Astro 118 – Physics of Planetary Systems
Problem Set #6, Winter 2014
Due in class on Friday, March 14, 2014

1) Read the short *Science* article "Exoplanet Habitability" by Sara Seager. In 1/2 to 2/3 of a page, describe what a biosignature gas is, why some biosignature gases may not be observable, and the possibility of biosignature gases as "false positives" sign for life. http://www.ucolick.org/~jfortney/classes/118/Seager13.pdf

2) From the definition of optical depth, τ, derive a variant on the simple equation of hydrostatic equilibrium where τ is the vertical coordinate, rather than height, z. Now, using that and our $T(\tau)$ equation, $T^4 = \frac{3}{4} T_{eq}^4 (\tau + 2/3)$, find the atmospheric pressure (in bars) at which the temperature T = 300 K, if T_{eq}=100 K (what one might expect at around 5 AU from the Sun), gravity=10 m/s^2, and opacity κ = 0.01 m^2/g.

3) Problem 9.5 from the textbook.

4) This problem is a generalization of the scale height (H) concept, where temperature changes with height, rather than being constant. Over a region where the temperature changes linearly with height and where $g(z)$ is a constant, show that the pressure, density, scale height, and radius are related by:

$$\frac{p}{p_o} = \left(\frac{H}{H_o}\right)^{-1/\beta} \quad \text{and} \quad \frac{n}{n_o} = \left(\frac{H}{H_o}\right)^{-(1+\beta)/\beta}$$

where $\beta = dH/dz$ and p_o, n_o, and H_o are the values at a starting distance z_o (for instance, at the surface).