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Chapter 1
VERTICAL STRUCTURE OF AN ATMOSPHERE

The simplest conceivable atmosphere is gravitationally bound (and therefore in hydrostatic equilibrium) and spherically symmetric. For the terrestrial planets, at least, solar radiation and the radiative properties of the atmospheric constituents fix the first-order description of the vertical thermal structure. An atmosphere controlled by sunlight can scarcely be spherically symmetric. Nevertheless, it is useful to think of a mean planetary atmosphere, with day–night and latitudinal variations occurring about the mean.

The *vertical structure* of an atmosphere is the run of pressure, temperature, density, and chemical composition with distance from the center of the planet (or with height above the surface). When these parameters are inferred in part from theory or when they are tabulated as mean or representative values, they constitute a *model atmosphere.*

Figure 1.1 shows the temperature profile for Earth's atmosphere; it serves to divide the atmosphere into different regions, where the controlling physics and chemistry differ.

The tropospheric temperature is governed by radiative and convective exchange. In the stratosphere trace amounts of O_3 are formed by sunlight; the remarkable ability of O_3 to absorb both ultraviolet and infrared radiation causes an inversion above the tropopause. The decrease in O_3 production and the increased rate of cooling to space by CO_2 reestablish a declining

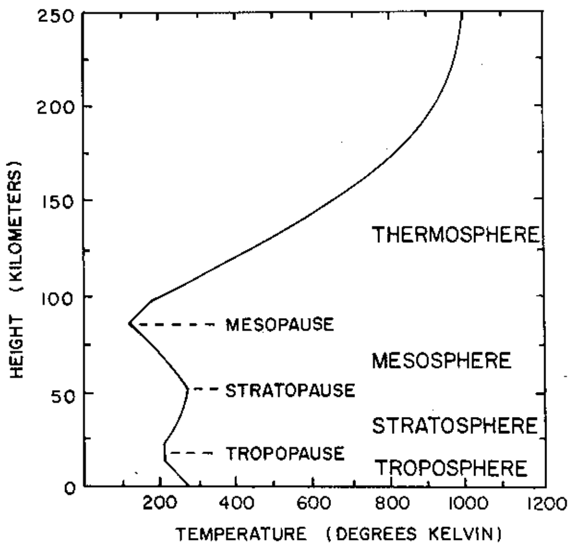

Fig. 1.1 Schematic temperature profile for Earth's atmosphere showing the various regimes defined by the temperature gradient.

temperature in the mesosphere. Finally, heating by O_2 photolysis and ionization increases the thermosphere temperatures to about 1000°K. We shall examine the main processes in these regions in turn.

1.1 Hydrostatic Equilibrium

The vertical distribution of pressure, temperature, and density in a static, spherical atmosphere with a specified composition are governed by three relationships. First we have hydrostatic equilibrium, in which the pressure gradient is

$$\frac{dp}{dr} = -\left(\frac{G\mathcal{M}}{r^2}\right)(MN) = -g(r)\rho \tag{1.1.1}$$

Here $\mathcal{M}$ is the mass of the planet and M is the mean mass of the molecules of a mixed atmosphere, N is their number density, ρ is the mass density, and r is the distance from the center of the (spherical) planet. Over height intervals Δr such that $\Delta r \ll r$, the local acceleration of gravity $g(r) \approx$ const.

For an equation of state, the perfect gas law is adequate:

$$p = NkT = \rho RT \tag{1.1.2}$$

where $R = k/M$ (erg/gm deg) is the gas constant appropriate to the atmospheric composition. Then hydrostatic equilibrium gives

$$\frac{dp}{p} = -\frac{G\mathcal{M}M}{kT}\frac{dr}{r^2} \approx -\frac{gM}{kT}\,dz \equiv -\frac{dz}{H} \tag{1.1.3}$$

where z is height above the surface and H the (pressure) scale height.

