

FORMATION OF THE GIANT PLANETS BY NUCLEATED INSTABILITY (STEVENS, 1982)

Case of mass M_c surrounded by a gaseous envelope, with $p(r)$, $\rho(r)$, $T(r)$ at position r in the envelope. $M_T = \text{total mass}$

$M_T = M_{env} + M_c$

IF NEGATIVE TRANSPORT DOMINATES: POSTERIOR MEAN

$\frac{dT}{dr} = \frac{-3k}{L(r)} \frac{16\sigma T^3}{4\pi r^2}$ A

ASSUME HSE:

$\frac{dp}{dr} = -\frac{GM_r}{r^2} \rho$ B

Divide A + B and assume $T \propto p$ at boundary $T_0 \propto p_0$

Let $p \propto T$

GIVES:

$T^4 \approx \frac{3kL}{16\pi\sigma M} p$

ASSUME IDEAL GAS: $p = \frac{\mu}{k} kT$ | let $M_T \approx M_c$

Approx. HSE + IDEAL GAS

$\frac{1}{p} \frac{dp}{dr} = -\frac{GM_r}{r^2}$

$T \approx \frac{MGM_r}{4\pi k r}$

$p \approx \frac{64\pi\sigma}{3kL} \left(\frac{MGM_r}{4\pi k r} \right)^4 \frac{1}{r^3}$

$$M_c = 0.38 \left(\frac{R}{k} \right)^{3/4}$$

Set $\frac{dM_c}{dM_T} = 0$ const: $M_T = \left(\frac{4R}{k} \right)^{3/4} M_c^{3/4}$

IF M_c BECOMES TOO LARGE NO SOLUTION FOR M_T IS POSSIBLE. HERE EQUILIBRIUM CANNOT BE MAINTAINED

SLIGHTLY VARYING CONSTANT WITH R IS COMPARED WITH CHANGING M_c OR M_T , BUT LAYS A $\alpha = M_T^{1/4}$ DEPENDENCE

$$M_T \approx M_c + \left(\frac{R}{k} \right)^{3/4} M_c^{3/4}$$

$$M_T = M_c + M_{EW}$$

$$L \propto \frac{GM_c}{M_c^{3/4}} \propto M_c^{1/4}$$

CONSTANT

$$L \approx \frac{GM_c}{R_c} \cdot \frac{dM_c}{dt}$$

$$\frac{4}{3} \pi R_c^3 \rho = M$$

ASSUME THAT L IS DUE TO THE ACCELERATION OF PLANETESIMALS

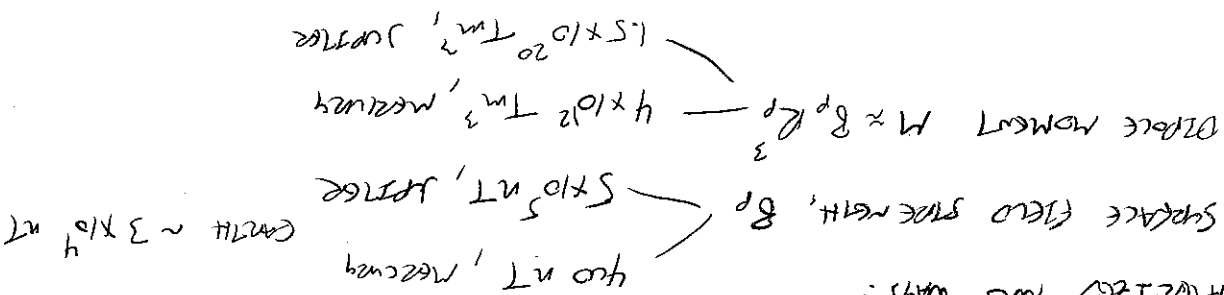
$$M_{EW} = \int_{R_{min}}^{R_c} 4\pi \rho(n) n^2 dn = \frac{32\pi L}{4k} \left(\frac{M GM_T}{4k} \right)^{3/4} \left(\frac{R_{min}}{R_c} \right)$$

FORMATION III

Solution relatively insensitive to outer boundary condition because of the low (Peters) dependence

$M_c \times M^{-1/2} \rightarrow$ Dissolution of polymer into the envelope makes forming the giant particle easier, but would also tend to decrease the core mass - below some critical estimate

PARAMETERIZED TWO WAYS:



PLANETS HAVE MAGNETIC FIELDS AND WE'D LIKE TO UNDERSTAND WHAT CAUSES, MAINTAINS, AND ENDS THEM, AND WHY THEY HAVE THE STRENGTHS THAT THEY DO.

