HW #2, WINTER 2014
ASTRO 118

$\boxed{2}$

A) $R = 1\ pc$ $\quad T = 30, 100\ K$

$n = 10^{10}/m^3$ $\quad$ PURE $H_2 \rightarrow \mu = 2$

TOTAL MASS OF CORE: $M = \frac{4}{3}\pi R^3 \rho$ $\rightarrow$ $M = \frac{4}{3}\pi \left(3.09 \times 10^{16}\ m\right)\left(3.34 \times 10^{-17}\ \frac{kg}{m^3}\right)$

WHERE $\rho = \underbrace{2 m_H n}$

$M \simeq 4 \times 10^{33}\ kg \simeq \boxed{2000\ M_\odot}$

$$R_{JEAN} = \left(\frac{15 kT}{4\pi G \mu m_H \rho}\right)^{\frac{1}{2}} = \left[\frac{15\left(1.38 \times 10^{-23}\ \frac{J}{K}\right)(30\ K)}{4\pi\left(6.67 \times 10^{-11}\right)(2)\left(1.67 \times 10^{-27}\ kg\right)\left(3.34 \times 10^{-17}\ \frac{kg}{m^3}\right)}\right]^{\frac{1}{2}}$$

$R_{JEAN, 30K} = 8.15 \times 10^{15}\ m = \underline{0.26\ pc}$

$R_{JEAN, 100K} = 1.49 \times 10^{16}\ m = \underline{0.48\ pc}$

SINCE THE CORE MASS IS $\sim 2000\ M_\odot$, AND THE JEANS LENGTH TO COLLAPSE IS SIMILAR TO THE CORES $\sim 1\ pc$ SIZE, THEN WHAT COLLAPSES WILL <u>NOT</u> BE A $1\ M_\odot$ STAR. IT WILL BE MUCH LARGER.

B) THE SAME METHOD AS IN PART A) IS USED

MASS OF KNOT $\simeq \underline{200\ M_\odot}$

$R_{JEANS, 30K} = 0.03\ pc$ — THE SUB-PART OF THE KNOT THAT WILL BE COLLAPSING WILL BE IN THE GENERAL RANGE OF <u>STELLAR MASSES</u>, SO A STAR WILL LIKELY FORM

$R_{JEANS, 200K} \simeq 0.07\ pc$

C) CASE B) IS THE MORE LIKELY CASE

$$t_{ff} = \sqrt{\frac{3\pi}{32 G\rho}} = \left[\frac{3\pi}{32(6.67\times10^{-11})(3.34\times10^{-15}\ \frac{kg}{m^3})}\right]^{\frac{1}{2}} = 1.15\times10^{12}\,s = 35{,}000$$

$$\approx \boxed{35{,}000 \text{ YEARS}}$$

D) INITIALLY, U IS SMALL, BUT IT BECOMES LARGE SINCE $T \rightarrow 3000\,K$

FROM THE VIRIAL THEOREM, $E = -U$, SO $|E| = |U|$. IF WE FIND U_{FINAL}, WE CAN THEN KNOW THE E EMITTED, TOO.

$$U = \frac{3}{2}\frac{M}{\mu m_H} kT \quad , \quad M = 200\,M_\odot \ \text{(FROM PART b)}$$

$$U = \frac{3}{2}\frac{4\times10^{32}\,kg}{(2)(1.67\times10^{-27})}(1.38\times10^{-23})(3000) = 7.4\times10^{39}\,J$$

$$L = \frac{\Delta E}{\Delta t} = \frac{E}{t_{ff}} = \frac{|U|}{t_{ff}} = 6.5\times10^{27}\,\frac{J}{s} \approx \boxed{17\,L_\odot}$$

AT IR WAVELENGTHS

$\Downarrow$

FROM WIENS LAW

E) $$\frac{\Omega_{GRAV}}{VOL} = \frac{-\frac{3}{5}\frac{GM^2}{R}}{\frac{4}{3}\pi R^3} = \frac{0.6\,\frac{(6.67\times10^{-11})(4\times10^{32})^2}{(3.086\times10^{15}\,m)}}{\frac{4}{3}\pi\,(3.086\times10^{15}\,m)^3} = \frac{2.07\times10^{39}}{1.23\times10^{47}} = 1.68\times10^{-8}\,\frac{J}{m^3}$$

$$E_{\substack{DENSITY\\B\text{-}FIELD}} = \frac{1}{2\mu_0}B^2 = 1.68\times10^{-8}\,\frac{J}{m^3}$$

$$B = \left[(1.68\times10^{-8})(2)(4\pi\times10^{-7})\right]^{\frac{1}{2}}$$

$$\boxed{B = 2.05\times10^{-7} \text{ TESLA}}$$

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2.2 — TEXTBOOK

$\boxed{b \geq 1 \text{ BY DEFINITION}}$

A) $M_J = \frac{4}{3}\pi \rho R_J^3$

$$\frac{1}{\alpha} M_J = \frac{4}{3}\pi \rho \left(\frac{R_J}{b}\right)^3$$

$$\frac{1}{\alpha} \cancel{M_J} = \cancel{\frac{4}{3}\pi \rho R_J^3} \frac{1}{b^3}$$

