GROUND TEMPERATURE, MORE

ANOTHER SIMPLE MODEL:

$$T_g^4 = T_{EQ}^4 \left(1 + \tfrac{3}{4}\tau_g\right)$$

EARTH, $\tau_g = 0.83 \rightarrow T_g = T_{EQ}\, 1.62^{1/4}$

$$\boxed{287 - 255}$$

30-40 DIFFERENCE LEADS TO $a_{HABITABLE}$

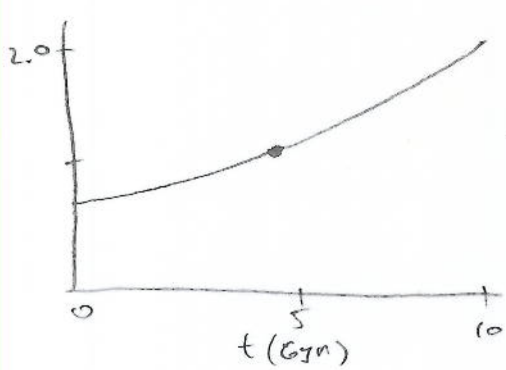
$$\boxed{a \approx 0.55 - 1.1 \text{ AU}}$$

VENUS, $\tau_g \approx 60$ $\quad T_{EQ} = 264 \rightarrow T_g = 750$ K

MARS, $\tau_g \approx 0.2$ $\quad T_{EQ} = 223 \rightarrow T_g = 233$ K

CHANGES WITH TIME

$$L(t) = \left[1 + \frac{2}{5}\left(1 - \frac{t}{t_\odot}\right)\right]^{-1} L_\odot$$



$t_\odot, L_\odot$ = CURRENT VALUES

STARS BRIGHTEN WITH TIME

$$P_{core} = \frac{\rho}{\mu_{core} m_H} k T_{core}$$

$P_{core} \approx$ CONSTANT — SAME MASS ABOVE

$\mu \uparrow$, SO $T_{core} \uparrow \rightarrow$ FASTER REACTION RATE

- LEADS TO THE FAINT YOUNG SUN PARADOX
  - STRONG EVIDENCE FOR LIQUID WATER AT 4 Gyr AGO
  - HOW IS THIS POSSIBLE WITH $L_* \approx 0.70 - 0.75\, L_\odot$?
  - NEED STRONGER GREENHOUSE EFFECT IN THE PAST
    - MORE CO_2? MORE CH_4?

UNDERSTANDING T_g THROUGH TIME

- A VERY DIFFICULT PROBLEM
- "GARBAGE IN – GARBAGE OUT"

- JEANS ESCAPE ATMOS. LOSS
- CONDENSATION OF H_2O
- H-LOSS DUE TO H_2O PHOTODISSOCIATION
- OXYGEN LOSS DUE TO OXIDATION
- LOSS OF CO_2 DUE TO WEATHERING
- SOLUBILITY OF GASES INTO WATER
- L_{SUN} EVOLUTION
- ALBEDO

$$A = f_{CLOUD} A_{CLOUD} + f_{ICE} A_{ICE} + f_{OCEAN} A_{OCEAN} + f_{ROCK} A_{ROCK}$$

- GREENHOUSE EFFECT – CO_2, H_2O, CH_4
- INITIAL CONDITIONS $\longrightarrow$ EARTH, INIT ATM = 85% CO_2, 10% CH_4, 5% N_2

HABITABLE ZONE INNER EDGE

HOTTER ATMOSPHERE W/ LITTLE TO NO H_2O CLOUDS

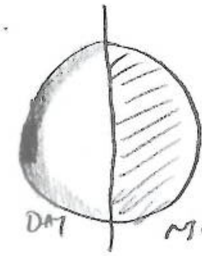
$\rightarrow H_2O$ VAPOR $\xrightarrow[UV]{} 2H + O \rightarrow$ LOSS OF H RAPIDLY $\rightarrow$ NO WATER

OUTER EDGE

CO_2 CONDENSATION INTO CLOUDS $\rightarrow$ LOSS OF GREENHOUSE EFFECT, + BRIGHT CLOUDS, LEADS TO COLD T_{SURF}

ATMOSPHERIC DYNAMICS

- ATMOSPHERIC CIRCULATION IS DRIVEN BY UNEVEN STELLAR HEATING, AS A FUNCTION OF LATITUDE & LONGITUDE.