A third relation must fix the temperature (cf. Section 1.2). If the mean mass and temperature are constant with height, we obtain the *barometric law*:

$$\begin{aligned} p(r) &= p(r_0)\exp\left(-\frac{G\mathcal{M}M}{kTrr_0}(r - r_0)\right) \\ &\approx p(r_0)\exp\left(-\frac{r - r_0}{H}\right) \\ p(z) &= p(z_0)\exp\left(-\frac{z - z_0}{H}\right) \end{aligned} \tag{1.1.4}$$

Thus the *pressure scale height* ($H = kT/Mg$) is an e-folding distance. In the general case the density distribution is

$$\begin{aligned} \frac{dN}{N} &= -\frac{dT}{T} - \frac{G\mathcal{M}M}{kT}\frac{dr}{r^2} \approx -\frac{dT}{T} - \frac{dz}{H} \\ &= -\left(\frac{1}{T}\frac{dT}{dz} + \frac{Mg}{kT}\right)dz = -\frac{dz}{H^*} \end{aligned} \tag{1.1.5}$$

which defines the *density scale height* H^*. The *integrated density* is the number of particles in a column above a specified height. It is, from (1.1.1),

$$\begin{aligned} \mathcal{N}(r) &\equiv \int_r^\infty N(r)\,dr = \int_0^{p(r)} \frac{r^2}{G\mathcal{M}M}\,dp \\ &\approx \frac{p(r)}{g(r)M} = N(r)H \end{aligned} \tag{1.1.6}$$

The integrated density is often written in terms of the height of a column under standard temperature and pressure conditions that would contain the same number of molecules or atoms. This is the equivalent thickness in "atmo-centimeters" (alternatively, "centimeter-atmospheres" or "centimeter-amagat"),

$$\xi = \frac{\mathcal{N}(z)}{N_0}\ \text{atm-cm} \tag{1.1.7}$$

where N_0 is Loschmidt's number (2.687×10^{19} cm^{-3}).

Without perfect mixing the mean mass M is a function of height, $M(z)$. At high altitudes the mixing processes are likely to be less important since the diffusion coefficient (Section 2.3.1) is large. Hence the atmosphere tends to separate out diffusively, and the composition changes as well through photochemical reactions. The gravity g is also a function of height [see (1.1.3)]. This effect is discussed in Section 7.1; the generalized barometric equation is (7.1.19).

1.2 Radiative Equilibrium

As a starting point we will regard the atmospheric temperature as governed by radiative equilibrium. Of course, it is not, but we will add convection later. In the ionosphere conduction becomes the dominant mechanism for heat transfer, and radiative equilibrium is not even a good starting approximation.

1.2.1 Equation of Radiative Transfer and Kirchhoff's Law

In a homogeneous medium (see Fig. 1.2) the monochromatic radiant intensity I_ν (measured in erg/cm^2 sec sr Hz) changes along distance ds (mea-

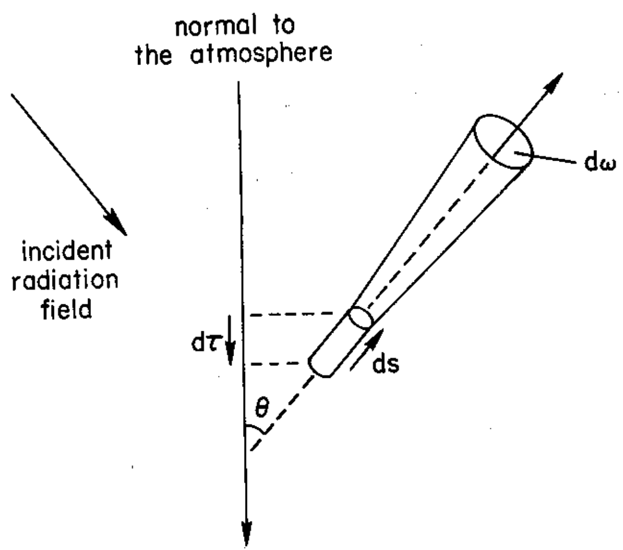

Fig. 1.2 Geometry for the equation of radiative transfer. The element ds is always positive and taken in the direction of propagation of a light ray.

sured in the direction of propagation of the light ray and always positive) by amount dI_ν given by

$$\frac{1}{\rho}\frac{dI_\nu}{ds} = -(\kappa_\nu + \sigma_\nu)I_\nu + j_\nu \tag{1.2.1}$$

where κ_ν is the mass absorption coefficient and σ_ν the mass scattering coefficient (both measured in cm^2/gm), and j_ν is the emission coefficient (erg/gm sec sr Hz). The emission j_ν may be due in part to scattering and in part to thermal excitation. The combination $\kappa_\nu + \sigma_\nu$ is the *extinction coefficient*.

