

MODELING IRRADIATED PLANETS

W

LONG HISTORY IN THE SOLAR SYSTEM

- ALL TERRESTRIAL PLANETS, TITAN

MAIN POINTS FROM LECTURE

PRE-DATA ERA = { SEAGER + CASSELAN, 1998, 2000
GUK. ET AL., 2000
BARBAN ET AL., 2001
SUDARSKY ET AL., 2000, 2003

SPITZER ERA = { FURNEY ET AL. | 2 PLANETS
SEAGER ET AL. | 3 DATA POINTS
SWANSON ET AL. | 4 PAPERS
BARMAN ET AL. | 2005

A BIG MESS SINCE THEN!

PROBLEMS WITH DIRECTLY APPLYING SOLAR SYSTEM WORK TO EXOPLANETS

SOLAR SYSTEM

- MUCH GREATER FOCUS ON DETAILED MEASUREMENTS
- FOR EXAMPLE → DERIVING DETAILS OF CLOUD T FOR SPECIFIC PLANETS RATHER THAN A GENERAL UNDERSTANDING
- THIS IS A PARTICULARLY BAD PROBLEM FOR THE PHOTOCHEMICALLY DERIVED HAZES
- T_{eff} RATHERLY MEASURABLE

MODELING IRRADIATED PLANETS

FROM THE EXOPLANET SIDE, WE'D LIKE INDICATORS OF

- $\log g$
- T_{eff} / FROM ONLY A VERY LIMITED AMOUNT OF INFORMATION

→ WE NEED A GENERAL UNDERSTANDING OF ALL PARAMETERS, SUCH THAT WE CAN APPLY METHODS TO A WIDE RANGE OF PLANETS

MISTAKES (PROBABLY) FROM EARLY WORK:

- INTERIOR TEMPERATURES THAT ARE TOO HOT
- WELL MIXED CLOUDS
- ASSUMPTION THAT A CLOUD IS A CLOUD — SILICATE CLOUDS SHOULD BE JUST LIKE NH_3 & H_2O CLOUDS — EVEN THOUGH THEY HAVE A LOWER ABUNDANCE

SUDAR 03 FIG. 16, 18

SEAGER 98, FIG. 2 & 3

BARANOV 01 FIG. 5 & 6

SUDAR 00, FIG 5

BROWN 01 FIG. 3

CH

BROWN 07 FIGURE 1 + 2

FARNETT 05 FIG. 2

GOUK. FIG. 1

HUBBARD FIG. 4 + 6

SS 00 FIG 2

SEAGER 05 FIG 2

$$p = nkT$$

$N = n_0 H \rightarrow$ LOWER $g \rightarrow$ LARGER H , LARGER COLUMN DENSITY AT GIVE P_0, n_0

\Rightarrow HIGHER GRAVITY PLANETS HAVE HIGHER GRAVITY PHOTOSPHERES

1st LAW OF THERMO: $dU = dQ - dW$

\swarrow \searrow \swarrow
 Δ INTERNAL ENERGY ENERGY ADDED BY HEATING ENERGY LOST VIA WORK DONE BY SYSTEM