$\Longrightarrow$ 

- 1ST ORDER: IS YOUR ATMOSPHERE DOMINATED BY THE RADIATIVE - EQUILIBRIUM SOLUTION OR IS IT DOMINATED BY CIRCULARIZATION?

$$\tau_{ADV} = \frac{L}{U} \approx \frac{R_p}{U} = \frac{10^5 \text{ km}}{1 \text{ km/s}} \approx 10^5 \text{ sec}$$ ADVECTIVE TIMESCALE

τ_{RAD} :

OUT OF RADIATIVE EQUILIBRIUM BY AN AMOUNT ΔT, LAYER THICKNESS IS Δz & ΔP

LAYER: $T + \Delta T$

$$\frac{\text{EXCESS } E}{\text{AREA}} = \boxed{\rho C_p \Delta T \Delta z}$$

RADIATE AS B.B. $= \sigma (T + \Delta T)^4 \approx \sigma T^4 \left(1 + \frac{4 \Delta T}{T}\right)$

$$\approx \sigma T^4 + \boxed{4 \sigma T^3 \Delta T}$$

$$\tau_{RAD} = \frac{\rho c_p \cancel{\Delta T} \Delta z}{4 \sigma T^3 \cancel{\Delta T}}$$

$$\frac{dP}{dz} = -\rho g$$

$$\Delta z \rho = \frac{\Delta P}{g}$$

$$\boxed{\tau_{RAD} \approx \frac{\Delta P \, C_p}{g \, 4 \sigma T^3}}$$ RADIATIVE TIMESCALE

DYNAMICS II

- PHOTOSPHERIC P IS A FUNCTION OF λ.
- WHETHER OR NOT ATMOSPHERE IS CLOSE TO RADIATIVE EQUILIBRIUM DEPENDS ON THE PRESSURE OF INTEREST

UPPER ATMOSPHERE : P low, $T \sim$ "HIGH"

SHORT $\tau_{RAD} \rightarrow$ RADIATIVE EQUIL. HOLDS

DEEP ATMOSPHERE : P HIGH, $T \sim$ "HIGH"

LONG $\tau_{RAD} \longrightarrow$ PLANET WELL HOMOGENIZED

(ALSO, WHEN OPTICALLY THICK, $\tau_{RAD} \propto P^2$

PLANET	τ_{RAD}	$\tau_{ADV,\,LON}$	ΔT_{LON}	$\tau_{ADV,\,LAT}$	ΔT_{LAT}
EARTH	WEEKS	1 DAY	~ 10 K	WEEKS	20–30 K
~~JUPITER~~	~~$\gtrsim$ DECADES~~	~~DECADES $\gtrsim$~~	~~$\gtrsim$ FEW K~~	~~$\gtrsim$ DECADES~~	~~$\gtrsim$ FEW K~~
VENUS	YEARS	DAYS	FEW K	WEEKS	FEW K

WELL HOMOGENIZED ($\tau_{RAD} \gg \tau_{ADV}$)

OK HOMOGENIZED ($\tau_{RAD} \gtrsim \tau_{ADV}$)

~~[illegible] HOMOGENIZED (τ_{RAD} [illegible]~~

SKIP TO P20

THERE ARE SOME COMPLEXITIES HERE, THOUGH

$\rightarrow$ FOR JUPITER, WE HAVE LONG τ_{RAD}, BUT ALSO $\rightarrow$

$\rightarrow$ RAPID ROTATION (SMALL DAY-NIGHT)

$\rightarrow$ ABSORPTION OF SUNLIGHT INTO THE CONVECTIVE INTERIOR (SMALL EQUATOR-TO-POLE)

SIMPLE ESTIMATE OF DAY-NIGHT CONTRAST

$$R_{DN} = \frac{\Delta T_{DAY-NIGHT}}{\Delta T_{RAD-EQ}} \approx 1 - e^{-\tau_{ADV}/\tau_{RAD}}$$

LARGE, SINCE T_{RAD-EQ} NIGHT IS 100-300 K

$\Downarrow$

1500 - 300 ≈ 1200

2000 ~ 300 ≈ 1700

$$\frac{\tau_{ADV}}{\tau_{RAD}} \gg 1 \Rightarrow R_{DN} = 1 \rightarrow \Delta T_{DAY-NIGHT} = \Delta T_{RAD-EQ}$$