Let us look at a few special cases. If emission and scattering back into the original beam are negligible, as in viewing a single star against a black sky, we have $j_\nu = 0$ and

$$I_\nu(s) = I_\nu(0)e^{-(\kappa_\nu+\sigma_\nu)\rho s} \tag{1.2.2}$$

which is *Lambert's exponential absorption law.*

If the source of emission includes scattering (as in the blue, sunlit sky), we have to specify a *scattering phase function,* $p(\cos\Theta)$, giving the angular distribution of scattered radiation. Thus, if j_ν is entirely due to scattering, the emission term is

$$j_\nu = \frac{\kappa_\nu + \sigma_\nu}{4\pi}\int I_\nu(\cos\Theta)p(\cos\Theta)\,d\Omega' \tag{1.2.3}$$

where the phase function itself is normalized so that integrated over a sphere it is

$$\frac{1}{4\pi}\int p(\cos\Theta)\,d\Omega' = \frac{\sigma_\nu}{\kappa_\nu+\sigma_\nu} \equiv \tilde{\omega}_\nu \tag{1.2.4}$$

SINGLE SCATTERING ALBEDO

which is called the *albedo for single scattering.*

For the scattering atmosphere, then, the equation of transfer is, for radiation in direction θ, ϕ,

$$\frac{dI_\nu(\theta,\phi)}{(\kappa_\nu+\sigma_\nu)\rho\,ds} = -I_\nu(\theta,\phi) + \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\nu(\theta',\phi')p(\theta,\phi;\theta',\phi')\sin\theta'\,d\theta'\,d\phi' \tag{1.2.5}$$

At the other extreme from a scattering atmosphere is one in *local thermodynamic equilibrium* (LTE). It is assumed that at each point a local temperature T can be defined so that the emission is given by Kirchhoff's law,

$$j_\nu = \kappa_\nu B_\nu(T) \tag{1.2.6}$$

where the Planck function is

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \qquad (1.2.7)$$

We then have, since σ_ν is assumed zero, the monochromatic radiation in LTE given by

$$\frac{dI_\nu(\theta, \phi)}{\kappa_\nu \rho \, ds} = -I_\nu(\theta, \phi) + B_\nu(T) \qquad (1.2.8)$$

The LTE approximation can never be exact, and one problem is ascertaining how inexact it is. In complete thermodynamic equilibrium, the temperature is everywhere the same; in the atmosphere the temperature has a definite gradient. Also, atmospheric emission is not Planckian at any point; the radiation field in the ultraviolet and infrared are not characteristic of the same T. Finally, the local kinetic temperature (given by the Maxwellian distribution law) is not the same as the effective Planckian temperature (defined by the radiation field). In a real situation we must usually treat scattering and thermal emission together. For combined isotropic scattering and thermal emission the transfer equation is

$$\frac{dI_\nu}{(\kappa_\nu + \sigma_\nu)\rho \, ds} = -I_\nu + \frac{\tilde{\omega}_\nu}{4\pi} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} I_\nu(\theta', \phi') \sin\theta' \, d\theta' \, d\phi' + (1 - \tilde{\omega}_\nu) B_\nu \qquad (1.2.9)$$

In the general case we define the source function (in the same units as I_ν) as the ratio of emission coefficient to opacity

$$\mathcal{J}_\nu = \frac{j_\nu}{\kappa_\nu + \sigma_\nu} \qquad (1.2.10)$$

Then the general equation of transfer is

$$\frac{dI_\nu}{(\kappa_\nu + \sigma_\nu)\rho \, ds} = -I_\nu + \mathcal{J}_\nu \qquad (1.2.11)$$

For isotropic scattering,

$$\mathcal{J}_\nu = \frac{\tilde{\omega}_\nu}{4\pi} \int I_\nu \, d\Omega \equiv \tilde{\omega}_\nu J_\nu \qquad (1.2.12)$$

where J_ν is the local mean intensity. For LTE,

$$\mathcal{J}_\nu = B_\nu(T) \qquad (1.2.13)$$

$d\tau = \kappa n \, ds$
$\frac{cm^2}{\text{MOLECULE}}$

and the combined case has

$$\mathcal{J}_\nu = \tilde{\omega}_\nu J_\nu + (1 - \tilde{\omega}_\nu) B_\nu \qquad (1.2.14)$$

