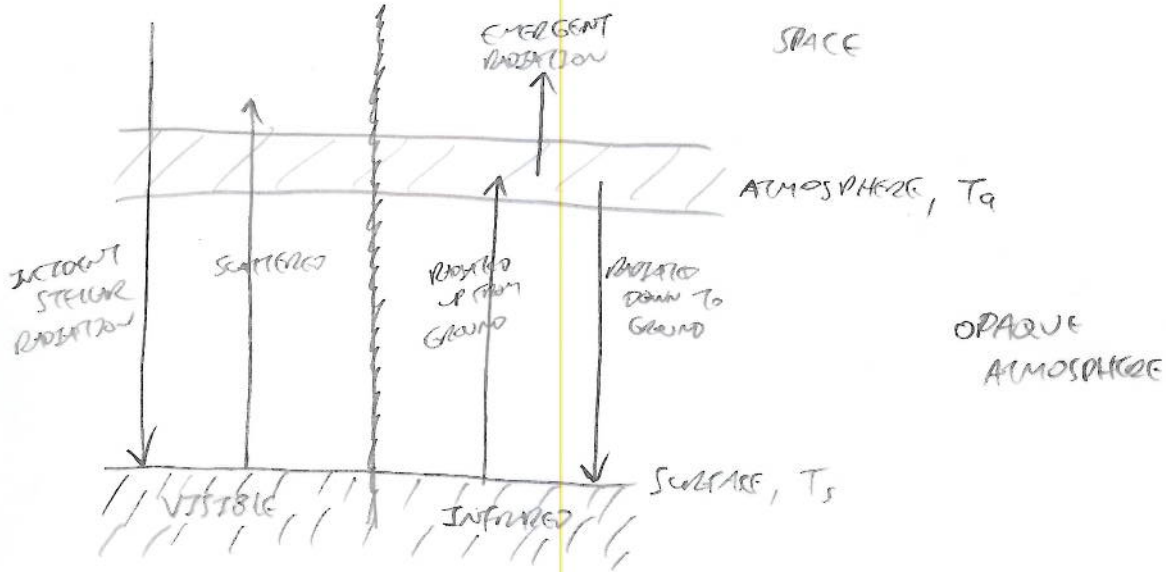
SURFACE TEMPERATURE FOR A SIMPLIFIED ATMOSPHERE

WE USED THE ENERGY BALANCE TO DERIVE T_{EQ}, WHICH IS THE T OF EMITTING LAYER OF OUR ATMOSPHERE

THIS CAN BE WRITTEN AS:

$$T_{EQ} = T_{EFF,*}\left(\frac{R_*}{2a}\right)^{1/2}(1-A_B)^{1/4}$$

WE CAN NOW MOVE ON TO LOOKING AT A SIMPLE GREENHOUSE ATMOSPHERE



1) ATMOSPHERE IS CLEAR IN VISIBLE, NO SCATTERING

2) GROUND HEATED AND EMITS IN THE IR, LONGER λ

3) ATMOSPHERE LAYER ABSORBS IR FLUX

4) GROUND ABSORBS ALL IR FLUX FROM ATMOSPHERE

SINCE OUR ATMOSPHERE IS DOING THE EMITTING $T_A = T_{EQ} = T_E$ EMISSION TEMPERATURE

$$\sigma T_A^4 = \sigma T_{EQ}^4 = \sigma T_E^4$$

FOR THE SURFACE

$$\sigma T_s^4 = \underbrace{\sigma T_*^4\left(\frac{R_*}{2a}\right)^2(1-A_B)}_{\sigma T_{EQ}^4} + \underbrace{\sigma T_A^4}_{\sigma T_E^4}$$

SURFACE TEMP II

THEREFORE THE SURFACE TEMP, T_s:

$$T_s = 2^{1/4} T_e$$

$$\boxed{T_s \approx 1.19\, T_e}$$

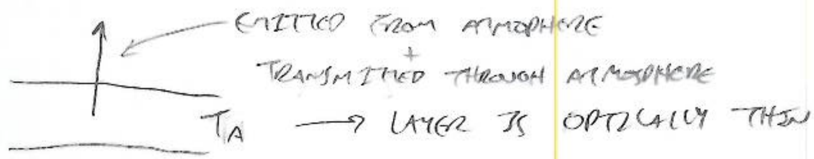
FOR EARTH, $T_e = 255\text{ K}$, $T_s = 303\text{ K}$

REAL $T_s = 280\text{ K}$

VENUS, $T_e = 230\text{ K}$, $T_s = 274\text{ K}$ (LOWER ALBEDO)

REAL $T_s = 730\text{ K}$

SLIGHTLY MORE REALISTIC MODEL IS THE LEAKY GREENHOUSE



$$\sigma T_E^4 = \underbrace{\sigma T_A^4}_{\text{ATMOSPHERE}} + \underbrace{\sigma T_s^4 (1-\alpha)}_{\text{FROM SURFACE}}$$

$0 < \alpha < 1$ → FRACTION OF RADIATION ABSORBED BY THE ATMOSPHERE LAYER

$$\sigma T_s^4 = \underbrace{\sigma T_*^4 \left(\frac{R_*}{2a}\right)^2 (1-A_B)^{1/4}}_{\sigma T_E^4} + \sigma T_A^4, \text{ AS BEFORE}$$

SOLVING FOR T_s YIELDS

$$\boxed{T_s = \left(\frac{2}{2-\alpha}\right)^{1/4} T_e}$$

$\alpha < 1$ WEAKENS GREENHOUSE EFFECT

$$T_s \geq T_e \geq T_a$$

- COULD BUILD UP A MORE REALISTIC RADIATIVE EQUILIBRIUM MODEL WITH MANY LAYERS WITH VARYING α, WAVELENGTH DEPENDENT α, BASED ON CHEMICAL ABUNDANCES AND CLOUDS