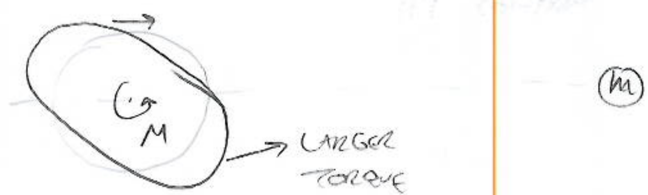
$$\frac{\tau_{ADV}}{\tau_{RAD}} \ll 1 \Rightarrow R_{DN} \rightarrow 0, \Delta T_{DAY-NIGHT} \rightarrow 0 \text{ (HOMOGENIZED)}$$

$$\frac{\tau_{ADV}}{\tau_{RAD}} = 0.3 \Rightarrow \Delta T_{DAY-NIGHT} \approx \tfrac{1}{4} \Delta T_{RAD-EQ}$$

CAN USE OBSERVATIONS TO ESTIMATE RADIATIVE + ADVECTIVE TIMESCALES → OR AT LEAST THEIR RELATIVE IMPORTANCE

TIDAL BULGE, II

SLOW ROTATION COMPARED TO ORBITAL PERIOD:



"SPIN UP" OF M → GAIN OF ANGULAR MOMENTUM

LOSS OF L OF m, ORBIT DECAYS

STAR "SPUN UP" AND PLANET MAY CRASH INTO STAR

STRONG EVIDENCE IN EXOPLANETARY SYSTEMS

$$\frac{1}{a}\frac{da}{dt} = -\left[\frac{63}{2}\left(GM_*^3\right)^{1/2}\frac{R_p^5}{Q_p M_p}e^2 + \frac{9}{2}\left(\frac{G}{M_*}\right)^{1/2}\frac{R_*^5 M_p}{Q_*}\right]a^{-13/2}$$

$$\propto a^{-13/2},\ R_p^5,\ R_*^5,\ \frac{1}{Q_p},\ \frac{1}{Q_*}$$

LARGE DEFORMATION: THE ROCHE LIMIT

DISTANCE TO BREAKUP:

$$\underbrace{F_{TIDAL} = F_{GRAVITY}}\quad \text{OR}\quad \left(F_{TIDAL} = F_{GRAVITY} + F_{STRENGTH}\right)$$

$$F_{G,u} = \frac{GMu}{r}$$

$$F_{T,u} = \frac{2GMur}{d^3}$$

$$d = r\left(2\frac{M}{m}\right)^{1/3}$$

$$M = \frac{4\pi\rho_M R^3}{3}$$

$$m = \frac{4\pi\rho_m r^3}{3}$$

$$\boxed{d = R\left(2\frac{\rho_M}{\rho_m}\right)^{1/3}}$$ FOR A RIGID OBJECT

A FEW NOTES ON TIDES

HABITABLE ZONE WILL INTERSECT W/ THE TIDAL LOCKING RADIUS

$$\boxed{a_{lock} = 0.027 \left(\frac{P_0 t}{Q}\right)^{1/6} M_*^{1/3}}$$

P_0 = INITIAL ROTATION PERIOD

$\boxed{\cancel{M_* \text{ IN } M_\odot}}$ UNITS: <u>CGS</u>

$Q = 2\pi \frac{E_0}{\Delta E}$ DISSIPATION QUALITY FACTOR

E_0 PEAK ENERGY STORED IN A CYCLE

ΔE = E DISSIPATED PER CYCLE

$Q_{EARTH} = 13$

$Q_{\text{ROCKY PLANETS}} \approx 100$

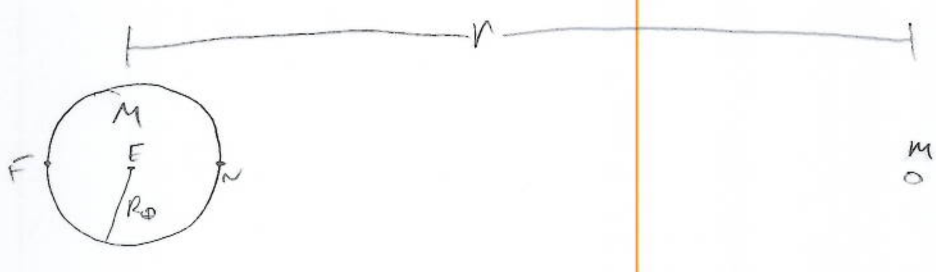
$Q_{\text{GAS GIANT}} \approx 10^5 - 10^6$

