Astronomy 220A
Problem Set #2: October 24, 2008
Due: Wednesday, November 5, 2008

1) Given a spherical equilibrium configuration of mass M and radius R, assume that the density distribution is given by $\rho(r)=\rho_c(1-r/R)$. Express all results below in terms of M and R.
a) Calculate the mass distribution m(r).
b) Calculate the total gravitational potential energy.
c) Calculate the pressure distribution P(r). It is fine to assume P(R)=0.
d) Assume an ideal gas equation of state. Calculate the total internal energy and show that the virial theorem is satisfied. Page 16 of Kippenhahn and Weigert may be helpful.

2) Free-free wavelength dependent opacity has the form, $\kappa_\nu \sim Z^2 \rho T^{-1/2} \nu^{-3}$. Using the definition of the Rosseland mean opacity κ (equation 5.19 in Kippenhahn and Weigert, and discussed in class), show that a factor ν^α in κ_ν gives a factor of T^α in κ. This directly leads to the $\kappa \sim T^{-7/2}$ dependence for free-free absorption.

3) Let's use some polytropes. As mentioned, contrary to all stellar structure textbooks, there *are* physical objects with polytrope index n=1. They are giant planets and brown dwarfs. In class we wrote that $\theta_1=(\sin \xi)/\xi$. Let's work with Jupiter, which has a radius of 70,000 km (1 R_J) and a mass of 1.9e30 g (1 M_J).
a) Using what is given above, solve for K.
b) Now solve for the central density, ρ_c.
c) Write out the density as a function of r, $\rho(r)$.
d) Plot pressure vs. density and density vs. radius for the polytrope Jupiter, and compare those to a "real" Jupiter model at http://www.ucolick.org/~jfortney/classes/Jup_Guillot99.dat Note that this model lacks a heavy element core, which is allowed, given the gravity field.
e) Using what you know, solve for the radius of a 50 M_J n=1 polytrope.

3) Develop a simple numerical procedure for calculating ε, the nuclear energy generation in ergs/g/s, as a function of density, temperature, and composition. This procedure will be part of the stellar model construction. Include hydrogen burning only, but consider both pp and CNO chains. You may assume that the reaction cycles have reached equilibrium. As a check, calculate the ratio of pp to CNO energy production for T_6=18, ρ=80 g/cm^3, X=0.70, Y=0.28. Include the program as part of your solution. You may approximate the contribution of the proton-proton II and III. (See Kippenhahn and Weigert Section 18.5.) Use weak screening for f_{11}. Assume X_{CNO} (CNO mass fraction) = 0.7 Z, or calculate it.

Problems 4 and 5 refer to the Numerical Recipes program "shootf."
4) Program the subroutines "load1" and "load2." The first should give the values of 4 dependent variables r, ℓ, P, T, at a mass point m slightly out from the center, to be used for starting the outward integrations. Print results. Submit program. Note that the central pressure P_c, the central temperature T_c, the total luminosity L, and the outer radius R are parameters that you guess in advance. Stellar M and μ are chosen. Recall that helpful surface conditions are that $L_{tot}=4\pi R^2 \sigma T_{eff}^4$ and $\kappa P=2g/3$.

5) Program the subroutine "derives." For an independent variable x (the mass coordinate) and the four dependent variables y_i at that point, the subroutine should calculate dy_i/dx. Print the values at the inner and outer boundaries. Submit your program.