Defining a *slant optical thickness* from s to s' as

τ

$$\tau_\nu(s, s') = \int_s^{s'} (\kappa_\nu + \sigma_\nu)\rho \, ds \qquad (1.2.15)$$

we can write down the formal solution to Eq. (1.2.11) as

$$I_\nu(s) = I_\nu(0) e^{-\tau_\nu(s,0)} + \int_0^s \mathcal{J}_\nu(s') e^{-\tau_\nu(s,s')} (\kappa_\nu + \sigma_\nu)\rho \, ds' \qquad (1.2.16)$$

If the source function is known, we have the solution for the radiation field. In practice, the solution is not so simple because $\mathcal{J}_\nu(s')$ depends on $I_\nu(s')$ directly or on $B_\nu(T)$ (which in turn depends on the heating from the radiation field) or both.

1.2.2 Monochromatic Radiative Equilibrium

In a plane-stratified atmosphere in which height z is measured upward, we measure optical depth τ downward and zenith angle of the direction of radiation flow θ from the upward vertical. Then $ds = \sec\theta \, dz$ is always positive and the vertical optical thickness is

$$d\tau_\nu = -(\kappa_\nu + \sigma_\nu)\rho \, dz \qquad (1.2.17)$$

Equation (1.2.8) for LTE conditions is then

$$\mu \frac{dI_\nu(\theta, \phi)}{d\tau_\nu} = I_\nu(\theta, \phi) - B_\nu(T) \qquad (1.2.18)$$

where $\mu = \cos\theta$. Integrating over a sphere we have

$$\frac{d}{d\tau_\nu}(\pi F_\nu) = 4\pi(J_\nu - B_\nu) \qquad (1.2.19)$$

where the mean intensity J_ν is given by (1.2.12) and the net flux across an area parallel to the surface is

$$\pi F_\nu = 2\pi \int_{-1}^{1} I_\nu(\mu)\mu \, d\mu \qquad (1.2.20)$$

We may obtain an approximate solution by the *two-stream approximation* (see Fig. 1.3). Suppose that the upward radiant intensity is $I_\nu(\mu, \tau) = I^+(\tau)$

FLUX $\equiv \pi F_\nu$

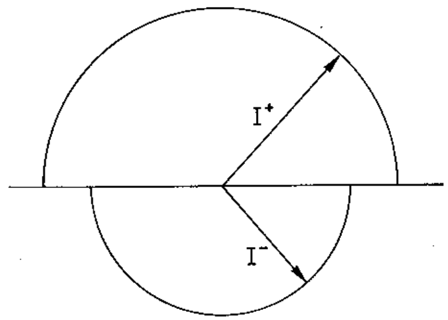

Fig. 1.3 The two-stream approximation considers the radiation field to be composed of two simple streams, one upward and one downward.

for $0 < \mu < +1$ and the downward radiation field is $I_\nu(\mu, \tau) = I^-(\tau)$ $(-1 < \mu < 0)$. Then the mean intensity at depth τ is

$$J_\nu \equiv \tfrac{1}{2} \int_{-1}^{1} I_\nu(\mu)\, d\mu = \tfrac{1}{2}(I^+ + I^-) \tag{1.2.21}$$

and the net flux is

$$\pi F_\nu = \pi(I^+ - I^-) \tag{1.2.22}$$

To obtain a second relation between mean intensity and flux, we multiply (1.2.18) by μ and integrate over a sphere. With (1.2.20) and (1.2.21) we have

$$2\pi \frac{d}{d\tau_\nu} \int_0^1 d\mu\, \mu^2(I^+ + I^-) = 2\pi \int_0^1 d\mu\, \mu(I^+ - I^-) \tag{1.2.23}$$

or

$$\frac{4\pi}{3} \frac{dJ_\nu}{d\tau_\nu} = \pi F_\nu \tag{1.2.24}$$

Substituting (1.2.19) for J_ν in this equation yields the flux equation

$$\frac{d^2 F_\nu}{d\tau_\nu^{\,2}} - 3F_\nu = -4 \frac{dB_\nu}{d\tau_\nu} \tag{1.2.25}$$