FAR LESS DISSIPATION PER ORBIT

PURE FLUID OBJECTS SHOULD BE $\sim 10^{12}$, SO IT ISN'T CLEAR WHY THE NUMBERS FOR GIANT PLANETS ARE AS <u>LOW</u> AS THEY ARE

MERCURY SHOULD BE TIDALLY LOCKED BUT PERTURBATIONS OF S.S. PLANETS LEAD TO 3:2 RESONANCE INSTEAD.

BASICS OF TIDES



$$a_{NE} = \frac{Gm}{(r - R_\oplus)^2} - \frac{Gm}{r^2} = \frac{Gm}{r^2}\left(1 + 2\frac{R_\oplus}{r}\right) - \frac{Gm}{r^2}$$

RELATIVE ACCELERATION BETWEEN N & E

SINCE $R_\oplus << r$

$$\boxed{a_{NE} = \frac{2GmR_\oplus}{r^3}}$$

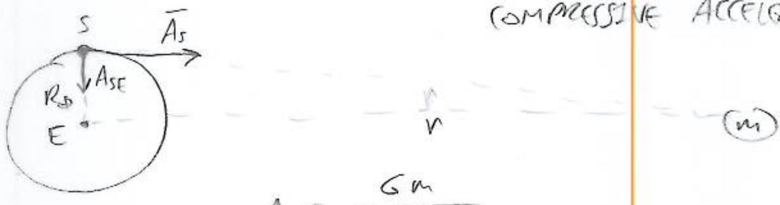
TOWARDS MOON

TIDES II

a_{EF} SAME MAGNITUDE, BUT AWAY FROM THE MOON.

MAKES A PROLATE PLANET

COMPRESSIVE ACCELERATIONS



$$A_s = \frac{Gm}{R_\oplus^2 + r^2}$$

$$A_{Em} = \frac{Gm}{R_\oplus^2 + r^2} \frac{r}{\left(R_\oplus^2 + r^2\right)^{1/2}} \overset{\text{SOME WORK}}{\approx} \frac{Gm}{r^2}$$

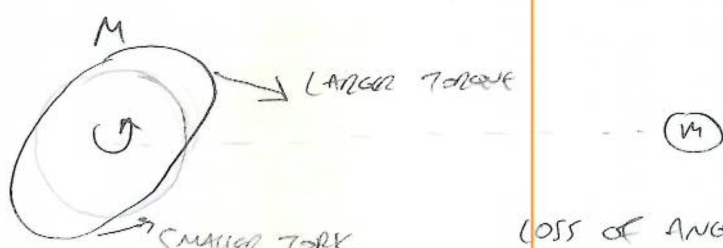
NO ACCELERATION TOWARDS MOON

$$A_{SE} = \frac{Gm}{R_\oplus^2 + r^2} \frac{R_\oplus}{\left(R_\oplus^2 + r^2\right)^{1/2}} \approx \boxed{\frac{Gm R_\oplus}{r^3}}$$

COMPRESSIVE ACCELERATION IS $\sim \frac{1}{2}$ EXTENSION ACCELERATION FOUND AT THE SUB-ANTI-MOON POINTS

TIDAL BULGE — EFFECT ON ORBITS

FAST ROTATION COMPARED TO ORBITAL PERIOD



LOSS OF ANGULAR MOMENTUM FROM M,
GAIN IN ORBIT OF m $\rightarrow$ m MOVES
FURTHER OUT WITH TIME $\rightarrow$ AS IN EARTH MOON

ROCHE LIMIT, II

IF YOU TAKE SATELLITE SHAPE DEFORMATION INTO ACCOUNT

— NO ANALYTIC SOLUTION

— FOR FLUID SATELLITE → $d \approx 2.44 R \left(\frac{\rho_M}{\rho_m}\right)^{1/3}$

