HW #2, WINTER 2012
ASTRO 118

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A) $R = 1\ pc$ $T = 30, 100\ K$

$n = 10^{10}/m^3$ PURE $H_2 \rightarrow \mu = 2$

TOTAL MASS OF CORE: $M = \frac{4}{3}\pi R^3 \rho \rightarrow M = \frac{4}{3}\pi\left(3.09\times10^{16}\ m\right)\left(3.34\times10^{-17}\ \frac{kg}{m^3}\right)$

WHERE $\rho = \underbrace{2 m_H n}$

$M \simeq 4\times10^{33}\ kg \simeq \boxed{2000\ M_\odot}$

$$R_{JEAN} = \left(\frac{15 kT}{4\pi G \mu m_H \rho}\right)^{\frac{1}{2}} = \left[\frac{15\left(1.38\times10^{-23}\ \frac{J}{K}\right)(30\ K)}{4\pi\left(6.67\times10^{-11}\right)(2)\left(1.67\times10^{-27}\ kg\right)\left(3.34\times10^{-17}\ \frac{kg}{m^3}\right)}\right]^{\frac{1}{2}}$$

$R_{JEAN,\ 30K} = 8.15\times10^{15}\ m = \underline{0.26\ pc}$

$R_{JEAN,\ 100K} = 1.49\times10^{16}\ m = \underline{0.48\ pc}$

SINCE THE CORE MASS IS $\sim 2000\ M_\odot$, AND THE JEANS LENGTH TO COLLAPSE IS SIMILAR TO THE CORES $\sim 1\ pc$ SIZE, THEN WHAT COLLAPSES WILL <u>NOT</u> BE A $1\ M_\odot$ STAR. IT WILL BE MUCH LARGER.

B) THE SAME METHOD AS IN PART A) IS USED

MASS OF KNOT $\simeq 200\ M_\odot$

$R_{JEANS,\ 30K} = 0.03\ pc$

$R_{JEANS,\ 200K} \approx 0.07\ pc$

THE SUB-PART OF THE KNOT THAT WILL BE COLLAPSING WILL BE IN THE GENERAL RANGE OF STELLAR MASSES

C) CASE B) IS THE MORE LIKELY CASE

$$t_{ff} = \sqrt{\frac{3\pi}{32 G\rho}} = \left[\frac{3\pi}{32(6.67\times10^{-11})(3.34\times10^{-15}\ \frac{kg}{m^3})}\right]^{\frac{1}{2}} = 1.15\times10^{12}\ s = 35{,}000$$

$$\approx \boxed{35{,}000\ \text{YEARS}}$$

D) INITIALLY, U IS SMALL, BUT IT BECOMES LARGE SINCE $T \rightarrow 3000K$

FROM THE VIRIAL THEOREM, $E = -U$, SO $|E| = |U|$. IF WE FIND U_{FINAL}, WE CAN THEN KNOW THE E EMITTED, TOO.

$$U = \frac{3}{2}\frac{M}{\mu m_H} kT \quad , \quad M = 200\ M_\odot$$

$$U = \frac{3}{2}\frac{4\times10^{32}\ kg}{(2)(1.67\times10^{-27})}(1.38\times10^{-23})(3000) = 7.4\times10^{39}\ J$$

$$L = \frac{\Delta E}{\Delta t} = \frac{E}{t_{ff}} = \frac{|U|}{t_{ff}} = 6.5\times10^{27}\ \frac{J}{s} \approx \boxed{17\ L_\odot}$$

AT IR WAVELENGTHS

E) $$\frac{\Omega_{GRAV}}{VOL} = \frac{\frac{-3}{5}\frac{GM^2}{R}}{\frac{4}{3}\pi R^3} = \frac{0.6\frac{(6.67\times10^{-11})(4\times10^{32})^2}{(3.086\times10^{15}\ m)}}{\frac{4}{3}\pi(3.086\times10^{15}\ m)^3} = \frac{2.07\times10^{39}}{1.23\times10^{47}} = 1.68\times10^{-8}\ \frac{J}{m^3}$$

$$E_{\substack{DENSITY\\B\text{-}FIELD}} = \frac{1}{2\mu_0}B^2 = 1.68\times10^{-8}\ \frac{J}{m^3}$$

$$B = \left[(1.68\times10^{-8})(2)(4\pi\times10^{-7})\right]^{\frac{1}{2}}$$

$$\boxed{B = 2.05\times10^{-7}\ \text{TESLA}}$$

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2.2 — TEXTBOOK

$\boxed{b \geq 1 \text{ BY DEFINITION}}$

A) $M_J = \frac{4}{3}\pi \rho R_J^3$

$$\frac{1}{\alpha} M_J = \frac{4}{3}\pi \rho \left(\frac{R_J}{b}\right)^3$$

$$\frac{1}{\alpha} \cancel{M_J} = \cancel{\frac{4}{3}\pi \rho R_J^3} \frac{1}{b^3}$$