The concept of radiative equilibrium means that the net flux divergence is everywhere zero, with no energy lost or supplied by convection or conduction. It does not mean that the flux πF_ν in each separate frequency everywhere is constant. Nevertheless, it is instructive to examine this case of *monochromatic radiative equilibrium* (MRE). With $dF_\nu/d\tau_\nu = 0$ everywhere, our equation for the thermal radiation versus τ_ν is

$$\frac{dB_\nu}{d\tau_\nu} = \tfrac{3}{4} F_\nu = \text{const} \tag{1.2.26}$$

In applying boundary conditions we have to be careful that they do not conflict with assumptions already introduced in the two-stream approximation. Let us suppose that the ground is a black body at temperature T_g and a cold black sky $(T = 0)$ lies above the atmosphere. With Eqs. (1.2.21) and (1.2.22) we can write

$$J_\nu = I^- + \tfrac{1}{2}F_\nu = I^+ - \tfrac{1}{2}F_\nu \tag{1.2.27}$$

Then the transfer equation (1.2.19) gives

$$\frac{dF_\nu}{d\tau_\nu} = 4(I^+ - B_\nu) - 2F_\nu = 4(I^- - B_\nu) + 2F_\nu = 0 \tag{1.2.28}$$

Hence the upward intensity at the ground is

$$I_g^{\,+} \equiv B_\nu(T_g) = B_\nu(T_1) + \tfrac{1}{2}F_\nu \tag{1.2.29}$$

where T_1 is the air temperature at the ground, and the downward intensity at the top of the atmosphere is

$$I_0^{\,-} \equiv 0 = B_\nu(T_0) - \tfrac{1}{2}F_\nu \tag{1.2.30}$$

where T_0 is the air temperature at $\tau = 0$.

The solution tells us that, to fit an LTE solution with the special case of MRE, there is a discontinuity in temperature at the ground, with $T_g > T_1$, and the air at the top approaches a value, $T_0 \neq 0$. The radiant flux leaving the atmosphere is

$$\pi I_0^{\,+} = \pi B_\nu(T_0) + \frac{\pi}{2} F_\nu = 2\pi B_\nu(T_0) \tag{1.2.31}$$

or twice what an opaque black body at temperature T_0 would emit.

The solution, then, from (1.2.26) and (1.2.30) is (see Fig. 1.4)

$$B_\nu(\tau_\nu) = B_\nu(T_0)(1 + \tfrac{3}{2}\tau_\nu) \tag{1.2.32}$$

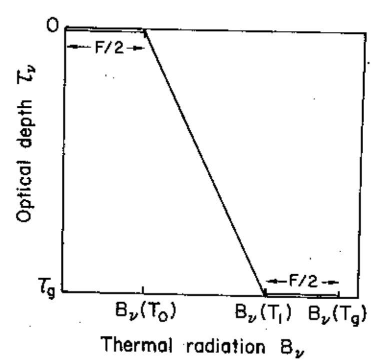

Fig. 1.4 The MRE solution for $T(\tau)$, presented as $B_\nu(T)$ vs. τ_ν. Note the discontinuity at the ground and the finite skin temperature at $\tau = 0$.

The atmospheric radiation πI_0^+ is thus characteristic of the thermal emission at $\tau_\nu = \frac{2}{3}$.

A common name for T_0 is the *Gold–Humphreys skin temperature*, or simply *skin temperature*. It can be derived on the assumption that a volume element in the stratosphere is heated by absorption of planetary radiation over a hemisphere and cooled by its own radiation over a sphere. By Kirchhoff's law, the absorption and emission are proportional as long as the same wavelength bands are involved, which is approximately true. Since the solid angles are in a ratio 1:2, the mean planetary temperature and the skin temperature are in the ratio $1:2^{-1/4}$ [cf. (1.2.45)]. The principal reason for deviations from this result is additional heating by solar radiation; a prominent example is ozone heating on the Earth.