$$dU = TdS - PdV$$

FINE, BUT WE'D PREFER TO LOOK AT CHANGES IN T & P, dT & dP, NOT dS & dV

LET US DEFINE G: $G = U + PV - TS$

\swarrow NOT A CONSERVED QUANTITY
 \searrow DOES NOT CORRESPOND TO A PHYSICAL PROPERTY

~~$dG = dU + PdV + VdP - TdS - SdT$~~

~~$dG = dU + PdV + VdP - TdS - SdT$~~

$dG = VdP - SdT$

EQUILIBRIUM CHEMISTRY — LET US ASSUME THAT A VERY LONG TIME HAS PASSED, SUCH THAT OUR SYSTEM HAS COME TO EQUILIBRIUM

AT EQUILIBRIUM, $dT = dP = 0$, THIS IMPLIES $dG = 0$

THE 2ND LAW OF THERMO GIVES US THAT

$$dS \geq \frac{dQ}{T}$$

S OF A CLOSED SYSTEM TENDS TO INCREASE OR STAY THE SAME

CHEMISTRY II

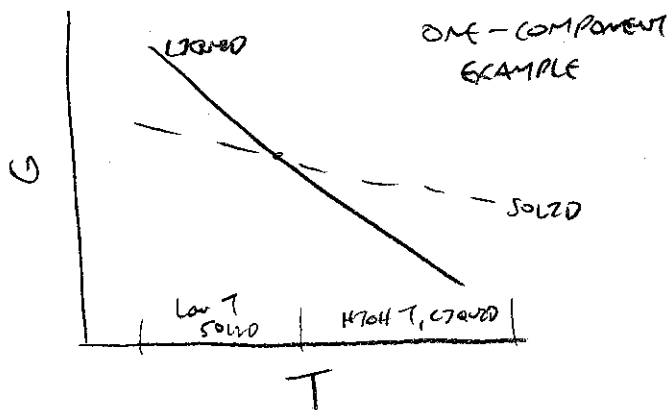
THIS GIVES US THAT $Tds \geq dU + PdV$

OR

$$dG \leq Vdp - SdT$$

→ ANY CHANGE TO THE SYSTEM (CHANGES IN ABUNDANCES) WILL TEND TO DRIVE dG TO SMALLER & SMALLER VALUES

→ AT SOME POINT G WILL REACH ITS MINIMUM.



MULTI-COMPONENT SYSTEM:

$$dU = Tds - PdV + \sum_i \mu_i N_i$$

↑
CHEMICAL POTENTIAL

- AMOUNT OF ENERGY ADDED TO A SYSTEM FOR PARTICLE i

$$G = \sum_i n_i \left[g_i^0(T) + RT \ln P_i \right]$$

↑ # OF MOLES ↑ TABULATED TEMPERATURE DEPENDENT G OF INDIVIDUAL GAS SPECIES PARTIAL PRESSURE

TO FIND ABUNDANCES

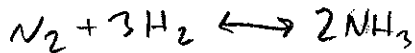
n_i, n_j, \dots

MINIMIZE G_{TOTAL}

$\frac{P_i}{P_{tot}}$ OF THE SPECIES

CHEMISTRY III

DEPARTURES FROM CHEMICAL EQUILIBRIUM



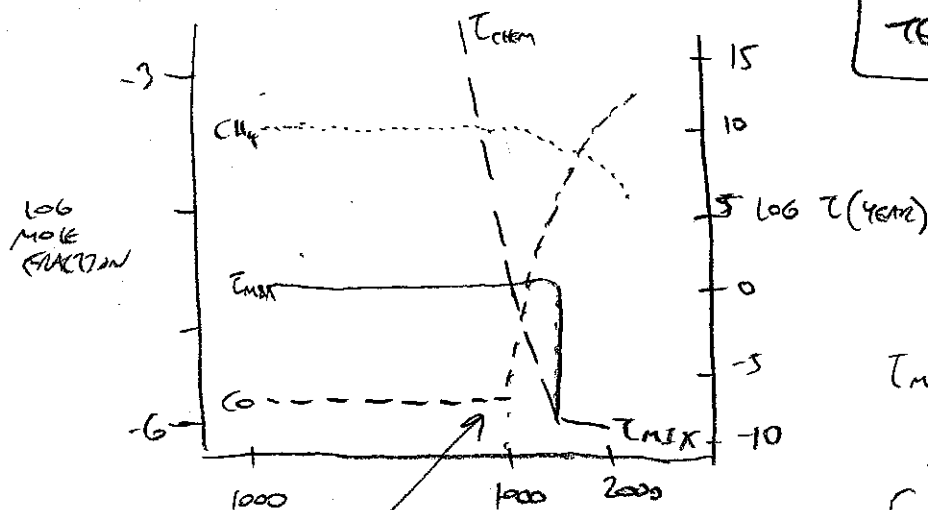
N_2 TRIPLE BOND AND CO DOUBLE BOND ARE HARD TO BREAK

- TIMESCALE TO BREAK APART N_2 AND CO ARE LONGER THAN MIXING TIME SCALE

IF $\tau_{MIX} < \tau_{CHEM}$, THEN EQUILIBRIUM HOLDS

IF $\tau_{MIX} > \tau_{CHEM}$, THEN WE SEE THE ABUNDANCE AT WHICH $\tau_{MIX} = \tau_{CHEM}$, THE "QUENCH LEVEL"

