Astro 118
HW#5
Winter 2014

① SHOW THAT $\frac{L_P}{L_*} = \frac{R_P^2}{4d^2}$

$$f_{EMIT} = \frac{L_{EMIT}}{4\pi D^2} = \frac{\cancel{4\pi} R_P^2 \sigma T_{EQ}^4}{\cancel{4\pi} D^2} = \frac{R_P^2 \sigma (1-A_B) L_*}{16\pi d^2 \sigma D^2}$$

T_{EQ}

$$f_{REFLECT} = \frac{L_* R_P^2 A_g q}{16\pi d^2 D^2}$$

- STELLAR FLUX FALLS OFF AS $\frac{1}{4\pi d^2}$
- FLUX TO OBSERVER FALLS OFF AS $\frac{1}{4\pi D^2}$

$$f_P = f_{EMIT} + f_{REFL} = \frac{L_* R_P^2 A_g q + R_P^2 (1-A_B) L_*}{16\pi d^2 D^2}$$

$$L_P = (f_P)(4\pi D^2) = \frac{L_* R_P^2 (A_g q + 1 - A_B)}{4d^2}$$

$$\frac{L_P}{L_*} = \frac{R_P^2}{4d^2}(A_g q + 1 - A_B)$$

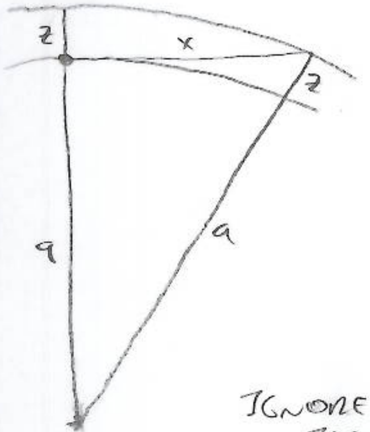
- $0 < q < 1$, GENERALLY

  $q = 1$ IS "FULL PHASE"
- A_B DEPENDS ON THE SPECTRUM OF THE PARENT STAR.
- A_g DOES NOT.

IF, AND ONLY IF, WE ASSUME $q = 1$ (OK, FINE) AND $A_g = A_B$ (NOT COOL), YOU GET

$$\boxed{\frac{L_P}{L_*} = \frac{R_P^2}{4d^2}}$$

② HORIZONTAL COLUMN DENSITY



$$n = n_o e^{-z/H} \xrightarrow{\text{THEREFORE}} \boxed{n = n_o e^{-x^2/2aH}}$$

WE KNOW THAT:

$$a^2 + x^2 = (a+z)^2$$

$$\cancel{a^2} + x^2 = \cancel{a^2} + z^2 + 2az$$

IGNORE z^2 TERM

$$z^2 \ll x^2$$

$$z^2 \ll 2az$$

$$\boxed{z = \frac{x^2}{2a}}$$

WE NOW HAVE n AS A FUNCTION OF x!

TOTAL COLUMN N FROM HORIZON TO HORIZON IS:

$$N_H = \int_{-\infty}^{\infty} n_o e^{-x^2/2aH}\, dx$$

$$N_H = n_o \sqrt{2\pi a H}$$

LOOKING IN ONE DIRECTION

$$\boxed{N_H = n_o \sqrt{\frac{\pi a H}{2}}}$$

③ A) T_{Eq} IS DEFINED BY AN ENERGY BALANCE OF ABSORBING ENERGY OVER πR_p^2 AND RADIATING IT OVER $4\pi R_p^2$ OR $2\pi R_p^2$. IN CLASS WE ASSUME ABSORBED ENERGY IS ALSO TRANSPORTED TO THE NIGHT SIDE, SO $4\pi R_p^2$ IS CORRECT FOR THE EMITTING SURFACE AREA. HOWEVER, IF THE NIGHT SIDE NEVER "SEES" THIS ENERGY, THEN IT CAN BE EMITTED ON THE DAY SIDE ONLY, MAKING $2\pi R_p^2$ APPROPRIATE.