1.2.3 Local Thermodynamic Equilibrium for a Gray Atmosphere Heated from the Ground

We are now ready to examine how the interchange of radiation governs the temperature of an atmosphere when there is no direct solar absorption by the atmosphere, when there is no conduction or convection, and when scattering can be neglected. The transfer equation (1.2.8) for LTE is

$$\frac{\mu\, dI_\nu}{\kappa_\nu \rho\, dz} = -I_\nu + B_\nu \tag{1.2.33}$$

Integrating over frequency we have

$$\frac{\mu}{\rho}\frac{d}{dz}\left(\int_0^\infty \frac{I_\nu}{\kappa_\nu}\, d\nu\right) = -I + B \tag{1.2.34}$$

where

$$I = \int_0^\infty I_\nu\, d\nu, \qquad B = \int_0^\infty B_\nu\, d\nu \tag{1.2.35}$$

By integrating over a sphere and setting the net flux constant, we have, analogously to Eq. (1.2.26),

$$-\frac{1}{\rho}\frac{d}{dz}\left(\int_0^\infty \frac{B_\nu}{\kappa_\nu}\, d\nu\right) = \tfrac{3}{4}F \tag{1.2.36}$$

where $F = \int F_\nu\, d\nu$. In the event $\kappa_\nu = \text{const}\,(=\kappa)$ the transfer equation (1.2.33) is

$$\mu \frac{dI}{d\tau} = I - B \tag{1.2.37}$$

and the thermal radiation is given by

$$\frac{dB}{d\tau} = \tfrac{3}{4}F \tag{1.2.38}$$

where

$$d\tau = -\kappa\rho\, dz \tag{1.2.39}$$

Hence the gray solution is analogous to that for MRE given above.

It would seem that, with a mean absorption coefficient properly defined, any nongray atmosphere could be treated with the gray solution. Thus writing

$$d\tau = -\langle\kappa\rangle \rho\, dz \tag{1.2.40}$$

a comparison of (1.2.36) and (1.2.38) suggests

$$\frac{1}{\langle\kappa\rangle} = \frac{1}{B}\int_0^\infty \frac{B_\nu}{\kappa_\nu}\, d\nu \tag{1.2.41}$$

where B is given by (1.2.35). This $\langle\kappa\rangle$ is the *Rosseland mean* used widely in astrophysics. The problem with it lies in the Eddington approximation. To conserve flux, the MRE equation (1.2.19) requires $J_\nu = B_\nu$ precisely. But in LTE there is a gradual shift with depth in the frequency distribution of the radiation, since temperature varies with depth, and J_ν cannot precisely equal B_ν to conserve the flux in each frequency interval. But if there is approximate equality over the spectrum, the Rosseland mean is a good approximation. It works well for stellar atmospheres, but is not good for most terrestrial situations. Since T and $B_\nu(T)$ vary with depth, $\langle\kappa\rangle$ cannot be treated as constant with depth. Thus iterations are required to compute $B_\nu(T)$ and $\langle\kappa\rangle$ as functions of depth. An alternative and generally better procedure is to put κ_ν on the right-hand side of (1.2.26) and integrate over ν. Comparison with (1.2.38) gives

$$\langle\kappa\rangle = \frac{1}{F}\int_0^\infty \kappa_\nu F_\nu\, d\nu \tag{1.2.42}$$

which is the *Chandrasekhar mean* coefficient. Its difficulty is that F_ν is not known in advance of solving the problem, requiring iterations, or other approximations.

We can now write the LTE gray-atmosphere solution and compare numerical results with temperatures in the Earth's atmosphere. Equation (1.2.38) gives a solution, similar to (1.2.32),

$$T^4(\tau) = T_0^{\,4}(1 + \tfrac{3}{2}\tau) \tag{1.2.43}$$

where T_0 is the temperature of the upper boundary. Here the integrated black-body intensity is

$$B(\tau) = \frac{\sigma}{\pi} T^4(\tau) \tag{1.2.44}$$

The total radiant flux from the Earth can be expressed in terms of a *mean planetary emission temperature*, obtained by integrating Eq. (1.2.31) over frequency:

$$T_e^{\,4} = 2T_0^{\,4} \tag{1.2.45}$$

For a rotating planet of radius R at a uniform temperature over the sphere this temperature is related to the incident solar flux by

$$4\pi R^2 \sigma T_e^{\,4} = (1 - \Lambda)\pi R^2(\pi \mathscr{F}_\odot) \tag{1.2.46}$$

where Λ is the effective planetary albedo and $\pi\mathscr{F}_\odot$ is the incident solar flux. With $\Lambda = 0.29$ for Earth, we find $T_e = 255°$K and a boundary temperature $T_0 = T_e/1.19 = 215°$K, which is close to the mid-latitude tropopause temperature. If we know the optical thickness of the atmosphere versus height, we can figure the temperature versus height—the lapse rate for radiative equilibrium. An example is given in Problem 1.3.