GIVEN THAT THIS IS EXTREMELY COMPLICATED, THERE HAS BEEN A WEIRD FASCINATION IN SIMPLE LAWS THAT CAN MATCH OBSERVATIONS, AND POTENTIALLY HAVE PREDICTIVE POWER.

RELEVANT PARAMETERS

- R_p
- R_c
- B_p
- B
- (core)
- Ω
- G_{core}
- q : CONVERTED ENERGY FLUX

ENERGY FLUX SCALING
 $B^2 \propto R_c^{2/3} \Omega^{1/3}$

CLASSIC NUMBER RULE
 $B^2 \propto \frac{P}{R^2}$

MAGNETIC BODIES LAW
 $B_p R_p^3 \propto (P R_p^5)^{1/3}$

CAN COMPARE TO OBSERVATIONS OR CAN COMPARE TO NUMERICAL RESULTS

DYNAMO II

SI units

for earth, $U \approx 10^{-4}$

$\Omega = 7 \times 10^{-5}$

$\rho \approx 10^4$

$B = 5 \times 10^{-3}$

$\lambda \approx 3 \times 10^5$ (length scale for spatial derivatives)

- viscous force
- LL
- lorentz force
- stresses
- inertial force

most common scales are

ELASTICITY & RHEOLOGY:

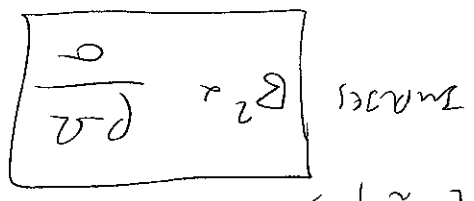
RETURN $\dot{\gamma}$ TO U VIA DIMITS LAW: $\dot{\gamma} = \sigma (\frac{E}{\eta} + \alpha \times B)$

$J \propto \sigma U B$

LORENTZ FORCE: $J \times B \propto \sigma U B^2$

(EMF) FORCE: $2 \rho \Omega U$

$\text{RATIO} = V = \frac{\sigma B^2}{2 \rho \Omega}$ EMF # (U DIMITS COST)



STRUCTURAL STRESS
THIS COST
+ SOLAR SYSTEM
DYNAMOS APPEAR
TO BE A1

V approx 0.1-1.0

A CERTAIN AMOUNT OF CONVECTIVE HEAT FLUX IS NEEDED TO MAINTAIN THE DYNAMO, SO IT IS A LITTLE ODD THAT IT DOESN'T COME INTO THE CLASSICAL SCALAR RELATION. PROBABLY WORKS AT SOME MINIMUM q_c IS MET

1) power/vol volume, $P = \rho_c / H_T \rightarrow$ T STATE HEIGHT

2) OMLIC DISSIPATION: $D = (\nabla \times \vec{b})^2 \propto \frac{\sigma M_o^2}{\lambda B^2} \times \frac{2M_o \rho_g}{\text{LENGTH OF THE FIELD}}$

$f_{ohm} =$ FRACTION OF AVAILABLE CURRENTLY = LOST TO OMLIC DISSIPATION

~~$f_{ohm} = \frac{E_m}{\rho_c} \times \frac{2M_o}{\rho_g} \times \frac{\lambda}{H_T}$~~

$f_{ohm} = \frac{D}{P} = \frac{\rho_c}{2M_o \rho_g^2} \times \frac{2M_o}{\rho_g} \times \frac{\lambda}{H_T}$

$E_m = \frac{2M_o}{B^2}$

$\frac{B^2}{2M_o} \propto f_{ohm} \left(\frac{\rho_c}{\lambda} \right) \frac{\lambda}{H_T}$

OMLIC DISSIPATION TIME $\tau_{ohm} = \frac{\rho_c}{E_m} = \frac{\lambda}{D}$

τ_{ohm} RELATED TO CHARACTERISTIC VELOCITY & DISTANCE $\tau_{ohm} = \frac{\lambda}{v} = \frac{\lambda}{\rho_c} = \frac{\lambda}{L}$

USE MIXING LENGTH TO GET U

$$U \propto \left(\frac{g L_c}{P H_T} \right)^{1/3} \quad H_T = \frac{c_p}{\alpha g}$$

Now Y_{SED} :

$$\frac{B^2}{2M_0} \propto f_{\text{form}} P^{1/3} \left(\frac{g_c L}{H_T} \right)^{2/3}$$

Let $L = R_c$

$$\frac{1}{H_T} \propto P R_c$$

$$Y_{SED} = B^2 \propto P R_c^{2/3} g_c^{2/3}$$

(CHRISTENSEN & ARBEQZ (2006))

INDEPENDENT OF Ω AND σ

(CAN BE COMPARED TO 1) MODELS

2) REAL PLANETS

3) FAULT CORRECTIVE STARS