$$b^3 = \alpha \Longrightarrow \boxed{b^3 \geq \alpha \text{ FOR COLLAPSE}}$$

B) $TV^{\gamma-1}$ = CONSTANT FOR AN ADIABATIC PROCESS

$T\left(\frac{1}{\rho}\right)^{\gamma-1}$ = CONSTANT

$$T_I\left(\frac{1}{\rho_I}\right)^{\gamma-1} = T_F\left(\frac{1}{\rho_I b^3}\right)^{\gamma-1} \qquad \gamma - 1 = \frac{5}{3} - 1 = \frac{2}{3}$$

$$T_I \cancel{\rho_I^{-2/3}} = T_F \cancel{\rho_I^{-2/3}} b^{-2}$$

$$\boxed{T_F = T_I b^2}$$

$M_J \propto T^{3/2}$ (EQ 2.16), SO THAT

$$\boxed{M_J \propto b^3}$$

THE JEANS MASS GOES UP

C) $\mu_{INIT} = 1.3$ $\qquad \frac{\mu_I}{\mu_{FINAL}} = \frac{13}{6}$

$\mu_{IONIZED} = 0.6$

INITIAL: $R_{J,I} = \left(\frac{15\, kT}{4\pi G \mu m_H \rho}\right)^{1/2} = \frac{X^{1/2}}{\mu_I^{1/2} \rho^{1/2}}$

FINAL: $\frac{R_J}{b} = \frac{X^{1/2}}{\mu_F^{1/2} (\rho b^3)^{1/2}} \Longrightarrow R_J = \frac{X^{1/2}}{\mu_F^{1/2} (\rho b^3)^{1/2}} \cdot b$

COMPRESSION AFFECTS DENSITY AS WELL

2.2

c) CONTINUED:

$$\frac{\cancel{X^{\frac{1}{2}}}}{\mu_I^{\frac{1}{2}}\,\cancel{\rho^{\frac{1}{2}}}} = \frac{\cancel{X^{\frac{1}{2}}}\,b\;b^{-\frac{3}{2}}}{\mu_F^{\frac{1}{2}}\,\cancel{\rho^{\frac{1}{2}}}}$$

$$\left(\frac{\mu_F}{\mu_I}\right)^{\frac{1}{2}} = b^{-\frac{1}{2}}$$

$$\left[\left(\frac{6}{13}\right)^{\frac{1}{2}}\right]^{-2} = \boxed{b = 2.17}$$

D) $L_I \propto R_I^2 T_I^4$

$$T\left(\frac{1}{\rho}\right)^{\gamma-1} = \text{CONST. AND FROM } b) \quad \underline{T_F = T_I b^2}$$

$$L_F \propto \left(\frac{1}{b} R_I\right)^2 \left(b^2 T_I\right)^4$$

$$\frac{L_F}{L_I} = \left(\frac{1}{b}\right)^2 \left(b^2\right)^4 = \boxed{\frac{L_F}{L_I} = b^6}$$

THE CLOUD WOULD BE ABLE TO RADIATE ENERGY AWAY VERY EFFICIENTLY, WHICH WILL AID IN COLLAPSE

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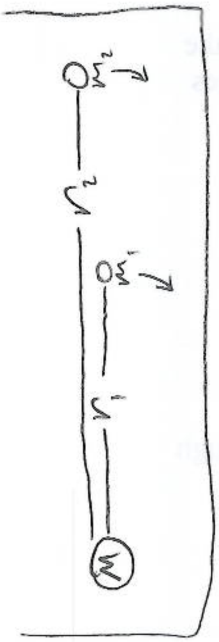
4) SHOW THAT 2.35 + 2.36 ARE TRUE:

A) $$L = (GM)^{1/2}\left(m_1 r_1^{1/2} + m_2 r_2^{1/2}\right)$$

IF $\Delta L = 0$, THEN $$\Delta L = \frac{\Delta L}{\Delta r_1}\Delta r_1 + \frac{\Delta L}{\Delta r_2}\Delta r_2 = 0$$

$$\Delta L = (GM)^{1/2}\left(m_1\left(\tfrac{1}{2}\right) r_1^{-1/2}\Delta r_1 + m_2\left(\tfrac{1}{2}\right) r_2^{-1/2}\Delta r_2\right) = 0$$

$$\boxed{m_1 r_1^{1/2}\Delta r_1 = -m_2 r_2^{-1/2}\Delta r_2}$$



B) $$E = -\frac{GM}{2}\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$\Delta E = \frac{\Delta E}{\Delta r_1}\Delta r_1 + \frac{\Delta E}{\Delta r_2}\Delta r_2 = -\frac{GM}{2}\left(\frac{-m}{r_1^2}\Delta r_1 \; \frac{m_2}{r_2^2}\Delta r_2\right)$$

BUT WE KNOW THAT $$\Delta r_2 = \frac{-r_1^{1/2} m_1 r_2^{1/2}\Delta r_1}{m_2}$$

SO THAT $$\Delta E = -\frac{GM}{2}\left[\frac{-m_1\Delta r_1}{r_1^2} + \frac{m_2 m_1 r_2^{1/2}\Delta r_1}{m_2 r_1^{1/2} r_2^2}\right]$$

$$= -\frac{GMm_1\Delta r_1}{2}\left(-\frac{1}{r_1^2} + \frac{1}{r_1^{1/2} r_2^{3/2}}\right)$$

$$= \frac{-GMm_1\Delta r_1}{2r_1^2}\left(-1 + \frac{1}{r_1^{-3/2} r_2^{3/2}}\right)$$

$$\boxed{\Delta E = \frac{-GMm_1\Delta r_1}{2r_1^2}\left[\left(\frac{r_1}{r_2}\right)^{3/2} - 1\right]}$$