The discontinuity between the air and surface temperature can be expressed from (1.2.29) as

$$\begin{aligned} T_s^{\,4} &= T^4(\tau_g) + \tfrac{1}{2}T_e^{\,4} \\ &= T_0^{\,4}(2 + \tfrac{3}{2}\tau_g) \\ &= T_e^4(1 + \tfrac{3}{4}\tau_g) \end{aligned} \tag{1.2.47}$$

where τ_g is the optical thickness at the ground. The deposition of sunlight striking the Earth is shown in Table 1.1.

Equations (1.2.43) and (1.2.47) illustrate how high temperatures can be attained near the ground if the infrared τ is large and heating is from below.

TABLE 1.1 *Approximate Percentage Deposition of Incident Solar Flux*[a]

Reflected to space	Absorbed by ground	Absorbed in troposphere	Absorbed above troposphere	Reradiated from ground to atmosphere
40	40	18	2	25

[a] The solar constant outside the atmosphere is 2 cal/min cm^2 = 1.39 × 10^6 erg/cm^2 sec; the global average heat received is one fourth this amount.

Heating from this combination of a transparent atmosphere in the visible, where the sun's energy is a maximum, and of high opacity in the infrared, where the Earth's Planckian curve peaks, is known as the *greenhouse effect.* It has long been thought that the trapping of infrared by glass is not the important thing in warming greenhouses. Rather, it is said, the glass roof merely keeps the warm air from convecting away. Purists have fought a steadily losing battle to replace "greenhouse effect" with a less picturesque term. We prefer to think of atmospheres as warming by the greenhouse effect, even if greenhouses do not.

If there is no internal heat source, the emission temperature T_e, computed from (1.2.46), will be equivalent to the measured *bolometric temperature* T_b, which is obtained by measuring the mean planetary flux from thermal emission over all frequencies and setting it equal to $\sigma T_b^{\,4}$. In the case of Jupiter and possibly other major planets, $T_b > T_e$, indicating internal generation of heat (cf. Section 1.8.3).

If an atmosphere's thermal emission is measured only in a narrow frequency interval, its intensity gives a *brightness temperature* T_B, defined by $I_\nu = B_\nu(T_B)$. If the atmosphere were gray, I_ν would be Planckian and T_B would be the same at all frequencies and the same as T_b. The brightness temperature of Venus in the microwave spectrum gave the first indication of its 750°K surface (cf. Section 1.8.1).

1.3 Convection in the Troposphere

As we have seen, a gray atmosphere in radiative equilibrium approaches a finite "skin temperature" at high altitude. This isothermal region is stable against convective circulation. At large τ, however, the radiative gradient dT/dz becomes steep (i.e., negatively large). Hence, an optically thick, gray atmosphere can be convectively unstable at low altitudes; the temperature distribution that radiative exchange tends to establish is then too steep to be hydrostatically supported.

If an element of gas moves adiabatically, the first law of thermodynamics requires that

$$C_v\,dT = -p\,dV \tag{1.3.1}$$

where C_v is the specific heat at constant volume (erg/gm°K). If V is the *specific volume* containing a gram of molecules, then the perfect gas law gives

$$dV = \frac{N_0 k}{p}dT - \frac{N_0 kT}{p^2}dp \tag{1.3.2}$$

where $N_0 = 1/M$ and M is the molecular mass. Since $C_p = C_v + N_0 k$ (erg/gm°K), we have the alternate thermodynamic relation

$$C_p\,dT = \frac{N_0 kT}{p}\,dp = V\,dp = \frac{1}{\rho}\,dp \tag{1.3.3}$$

With hydrostatic equilibrium, (1.1.1), the first law thus gives the *dry adiabatic temperature gradient*,

$$\frac{dT}{dz} = -\frac{g}{C_p} = -\frac{\gamma - 1}{\gamma}\frac{gM}{k} \tag{1.3.4}$$

where $\gamma = C_p/C_v$. For the Earth's troposphere this lapse rate (the negative of the temperature gradient) is 9.8°K/km.