$\frac{\text{ENERGY}}{s}$ ABSORBED: $\boxed{\frac{L_*}{4\pi d^2}(1-A_B)\pi R_p^2} = \frac{\text{ENERGY}}{s}$ EMITTED : $\boxed{2\pi R_p^2 \sigma T_{Eq}^4}$

$$T_{Eq} = \left[\frac{(1-A_B)L_*}{8\pi d^2 \sigma}\right]^{1/4} = \boxed{2^{1/4} \text{ LARGER}} \text{ THAN "FAST" ROTATOR CASE}$$

③ B) $\frac{E}{s}$ ABSORBED $= \frac{L_*}{4\pi d^2} \pi R_p^2 (1-A_B) = \frac{(1-A_B) L_*}{4} \left(\frac{R_p}{d}\right)^2$, BUT NOT RELEVANT HERE!

$$\frac{E}{s}\ \text{EMITTED} = \overbrace{2\pi R_p^2 \sigma T_{DAY}^4}^{\text{DAY}} + \overbrace{2\pi R_p^2 \sigma T_{NIGHT}^4}^{\text{NIGHT}}$$

$$\beta = \frac{\cancel{2\pi R_p^2 \sigma} T_N^4}{\cancel{2\pi R_p^2 \sigma} T_N^4 + \cancel{2\pi R_p^2 \sigma} T_D^4}$$

$0 \le \beta \le 1$

$\beta = \frac{1}{2}$ IF $T_N = T_D$

$\beta = 1$ IF $T_N \gg T_D$

$\beta = 0$ IF $T_N \ll T_D$

$$\beta = \frac{T_N^4}{T_N^4 + T_D^4} = \frac{273^4}{273^4 + 373^4} = \boxed{0.223}$$

4) Will be updated later in the week, sorry!

2) Calculate the expected increase in the global average temperature of the Earth at a full Moon (when we see the moon at full phase) compared to a new Moon (when we see no light from the moon). Neglect eclipses and orbital eccentricity. The radius of the Moon is 1700 km, the semi-major axis of the Moon's orbit is 384,000 km, and its geometric albedo in a very wide visible bandpass is 0.10. Earth's Bond albedo is 0.3. *(15 points)*

$$T_{EQ} = \left[\frac{(1-A_B) L_\star}{16\pi d^2 \sigma}\right]^{1/4} = \left[\frac{(1-A_B) F_{INC}}{4\sigma}\right]^{1/4}$$

F_{INC} = INCIDENT FLUX

$F_{INC} = \frac{L_\star}{4\pi d^2} = 1370 \frac{W}{m^2}$

$\boxed{T_{EQ,\ NORMAL} = 255.00218}$ FROM PUTTING IN THE NUMBERS

$\boxed{\rightarrow \text{NEW MOON CASE}}$ $\boxed{F_{INC} = 1370 \frac{W}{m^2}}$

T_{EQ}, WITH FULL MOON

$$f_{REFLE} = \frac{1}{4\pi D^2} A_g \pi R_{MOON}^2 \boxed{\frac{L_\star}{4\pi d^2}}$$

$$= \frac{1}{4\pi (380{,}000 \times 10^3)^2} (0.1)(\pi)(1700 \times 10^3)^2 \left(1370 \frac{W}{m^2}\right)$$

$$\boxed{f_{REFLEC} = 0.00068 \frac{W}{m^2}}$$

T_{EQ}, FULL MOON: $F_{INC} = 1370.00068 \frac{W}{m^2}$

$$\boxed{T_{EQ} = 255.00221\ K}$$

$\boxed{0.00003K \text{ DIFFERENCE}}$

	M(M⊕)	R(R⊕)	COMPOSITION
(5) 55 CAN e	8.6	2.0	100 % ROCK
GJ 1214b	6.6	2.7	AROUND 100 % WATER, OR, LESS WATER, BUT WITH H/He
HD 97658b	6.4	2.9	CAN'T BE PURE WATER — NEEDS SOME H/He ATMOSPHERE
KEPLER 10b	4.5	1.4	50% ROCK, 50 % IRON
KEPLER 11b	4.3	2.0	50% WATER, 50% ROCK

DEPENDING ON M & R, YOUR RESULTS MAY VARY