$\tau_{MIX, CONVECTIVE} \ll \tau_{MIX, RADIATIVE}$



τ_{CHEM} IS EXTREMELY TEMPERATURE DEPENDENT

$$\tau_{MIX, RAD} \approx \frac{H^2}{K_{22}}$$

$$K_{22} = \text{EDDI DIFFUSION COEFFICIENT} = \frac{cm^2}{s}$$

$$\sim 10^4 \frac{cm^2}{s}$$

$$10^{10} \frac{cm^2}{s}$$

$$\tau_{MIX, CONV} = \frac{H}{V_c}$$

CONVECTIVE VELOCITY

QUENCH LEVEL

CHEMISTRY IV

PHOTOCHEMISTRY

$$\frac{dn_i(z)}{dt} + \frac{d\phi_i(z)}{dz} = P_i(z) - n_i(z)L_i(z)$$

i = SPECIES IN QUESTION

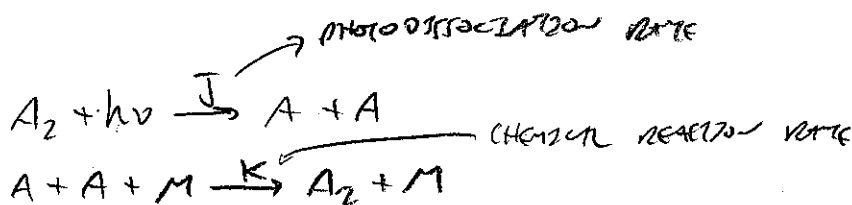
n_i = # DENSITY

$P_i(z)$ = PRODUCTION RATE \rightarrow COUPLED TO OTHER i, k, l, m SPECIES

$L_i(z)$ = LOSS RATE

$\phi_i(z)$ = VERTICAL MOVEMENT OR "FLUX"

EXAMPLE
IGNORE FLUX



$$\frac{dA}{dt} = +J[A_2] - K[A][A] = 0$$

$$\frac{dA_2}{dt} = -J[A_2] + K[A][A] = 0$$

[] = CONCENTRATION

PHOTO. CHEM. EQUIL. ASSUMED $\Rightarrow \frac{dA}{dt} = \frac{dA_2}{dt} = 0$

$$\Sigma A + 2A_2 = A_{\text{TOTAL}} \quad (\text{FIXED AMOUNT OF } A)$$

$$[A] = \sqrt{\frac{J[A_2]}{K}}$$

$$A + 2A_2 = A_{\text{TOTAL}}$$

\uparrow
INPUT INTO MODEL

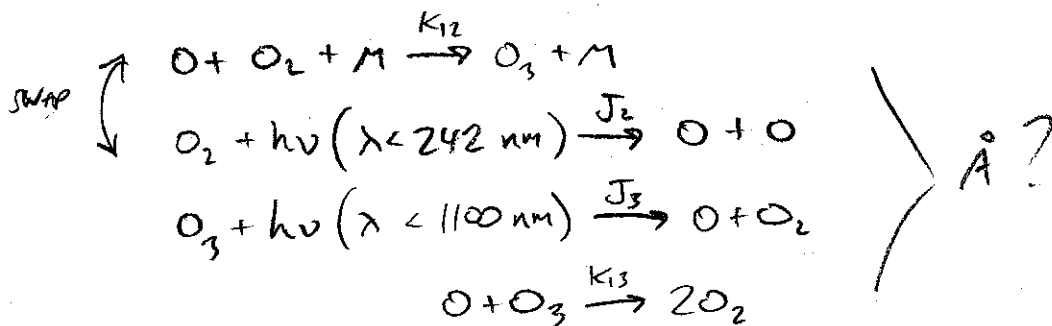
CHEMISTRY II

FLUX INCIDENT FROM ALL DIRECTIONS

$$J(z) = \frac{\#}{s} = \int_0^{\infty} \underbrace{\sigma(\nu)}_{\text{ABS X-SECTION}} \underbrace{F_{\text{TOT}}(\nu)}_{\text{LARGE IN UV, GENERALLY}} d\nu$$

$\frac{\text{PHOTONS}}{\text{m}^2 \cdot \text{s} \cdot \text{Hz}}$

FOR OZONE ON EARTH



$$[\text{O}_3] \approx \sqrt{\frac{J_2 K_{12}}{J_3 K_{13}}} [\text{M}] [\text{O}_2]$$

→ If J_3 large (O_3 destruction) $[\text{O}_3]$ is low
(HIGH IN THE ATM. WHERE UV FLUX IS HIGH)

→ If J_2 large (O_2 destruction) $[\text{O}_2]$ is high

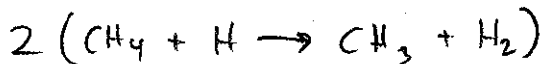
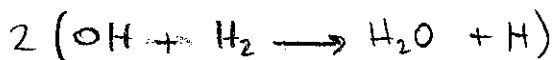
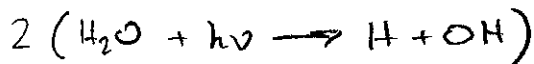
- FIRST STEP IN O_3 FORMATION PROCESS
- LOW IN ATM. WHERE LONGER λ PHOTONS PENETRATE

