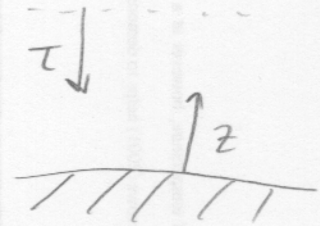
2) $\frac{dP}{dz} = -\rho g$ $\qquad d\tau = \kappa \rho \, ds$, GENERALLY

$d\tau = -\kappa \rho \, dz$, IF z IS + UPWARDS AND τ IS + DOWNWARDS



$$\frac{dP}{d\tau} = \frac{-\rho g}{-\kappa \rho}$$

$$\boxed{\frac{dP}{d\tau} = \frac{g}{\kappa}}$$

$$T^4 = \frac{3}{4} T_{EQ}^4 \left(\tau + \frac{2}{3}\right)$$

$$300^4 = \frac{3}{4} 100^4 \left(\tau + \frac{2}{3}\right)$$

$$108 = \tau + \frac{2}{3}$$

$$\boxed{\tau = 107.33}$$

$$\frac{dP}{d\tau} = \frac{g}{\kappa}$$

$$\int dP = \int \frac{g}{\kappa} d\tau$$

$$P = \frac{g}{\kappa} \tau$$

$$P = \frac{10 \frac{m}{s^2}}{0.01 \frac{m^2}{g}} \tau$$

$$P = 107.33 \times 10^3 \text{ Pa}$$

$$\boxed{P = 1.07 \text{ BARS}}$$

OK, I JUST REALIZED THAT INSTEAD OF $\kappa = 0.01 \frac{m^2}{kg}$ I WROTE $0.01 \frac{m^2}{g}$, WHICH IS NOT WHAT I INTENDED. SO I GAVE FULL CREDIT TO A LOT OF ANSWERS....

3) $a_{lock} = 0.027 \left(\frac{P_o t}{Q}\right)^{1/6} M_*^{1/3}$ (cgs)

$t = 4$ Gyr $= 1.26 \times 10^{17}$ s

$P_o = 13.5$ Hours $= 48,600$ s

$Q = 10$ or 100

$M_* = 0.2\ M_\odot = 3.98 \times 10^{32}$ g

A) $\boxed{a_{lock} = 5.8 \times 10^{12} \text{ cm} = 0.38 \text{ AU} \quad Q=10}$

B) $\boxed{a_{lock} = 3.9 \times 10^{12} \text{ cm} = 0.26 \text{ AU} \quad Q=100}$

C) HABITABLE ZONE

WE WANT $T_g = 273 - 373$ K

$T_g^4 = T_{EQ}^4 \left(1 + \frac{3}{4}\tau_g\right)$ $\quad \tau_g = 0.83$ FOR EARTH-LIKE ATMOSPHERE

GROUND TEMP. RANGE FOR LIQUID WATER

$T_g = 273\text{K} \rightarrow \boxed{T_{EQ} = 242 \text{ K}} \rightarrow$ COLDEST

$T_g = 373\text{K} \rightarrow \boxed{T_{EQ} = 330 \text{ K}} \rightarrow$ HOTTEST

$$a = \left[\frac{(1-A_B) L_*}{16\pi\sigma T_{EQ}^4}\right]^{1/2}$$

$A_B = 0.3$, $\sigma = 5.67 \times 10^{-5}$ in cgs

$L_* = 0.005\ L_\odot = 1.91 \times 10^{31}$ erg/s

$$a_{FAR} = \left[\frac{(1-0.3)\ 1.91\times10^{31}}{16\pi\ \sigma (242)^4}\right]^{1/2} = 1.17 \times 10^{12} \text{ cm} = \boxed{0.078 \text{ AU}}$$

$$a_{CLOSE} = \left[\frac{(1-0.3)\ 1.91\times10^{31}}{16\pi\ \sigma (330)^4}\right] = 6.28 \times 10^{11} \text{ cm} = \boxed{0.042 \text{ AU}}$$

ALL HABITABLE ZONE PLANETS AROUND THIS STAR WILL BE TIDALLY LOCKED. THIS COULD BE A BAD THING, AS THE ATMOSPHERE COULD FREEZE OUT ON THE NIGHT SIDE. ALSO, THERE COULD BE NO PHOTOSYNTHESIS ON THE NIGHT SIDE.

4) $T = (\text{constant})(r)$, so that $\frac{dT}{T} = \frac{dr}{r}$

A) We know that hydrostatic equilibrium holds:

$$\frac{dP}{dr} = -g\rho \quad , \quad \text{and ideal gas: } P = \frac{\rho}{\mu m_H} kT$$

$$\Downarrow \qquad H = \frac{kT}{\mu m_H g} \qquad \beta = \frac{dH}{dr}$$

$$\frac{dP}{dr} = -\frac{P \mu m_H}{kT} g$$

$$\frac{dP}{P} = -\frac{dr}{H} = \frac{-1}{\beta}\frac{dH}{H} \Rightarrow \underbrace{\int_{P_0}^{P} \frac{dP}{P} = -\frac{1}{\beta}\int_{H_0}^{H} \frac{dH}{H}}$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{1}{\beta}\ln\left(\frac{H}{H_0}\right)$$

$$\frac{P}{P_0} = e^{-\frac{1}{\beta}\ln\left(\frac{H}{H_0}\right)} = e^{\ln\left(\frac{H}{H_0}\right)^{-\frac{1}{\beta}}}$$

$$\boxed{\frac{P}{P_0} = \left(\frac{H}{H_0}\right)^{-\frac{1}{\beta}}}$$

B) Since $H = (\text{constant})(T)$, then $\frac{dH}{H} = \frac{dT}{T}$

$P = nkT$, so that:

$$\frac{dP}{P} = \frac{dn}{n} + \frac{dT}{T}$$

$$\frac{dn}{n} = \frac{dP}{P} - \frac{dH}{H}$$

4) 3) CONT...

$$\frac{dn}{n} = \underbrace{\frac{-dH}{\beta H}}_{\frac{d\rho}{\rho}} - \frac{dH}{H} = -\left(\frac{1}{\beta}+1\right)\frac{dH}{H}$$

$$\int_{n_0}^{n} \frac{dn}{n} = -\left(\frac{1}{\beta}+1\right)\int_{H_0}^{H} \frac{dH}{H}$$

$$\ln\left(\frac{n}{n_0}\right) = -\left(\frac{1+\beta}{\beta}\right)\ln\left(\frac{H}{H_0}\right)$$

$$\frac{n}{n_0} = e^{-\left(\frac{1+\beta}{\beta}\right)\ln\left(\frac{H}{H_0}\right)}$$

$$\boxed{\frac{n}{n_0} = \left(\frac{H}{H_0}\right)^{-\left(\frac{1+\beta}{\beta}\right)}}$$