ASTRO 118
HW # 3, WINTER 2012

2) BOOK # 3.3

~~$a = \frac{v^2}{R}$~~

~~$a_{GRAV} = \frac{GM}{R^2} = \frac{v^2}{R} \Rightarrow v = \sqrt{\frac{GM}{R}}$~~

$$P^2 = \frac{4\pi^2 a^3}{G(M_1 + m_2)} \approx \frac{4\pi^2 a^3}{GM}$$

TWO PARENT BODIES AT SUN($M_\odot$) AND PLANET(M_p)

$$\frac{4\pi^2 a^3}{GM_*} = \frac{4\pi^2 r_H^3}{GM_p} \Rightarrow \boxed{r_H = a\left(\frac{M_p}{M_\odot}\right)^{\frac{1}{3}}}$$

3)

A) $L = (GM)^{1/2}(m_1 r_1^{1/2} + m_2 r_2^{1/2})$

IF $\Delta L = 0$, THEN $\Delta L = \frac{\Delta L}{\Delta r_1}\Delta r_1 + \frac{\Delta L}{\Delta r_2}\Delta r_2 = 0$

$$\Delta L = (GM)^{1/2}\left(m_1\left(\tfrac{1}{2}\right)r_1^{-1/2}\Delta r_1 + m_2\left(\tfrac{1}{2}\right)r_2^{-1/2}\Delta r_2\right) = 0$$

$$\boxed{m_1 r_1^{-1/2}\Delta r_1 = -m_2 r_2^{-1/2}\Delta r_2}$$

B) $E = \frac{-GM}{2}\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$

$$\Delta E = \frac{\Delta E}{\Delta r_1}\Delta r_1 + \frac{\Delta E}{\Delta r_2}\Delta r_2 = -\frac{GM}{2}\left(\frac{-m_1}{r_1^2}\Delta r_1 - \frac{m_2}{r_2^2}\Delta r_2\right)$$

3) B)

BUT WE KNOW THAT $\Delta r_2 = \dfrac{-r_1^{-1/2} m_1 r_2^{1/2} \Delta r_1}{m_2}$

SO THAT $\Delta E = -\dfrac{GM}{2}\left(\dfrac{-m_1 \Delta r_1}{r_1^2} + \dfrac{\cancel{m_2} m_1 r_2^{1/2} \Delta r_1}{\cancel{m_2} r_1^{1/2} r_2^2}\right)$

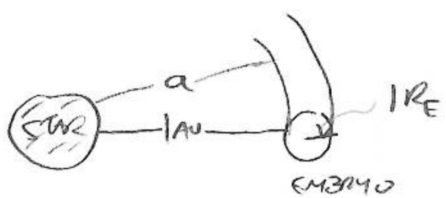
$$= \frac{-GMm_1 \Delta r_1}{2}\left(\frac{-1}{r_1^2} + \frac{1}{r_1^{1/2} r_2^{3/2}}\right)$$

$$= \frac{-GMm_1 \Delta r_1}{2 r_1^2}\left(-1 + \frac{1}{r_1^{-3/2} r_2^{3/2}}\right)$$

$$\boxed{\Delta E = \frac{-GMm_1 \Delta r_1}{2 r_1^2}\left(\left(\frac{r_1}{r_2}\right)^{3/2} - 1\right)}$$

4) THIS WAS MEANT TO BE A PROBLEM W/O INTEGRALS

A)



$\sigma = \pi R_E^2$

$\dfrac{\text{VOLUME SWEPT}}{\text{TIME}} = \sigma\,(m^2)\; v\left(\dfrac{m}{s}\right)$

$\dfrac{\text{VOL}}{\text{YEAR}} = \pi R_E^2 \; \dfrac{2\pi a}{1 \text{ YEAR}}$

$\dfrac{\text{MASS}}{\text{YEAR}} = \dfrac{\text{VOL}}{\text{YEAR}}(\rho) = \dfrac{2\pi^2 R_E^2 a \left(10^{-9} \frac{g}{cm^3}\right) cm^3}{\text{YEAR}} = \boxed{(2.95 \times 10^5) R_E^2 \; \frac{g}{\text{YEAR}}}$

$a = 1.49 \times 10^{13}$ cm

4) B) $\rho_{EMBRY} = 3 \frac{g}{cm^3}$

$M_E = \frac{4}{3}\pi R_E^3 \rho_E$

SET $M_{GAS} = M_E$

$\frac{4}{3}\pi R_E^{\cancel{3}} \rho_E = 2.95 \times 10^5 (\cancel{R_E^2})$ g IN GRAMS & CENTIMETERS

$R_E \approx 23{,}500$ cm

$\boxed{R_E \approx 235 \text{ m}}$