For saturated air the first law includes the latent heat released by water condensing:

$$C_v\,dT = -p\,dV - L\,dw_s, \qquad C_p\,dT = \frac{1}{\rho}\,dp - L\,dw_s \tag{1.3.5}$$

where w_s is the mass of saturated water per mass of air and L is the latent heat of vaporization. The *saturation adiabatic lapse rate* is then

$$-\frac{dT}{dz} = \frac{g/C_p}{1 + (L/C_p)(dw_s/dT)} \tag{1.3.6}$$

The wet lapse rate may be about half the dry rate or around 5°K/km. Since convection is partly moist and partly dry, the troposphere has an average value of 6.5°K/km. This value characterizes the static stability of the Earth's large-scale weather systems (see Section 2.2.4).

The temperature distribution in radiative-convective equilibrium is thus simply the adiabatic curve at low altitudes, merging into the purely radiative one at higher levels (see Fig. 1.5). Of an infinite number of parallel adiabatic atmospheres we must select the one that emits the same upward radiant flux as the radiative atmosphere itself. For example, curve 1 in Fig. 1.5 is tangent to the radiative curve and would not require a temperature discontinuity. But the adiabatic curve is everywhere below the radiative one and it clearly cannot supply the radiant flux required above point A to support the temperature there in radiative equilibrium.

A self-consistent solution is an adiabatic curve displaced to the right by an amount such that the flux boundary condition is satisfied. The temperature discontinuity at the ground must also be removed, since it is unstable convectively. The flux given by the various adiabatic distributions is readily computed from the formal solution to the transfer equation (1.2.16).

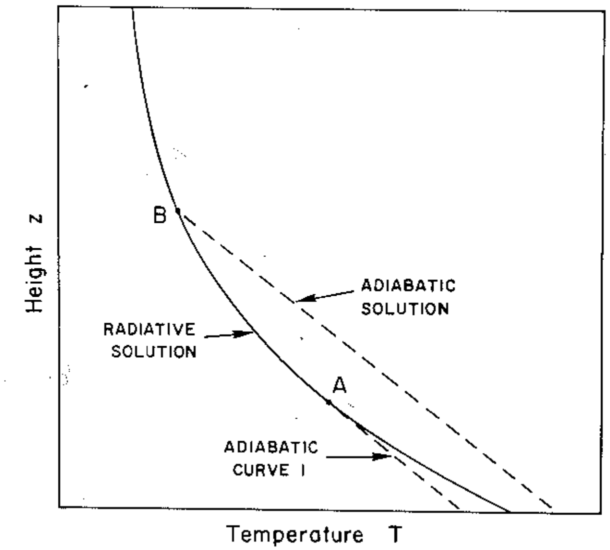

Fig. 1.5 When convection is present the adiabatic portion of the curve not only extends to a point where the radiative solution is convectively stable; it must also supply the thermal radiation to sustain the profile above "*B*." Point *B* is called the *radiative-convective boundary*. The tropopause, defined as the base of the first isothermal region, is just above the top of the figure.

1.4 Latitudinal Variations of the Tropopause and Departures from Grayness

The radiative-convective model with a gray atmosphere reproduces the main features of the troposphere temperature distribution, with the radiative "skin temperature" being identified with the near-isothermal region of the tropopause. This happy state of affairs does not hold, however, when we examine some of the finer features. Figure 1.6 shows isotherms on meridional cross sections for the various seasons. The heavy line shows the location of the tropopause. It is apparent from (1.2.45) and (1.2.46) that in the gray atmosphere, heated from below and without dynamical interchange latitudinally, the tropopause would be warmer in the tropics than in the Arctic simply because the ground temperature is higher. In fact, the tropic tropopause is around 15 km with $T \approx 195$°K, whereas over the polar cap it is as low as 8 to 10 km and $T \approx 225$°K.

Why should the tropopause be lower and warmer in the Arctic? There are several contributing factors. Quite likely the most important is the fact that the atmosphere is not only nongray but the distribution of the infrared active gases (CO_2, H_2O, and O_3) varies with latitude.