PLANETARY INTERIORS

THINKING ABOUT PLANETS AS A COLLECTION OF ATOMS & ELECTRONS

THE ENERGIES INVOLVED

$K.E.: \quad KE_{e^-} >> KE_{p}$ (ELECTRONS & PROTONS)

AROUND CENTER OF MASS, $P_{e^-} = P_p$ $\qquad KE = \frac{P^2}{2m}$

MOMENT OF INERTIA — A MEASURE OF CENTRAL CONDENSATION OF A PLANET

$$I = \int_0^M r^2 \, dm$$

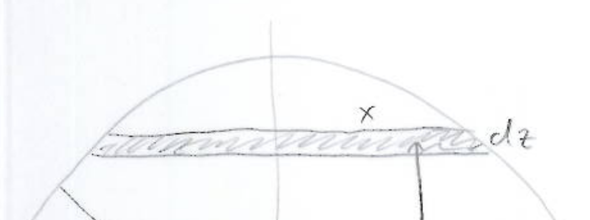
DISK: $dm = \rho \, dA = \rho \, 2\pi r dr$

$$I_{DISK} = 2\pi\rho \int_0^R r^3 dr = \frac{1}{2}\pi\rho R^4 \qquad \rho = \frac{M}{\pi R^2}$$

$$\boxed{I_{DISK} = \frac{1}{2} M R^2}$$

SPHERE, UNIFORM

$$dI = \frac{1}{2}\pi\rho\left(R^2 - z^2\right)^2 dz$$



$$x^2 = R^2 - z^2$$

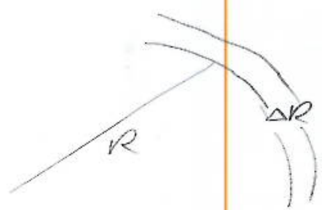
$$I_{SPHERE} = \frac{1}{2}\pi\rho\int_{-R}^{R}\left(R^2 - z^2\right)^2 dz = \frac{8}{15}\pi\rho R^5 = \boxed{\frac{2}{5} M R^2}$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

MOMENT OF INERTIA, II

$$\frac{dI}{dR} = \frac{8}{3}\pi\rho R^4$$

M of I of A SHELL:



$$I_{SHELL} = \frac{8}{3}\pi\rho R^4 \Delta R$$

$$\boxed{I_{SHELL} = \frac{2}{3} M_{SHELL} R^2}$$

$$M_{SHELL} = \rho V_{SHELL}$$

PLANET	I/MR^2
JUPITER	0.264
SATURN	0.220
URANUS	0.230
NEPTUNE	0.241
EARTH	0.3307
MOON	0.391
MARS	0.365
GANYMEDE	0.31
CALLISTO	0.3549
TITAN	0.34

GENERALIZED: $\rho(r)$

$$\boxed{I = \frac{8}{3}\pi \int_0^R \rho(r) r^4 \, dr}$$

PLANETARY INTERIORS

PLANETS: A HIGH-PRESSURE COLLECTION OF ATOMS & ELECTRONS

DERIVE A MASS-RADIUS RELATION FOR PLANETS

ENERGIES INVOLVED

$$K.E.: \; KE_{e^-} \gg KE_{p}$$

AROUND CENTER OF MASS, $P_{e^-} = P_p$ $\quad KE = \frac{P^2}{2m}$

PLANET MASS: $M_p = N_p$ ATOMS

$$N_p = \frac{M_p}{A\, m_p} \quad A = \text{AVERAGE ATOM MASS}$$

$$\# \text{ of } e^- = Z N_p$$

$$\text{VOLUME} = V_p$$

PLANETARY INTERIORS II

- e^- ARE FERMIONS, SO THEY OBEY THE EXCLUSION PRINCIPLE
- CAN'T SQUISH THEM TOGETHER
- T SO LOW IN PLANETS THAT EXCLUSION PRINCIPLE APPLIES
  - BASICALLY ZERO TEMPERATURE — KE $\not\propto$ TEMPERATURE

$$V_p = \frac{V_P}{ZN_P} = \frac{4\pi R_P^3}{3ZN_P} = \text{VOL PER } e^-$$

IF CELL IS SPHERICAL, $d = \left(\frac{3V_p}{4\pi}\right)^{1/3} = \left(\frac{1}{ZN_P}\right)^{1/3} R_P = \left(\frac{Am_p}{ZM_P}\right)^{1/3} R_P$