SUCH A SMALL NETWORK IS REALLY AN EXCEPTION, RATHER THAN THE USUAL CIRCUMSTANCE

→ VAST EQUATION NETWORKS

PHOTOCHEMISTRY

METHANE DESTRUCTION IN COOL JOVIAN PLANETS



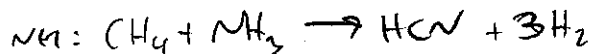
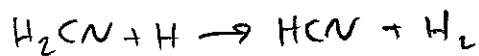
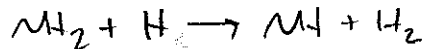
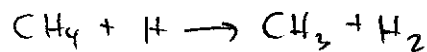
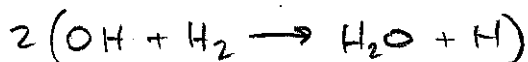
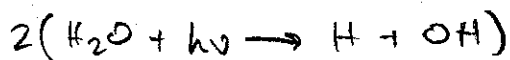
ETHYLENE

C_2H_6 - ETHANE

ALSO BROKEN UP, LEADS
DOWN C_2H_x PATHWAY
TO HAZES

ACETYLENE

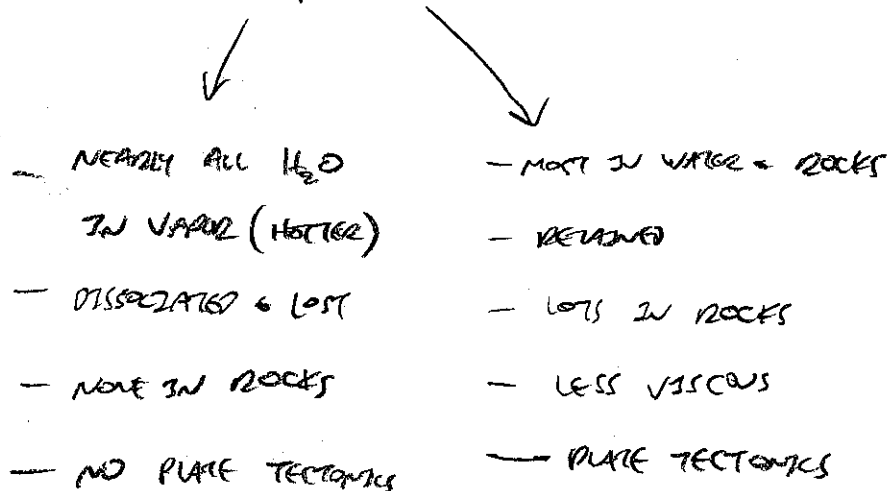
ALSO CROSS-REACTIONS W/ NITROGEN



PHOTOCHEMISTRY II

- WATER DESTRUCTION IS A COMMON FEATURE FOR IRRADIATED GAS GIANTS
- GAS GIANTS HAVE LARGE RESERVOIRS OF THERMOCHEMICAL PRODUCTS, SO THAT THERE CAN BE RECYCLING OF PHOTO-CHEMICAL DERIVED PRODUCTS
 - THERMOCHEM EQUIL. IS GENERALLY MORE IMPORTANT THAN PHOTOCHEM.
- IN TERRESTRIAL PLANETS, THE OPPOSITE IS TRUE — NO VAST INTERIOR OF GAS → JUST A SURFACE.

PARADIGM OF VENUS/EARTH DICHOTOMY



CLOUDS I

1) WHEN GASES COOL, THEY CONDENSE TO FORM LIQUIDS AND SOLIDS

2) WHEN THIS HAPPENS IS GOVERNED BY THE CLAUSIUS-CLAPEYRON RELATION

$$\left[\frac{dp}{dT} = \frac{L}{T \Delta v} \right]$$

↑ SPECIFIC LATENT HEAT
↓ SPECIFIC VOLUME, $\frac{1}{\rho}$

SLOPE OF THE
COEXISTENCE CURVE

DERIVATION

$$ds = ds(V, T) = \frac{ds}{dV} dV + \frac{ds}{dT} dT$$

URING A PHASE CHANGE $dT=0$

$$ds = \frac{ds}{dV} dV$$

MAXWELL RELATION: $\left(\frac{dV}{ds} \right)_P = \left(\frac{dT}{dP} \right)_S$

