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ASTRO 220A,

HW #3 SOLUTIONS

2) OUR FORMULAE FOR THE TEMPERATURE DEPENDENCE OF THE REACTIONS, ν, AND THE ENERGY AT THE GAMOW PEAK, E_0, CAN BE FOUND IN K&W, AND ARE:

$$W = Z_j^2 Z_k^2 \frac{A_j A_k}{A_j + A_k}$$

$$E_0 = 5.665\, W^{1/3}\, T_7^{2/3}$$

$$\nu = 6.574\, W^{1/3}\, T_7^{-1/3} - \frac{2}{3}$$

$$\langle \sigma v \rangle = \langle \sigma v \rangle_0 \left(\frac{T}{T_0}\right)^{\nu}$$

A) $p + p \longrightarrow W = \frac{1}{2}$

AT $T_6 = 10$, $\boxed{E_0 = 4.50 \text{ keV} \quad \nu = 4.55}$

AT $T_6 = 15$, $\boxed{E_0 = 5.89 \text{ keV} \quad \nu = 3.89}$

B) ${}^7Be(p,\gamma)^8B$

$$W = 4^2 \cdot 1^2 \left(\frac{(7)(1)}{7+1}\right) = 14$$

AT $T_6 = 15$, $\boxed{E_0 = 17.89 \text{ keV} \quad \nu = 13.17}$

2) c) $^{14}N(p,\gamma)^{15}O$

$$W = 7^2 \cdot 1^2 \frac{(14)(1)}{(14+1)} = 45.73$$

At $T_6 = 15$, $\boxed{E_0 = 26.55 \text{ keV} \quad \tau = 19.87}$

At $T_6 = 25$, $\boxed{E_0 = 37.31 \text{ keV} \quad \tau = 16.65}$

3) The minimum temperature required to fuse hydrogen is T~4 × 10^6 K. Now consider a fully convective, fully ionized, solar composition object at the star/brown dwarf dividing line. Let's assume that for fusion to occur, the object can't be supported by degeneracy, i.e., gas pressure must be more important that degeneracy pressure. With this condition, what is the transition mass between stars and brown dwarfs? How does the transition mass change at Z=0.0 object?

I GAVE FULL CREDIT FOR A LOT OF ANSWERS. THE POINT THAT MAKES THIS EASIER IS TO REMEMBER THAT A FULLY CONVECTIVE STAR IS AN $n = \frac{3}{2}$ POLYTROPE. IF YOU USE $n=1$ (FOR BROWN DWARFS) THAT WORKED OUT O.K. TOO.

$$\bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3} \qquad \frac{\rho_c}{\bar{\rho}} = 5.99 \text{ FOR } n = \tfrac{3}{2} \text{ POLYTROPE}$$

TRANSITION: $P_{GAS} = P_{DEGENERACY}$

$$\frac{\rho}{\mu m_p} kT \qquad\qquad 1.003 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{\frac{5}{3}}$$

$$\rho_c = \mu_e^{\,5/2} \left(\frac{kT}{\mu m_p \; 1.003 \times 10^{13}}\right)^{3/2}$$

$$\mu = 0.6$$
$$\mu_e = \frac{2}{1+X} \approx 1.2$$

$$\rho_c \approx 7.25 \times 10^{-8} \left(T^{3/2}\right)$$

$$T \sim 4 \times 10^6 \text{ K}$$

$$\boxed{\rho_c \approx 580 \ \tfrac{g}{cm^3}}$$

THIS YIELDS $M = \bar{\rho}\,\frac{4}{3}\pi R^3$

$$M = \left(\frac{\rho_c}{5.99}\right)\frac{4}{3}\pi R^3 \simeq \boxed{405\ R^3}$$

FOR A POLYTROPE, $R = r_n \xi_1$

$$R = \underbrace{\left(\frac{(n+1) P_c}{4\pi G \rho_c^2}\right)^{1/2}}_{r_n} \xi_1$$

$\xi_1 \approx 3.65$ FOR $n = \frac{3}{2}$

$$R = \left[\frac{(n+1) \frac{\cancel{\rho_c}}{\mu m_p} kT}{4\pi G \rho_c^{\cancel{2}}}\right]^{1/2} (3.65)$$

$$= \left[\frac{\left(\frac{5}{2}\right)}{4\pi G} \frac{kT}{\mu m_p \rho_c}\right]^{1/2} (3.65)$$

$$= \left(3.98 \times 10^{14} \frac{T}{\rho_c}\right)^{1/2} (3.65)$$

$$R = 6.0 \times 10^{9} \text{ cm}$$

$$M = 405 R^3 \approx 8.9 \times 10^{31} \text{ g} \approx \boxed{0.045\ M_\odot}$$

AT $Z=0$, $\mu \downarrow$, SO $\rho_c \uparrow$, SO $\boxed{M \uparrow}$

"REAL" VALUE IS $\boxed{0.075\ M_\odot}$