- WHAT IS THIS KE?

$$E_k = \frac{p^2}{2m_e} \qquad p = \frac{h}{\lambda}$$

$$\lambda \approx 2\pi d$$

$$E_k = \frac{h^2}{2m_e}\frac{1}{4\pi^2 d^2}$$

$$\sum E_k = E_{k,TOT} = E_k Z N_p$$

$$= \frac{h^2}{8\pi^2 m_e m_p^{5/3}}\frac{M_P^{5/3} Z^{5/3}}{A^{5/3} R_P^2}$$

$$d^2 = \left(\frac{Am_p}{ZM_P}\right)^{2/3} R_P^2$$

$$N_P = \frac{M_P}{Am_p}$$

$$\boxed{E_{k,TOT} \approx 9.8\times10^6 \frac{M_P^{5/3} Z^{5/3}}{A^{5/3} R_P^2}}$$

$\downarrow$

γ_K

INTERIORS III

2ND ENERGY: ELECTROSTATIC POTENTIAL

$$E_e \approx -a \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{d} = -a \frac{Z^{4/3} e^2}{4\pi\epsilon_0} \frac{M_p^{1/3}}{A^{1/3} m_p^{1/3} R_p} \qquad a \simeq 1$$

$$\sum E_e = E_{e,tot} = E_e N_p Z$$

$$\approx -a \frac{e^2}{4\pi\epsilon_0 m_p^{4/3}} \frac{M_p^{4/3} Z^{2/3}}{A^{4/3} R_p}$$

$$\boxed{E_{e,tot} \approx \underset{\gamma_e}{\underset{\downarrow}{1.16\times10^{8}}} \frac{M_p^{4/3} Z^{2/3}}{A^{4/3} R_p}}$$

3RD ENERGY: GRAVITATIONAL POTENTIAL

$$\boxed{E_{g,tot} \simeq -b \frac{GM_p^2}{R_p} \approx \underset{\gamma_g}{\underset{\downarrow}{6.0\times10^{-11}}} \frac{M_p^2}{R_p}}$$

VIRIAL THEOREM: $2KE = -PE$, SO THAT

$$2\gamma_K \frac{M_p^{5/3} Z^{5/3}}{A^{5/3} R_p^2} \approx \gamma_e \frac{M_p^{4/3} Z^{2/3}}{A^{4/3} R_p} + \gamma_g \frac{M_p^2}{R_p}$$

$$\boxed{\frac{1}{R_p} \approx \underbrace{\frac{\gamma_e}{2\gamma_K}}_{5.9} \frac{A^{1/3} Z^{2/3}}{M_p^{1/3}} + \underbrace{\frac{\gamma_g}{2\gamma_K}}_{3.0\times10^{-18}} \frac{M_p^{1/3} A^{5/3}}{Z^{5/3}}}$$

INTERIORS IV

AT LOW M_p, LEFT SIDE DOMINATES:

$$R_p \propto M_p^{1/3} \longrightarrow \text{NORMAL, UNCOMPRESSIBLE MATTER}$$

AT HIGH M_p, RIGHT SIDE DOMINATES,

$$R_p \propto M_p^{-1/3} \longrightarrow \text{RADIUS } \underline{\text{DECREASES}} \text{ WITH INCREASING MASS}$$

FIGURES – COLE BOOK
– FORTNEY / SEAGER

CENTRAL PRESSURE OF A PLANET

$$\frac{dP}{dr} \approx \frac{P_{core} - P_{surf}}{R_p - 0} \approx \frac{-P_{core}}{R_p} = -g\rho$$

$$\approx \frac{GM_p}{R_p^2}\bar{\rho}_p$$

$$\boxed{P_{core} \approx G\frac{M_p \bar{\rho}_p}{R_p}}$$

$P_{core,\,EARTH} \approx 3.4 \times 10^{11}$ Pa $= 3.4$ MBAR

$P_{core,\,JUPITER} \approx 2.5 \times 10^{12}$ Pa $= 25$ MBAR

$P_{core,\,NEPTUNE} = 4.4 \times 10^{11}$ Pa $= 4.4$ MBAR

REAL VALUES ARE $\approx 2\times$ LARGER

$P_{core,\,MOON} = 1 \times 10^{11}$ Pa $= 0.1$ MBAR