$$ds = \left(\frac{dP}{dT} \right) dV$$

SINCE P & T ARE CONSTANT $\frac{dP}{dT}$ IS NOT A FUNCTION OF V ,

SO $\frac{dP}{dT} \rightarrow \frac{dP}{dT}$

INTEGRATE BOTH SIDES

$$s_2 - s_1 = \frac{dP}{dT} (V_2 - V_1)$$

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V}$$

CLOUDS II

$$\text{1st law: } dU = dQ - dW = Tds - PdV$$

$$dU + PdV = \underbrace{dh}_{\text{ENTHALPY}} = Tds$$

$$\Rightarrow ds = \frac{dh}{T} \Rightarrow \Delta S = \frac{\Delta h}{T}$$

$$\frac{dp}{dT} = \frac{\Delta h}{T \Delta V}$$

$$\Delta h = L$$

$$\boxed{\frac{dp}{dT} = \frac{L}{T \Delta V}}$$

WE CAN MAKE SOMETHING EVEN MORE USEFUL WITH THE IDEAL GAS

LAW AND THAT $V_{\text{GAS}} \gg V_{\text{LIQUID}}$

$$\Delta V = V_2 - V_1 \approx V_2 \approx \frac{RT}{P}$$

$$\frac{dp}{dT} = \frac{L P}{T^2 R}$$

$$\frac{dp}{P} = \frac{L}{R} \frac{dT}{T^2}$$

$$\ln P = -\frac{L}{R} \frac{1}{T} + C$$

$$\text{OR } \ln \frac{P_1}{P_2} = \frac{L}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

CLOUDS III

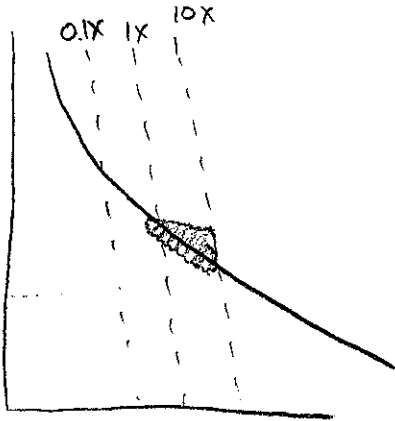
FOR A GAS SPECIES THAT MIGHT CONDENSE:

$$\frac{dp_v}{dT} = \frac{L}{T \Delta V}$$

$$P_g(T) = X_j P(T) \geq P_{v,j}(T)$$

PARTIAL PRESSURE
MIXING RATION
GAS PRESSURE
SATURATION VAPOR PRESSURE

RELATED TO HUMIDITY OR THE DEW POINT



- HIGHER T CAN SUPPORT MORE VAPOR
- INITIAL CONDENSATION IS A FUNCTION OF METALLICITY
- FOR, FOR INSTANCE, U & N → NOT CLEAR TO WHAT DEPTH NEEDS TO PROBE TO REACH THE CLOUD BASE

CLOUD VERTICAL EXTENT

LET'S INTEGRATE C-C RELATION FROM BASE TEMPERATURE T_c TO LOWER T, HIGHER UP

$$\frac{dp_v}{dT} = \frac{L p_v}{R T^2} \Rightarrow p_v(T) = p_v(T_c) \exp\left[\frac{-L(T_c - T)}{R T_c T}\right]$$

$$\text{LET } T \sim T_c \Rightarrow T_c T \approx T_c^2$$

(SPECIFIC GAS CONSTANT)
 $R_v = \frac{R}{\text{MASS OF A MOLE}}$

$$\frac{dT}{dz} = \frac{-g}{C_p} \quad (\text{ASSUME CONVECTIVE ATMOSPHERE})$$

CLOUDS, EXTENT II

FROM T -PROFILE: $T_c - T = \frac{g}{c_p} (z - z_c)$

$$P_v(z) = P_v(T_c) e^{-(z - z_c)/H_c}$$

$$H_c = \frac{R_v T_c^2 c_p}{g L} = \text{DISTANCE OVER WHICH } P_v \text{ DROPS BY } e.$$

$$H = \frac{RT}{\mu g}$$

$$\frac{H_c}{H} = \frac{R_v T_c^2 c_p \mu g}{g L R T}$$

$$\boxed{\frac{H_c}{H} = \frac{R_v \mu c_p T_c}{R L}}$$

$\approx 0.05 - 0.2$ IN
THE SOLAR SYSTEM

THE HOT JUPITER / BROWN DWARF EXAMPLE

- MOST OF THE CONDENSATES COME ABOUT FROM CHEMICAL REACTIONS WHERE THE STABLE PHASES ARE SOLIDS

Ca-Ti-Al SOLIDS

Mg_2SiO_4 , $MgSiO_3$ SOLIDS

- $Fe(g) \rightarrow Fe(l)$ IS A COUNTER-EXAMPLE