Astronomy 220A
Problem Set #4, Due Monday, November 22, 2010

1) We can make a rough estimate of how much the envelope of a red giant should expand as a result of the contraction of its core based on conservation of energy. Consider a star of mass M and initial radius R, with a core of mass M_c and radius R_c. We will focus on the phase when there are no nuclear reactions in the helium core, and when hydrogen burning in the shell above either does not occur or occurs too slowly to make a significant contribution to the energy budget.

a) Compute the gravitational binding energy and the total energy of the core, assuming that the electrons are non-relativistic and that radiation pressure is insignificant. Your answer will involve a constant of order unity that depends on the density distribution within the core, which you can simply write as α_c.

b) Compute the gravitational binding energy and the total energy of the envelope by itself. Again, you will have a constant of order unity, which you can write as α_e. You may neglect changes in the structure of the envelope due to the gravitational pull of the core.

c) Compute the potential energy associated with gravitational attraction between the core and the envelope, assuming $R >> R_c$. This will involve yet another constant α_{ce}.

d) Now suppose that the core contracts from its initial radius $R_{c,0}$ to a smaller radius $R_{c,1}$. This causes the envelope to expand from its initial radius R_0 to a new radius R_1. Assuming that the total energy content of the star is conserved in the process, compute R_1/R_0 in terms of R_0, $R_{c,0}$, $R_{c,1}$, M_c, and M. For convenience, you may set all the factors equal to 1.

e) Evaluate the ratio of the new and old radii, R_1/R_0, for a core near the Schonberg-Chandrasekhar limit, $M_c/M \approx 0.1$ with an initial radius $R_{c,0}/ R_0 = 0.02$ that shrinks by a factor of 2, so that $R_{c,1}/ R_0 = 0.01$.

2) Determine the ratio of the number of photons to the number of neutrinos emitted by the Sun per second. Recall that during the creation of one alpha particle that about 26.7 MeV of energy is released, and neutrinos carry away only about 2% of this energy. Assume that the Sun emits that energy as a blackbody when you estimate its average energy per (blackbody) photon.

3) In class we discussed homologous contraction and derived the how such a cloud contracts with time. Let's assume we have dense core of giant molecular cloud composed of 100% H_2 at an initial density ρ_0=3 x 10^{14} g/cm^3, that is constant throughout the core.

a) Calculate the free-fall time, t_{ff}.

b) Plot r/r_0 and $\log_{10}(\rho/\rho_0)$ at a function of time, t.

4) Integrate the 4 structure equations from the surface inward to the chosen fitting point and outward from the center to the fitting point. What are the fitting point values of (inward and outward) of P, T, r, and l? For this problem, only the data from your first trial integration are required. However, you should develop the procedure for adjusting the four input parameters to get a better fit. Check out the stellar model assignment and the Numerical Recipes shootf routine.