$$b^3 = \alpha \longrightarrow \boxed{b^3 \geq \alpha \text{ FOR COLLAPSE}}$$

B) $TV^{\gamma-1}$ = CONSTANT FOR AN ADIABATIC PROCESS

$$T\left(\frac{1}{\rho}\right)^{\gamma-1} = \text{CONSTANT}$$

$$T_I\left(\frac{1}{\rho_I}\right)^{\gamma-1} = T_F\left(\frac{1}{\rho_I b^3}\right)^{\gamma-1} \qquad \gamma - 1 = \frac{5}{3} - 1 = \frac{2}{3}$$

$$T_I \cancel{\rho_I^{-2/3}} = T_F \cancel{\rho_I^{-2/3}} b^{-2}$$

$$\boxed{T_F = T_I b^2}$$

(EQ 2.16)

$M_J \propto T^{3/2}$, SO THAT

$$\boxed{M_J \propto b^3}$$

THE JEANS MASS GOES UP

C) $\mu_{INIT} = 1.3$ $\qquad \frac{\mu_I}{\mu_{FINAL}} = \frac{13}{6}$

$\mu_{IONIZED} = 0.6$

INITIAL: $$R_{J,I} = \left(\frac{15 kT}{4\pi G \mu m_H \rho}\right)^{1/2} = \frac{X^{1/2}}{\mu_I^{1/2}\rho^{1/2}}$$

FINAL: $$\frac{R_J}{b} = \frac{X^{1/2}}{\mu_F^{1/2}(\rho b^3)^{1/2}} \Rightarrow R_J = \frac{X^{1/2}}{\mu_F^{1/2}(\rho b^3)^{1/2}} \cdot b$$

COMPRESSION AFFECTS DENSITY AS WELL

2.2

c) CONTINUED:

$$\frac{\cancel{x^{1/2}}}{\mu_I^{1/2}\,\cancel{\rho^{1/2}}} = \frac{\cancel{x^{1/2}}\,b\,b^{-3/2}}{\mu_F^{1/2}\,\cancel{\rho^{1/2}}}$$

$$\left(\frac{\mu_F}{\mu_I}\right)^{1/2} = b^{-1/2}$$

$$\left[\left(\frac{6}{13}\right)^{1/2}\right]^{-2} = \boxed{b = 2.17}$$

D) $L_I \propto R_I^2 T_I^4$

$T\left(\frac{1}{\rho}\right)^{\gamma-1}$ = CONST. AND FROM b) $\underline{T_F = T_I b^2}$

$$L_F \propto \left(\frac{1}{b} R_I\right)^2 \left(b^2 T_I\right)^4$$

$$\frac{L_F}{L_I} = \left(\frac{1}{b}\right)^2 \left(b^2\right)^4 = \boxed{\frac{L_F}{L_I} = b^6}$$

THE CLOUD WOULD BE ABLE TO RADIATE ENERGY AWAY VERY EFFICIENTLY, WHICH WILL AID IN COLLAPSE

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PRESSURE SUPPORT GRAVITY

A) $\frac{dP}{dr}$ VS. $\frac{-GM\rho}{r^2}$

ASSUME ISOTHERMAL, UNIFORM CONTRACTION

(IDEAL GAS)

$$\frac{dP}{dr} \sim \frac{P}{r} \sim \frac{\rho kT}{r} \sim \frac{r^3}{r} \sim \boxed{r^2}$$

$$\frac{-GM\rho}{r^2} \sim \frac{r^3}{r^2} \sim \boxed{r}$$

$$\text{RATIO} = \frac{\frac{dP}{dr}}{\frac{-GM\rho}{r^2}} \sim \frac{r^2}{r} \sim \boxed{r}$$

AS $r \to 0$, THE RATIO OF THESE TWO TERMS ALSO GOES TO ZERO, MEANING THAT THE PRESSURE SUPPORT WILL BECOME EVEN WEAKER AS THE CLOUD COLLAPSES.

B) NOW $\frac{dP}{dr} \sim \frac{P}{r} \sim \frac{\rho kT}{r} \sim \frac{r^3 r^{-a}}{r} \sim r^2 r^{-a}$

$$\text{RATIO} = \frac{\frac{dP}{dr}}{\frac{-GM\rho}{r^2}} \sim \frac{r^2 r^{-a}}{r} \sim r\, r^{-a}$$

IF $\boxed{a > 1}$ THEN AS $r \to 0$, THE RATIO WOULD INCREASE WHICH WOULD HELP TO STOP THE COLLAPSE. SO IF $T \propto r^{-1}$ (OR LARGER), FREE-FALL CONTRACTION WOULD BE HALTED.