Laboratory Report 1
HALL EFFECT IN PLASMA

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ABSTRACT

In this experiment, the Hall Effect is measured in a low-density plasma. By controlling external parameters, such as pressure, magnetic field and discharge current, we will measure the Hall Voltage and determine various properties of the electron gas. The quantities that will be computed from our independent and measured parameters are the number density of electrons, temperature of the electron gas, collision cross section, electron drift speed, average and RMS electron velocities, as well as the Hall Field.

1. BACKGROUND & MOTIVATION

The Hall Effect describes an electric field, $\vec{E}_H$, induced by moving charges, I_D, in a non-parallel magnetic field. These moving charges feel a force characterized by the Lorentz Force:

$$\vec{F} = q(\vec{E} + \vec{u}_q \times \vec{B}) \tag{1}$$

Applying a magnetic field, $\vec{B}$, to a current, $\vec{I}_D$, results in the separation of charge in the direction perpendicular to these two fields. This polarization establishes a perpendicular electric field, $\vec{E}_H$, which builds up and reaches equilibrium when the electric force, $q\vec{E}_H$, exactly counters the magnetic force of (1):

$$q\vec{E}_H = -q\vec{u}_q \times \vec{B} \tag{2}$$

We examine the Hall Effect in plasma, rather than in metal, because the convenient properties of plasma result in stronger, more easily observable effects. Plasma is an electrically neutral ionized gas consisting of free electrons and ions. The low number density of electrons in a plasma allows for drift velocities much higher than would be seen in metal carrying an equivalent amount of current. The relationship between electron drift velocity, $\vec{u}_e$, electron density, n_e, and current density, $\vec{j}$, is given by:

$$\vec{u}_e = \frac{-\vec{j}}{en_e} \tag{3}$$

As can be inferred directly from (2), these high drift velocities result in a stronger Hall Field.

2. EQUIPMENT & METHOD

For this experiment, we use a gas mixture of 98.9% helium, 1% argon, and 0.1% nitrogen. The majority of the free electrons in the gas will be supplied by ionized argon. A large potential difference is then set up across the 75mm length of the tube and, with adequate voltage and pressure, an electric field is established. Since the gas possesses some resistance, discharge current is established when the frictional force is balanced by the electric force:

$$\vec{F}_r = \vec{F}_q$$

$$-m_q \nu_q \vec{u}_q = q\vec{E}$$

Once a steady current is achieved, a magnetic field can be applied perpendicular to the 8mm radius of the discharge tube. From (2), an electric field is induced by the separation of charge caused by the magnetic force. Once the electric field reaches equilibrium, thus the Hall Field has been established, the Hall Voltage can be measured between probes 1 and 2 using a voltmeter, as shown in Figure 1. Similarly, the ohmic voltage, spanning the 75mm length of the tube, can be measured between probes 2 and 3.

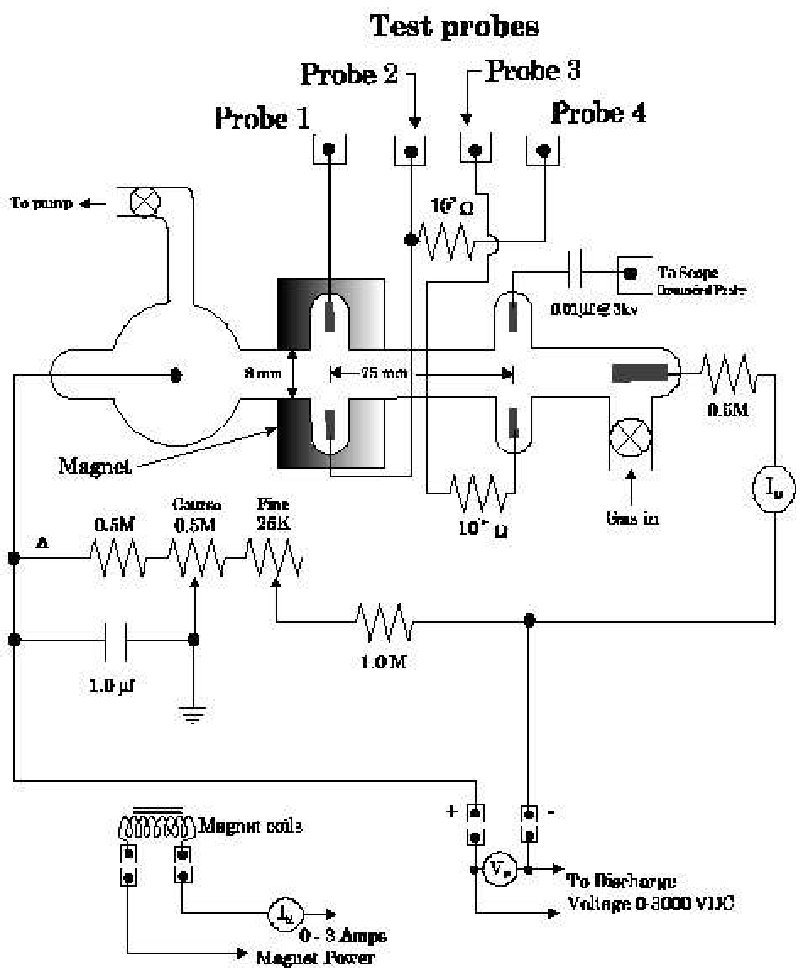

Fig. 1.— Hall Effect tube and magnet electronic diagram.

Since plasma is capable of supporting many wave types, it is susceptible to instabilities, oscillations and striations. Minute charges in current or gas density result in significant voltage oscillations. In order to protect our measuring devices, low-pass filters were used. Also, to ensure accurate data collection, pressure was held constant while taking measurements. Oscillations were observed on an oscilloscope, with care to keep fluctuations below 50mV, peak to peak.

Our procedure began with a check of electrical and gas flow connections to ensure all were off and closed, respectively. The pump was turned on and the pump-out valve opened, shown in Figure 1, to remove any gas occupying the tube. After a moment, the vacuum gauge declined and rested at 3.5 torr. Next, we opened the main and the coarse valve on the helium supply tank to provide gas for the experiment, although it was not yet given access to the tube. Then, the pump-out valve was closed, and the out-flow valve opened. We careful adjusted the in-flow valve to establish a flow of gas from the tank into the tube. In addition, we adjusted the regulator valve to increase the pressure above the blue mark on the gauge (set at $5\frac{lb}{in^2}$). Once the pressure increase was established in the tube, adjustments were made using the out-flow valve; the pressure in the tube was kept between 15 and 30 torr.

After a few minutes, to ensure steady flow, the high voltage supply was turned on (>2000 volts). Next, the oscilloscope was attached to observe fluctuations; pressure, voltage and flow rate were adjusted until oscillations resided below 50 mV, peak-peak. The probe circuits were then turned on, shorted and zeroed. Probe 2 was adjusted to be referenced to ground potential. Each time a change was made in conditions, and a subsequent measurement recorded using the voltmeter, probe 2 was re-referenced to ground.

3. DATA COLLECTION & ANALYSIS

Data was acquired to establish relationships between pressure, voltage, current and magnetic field strength. For varying pressures, discharge voltage was measured as a function of discharge current, magnetic field strength was measured as a function of magnet current, and the Hall Field was measured as a function of magnetic field strength. These relations provided a framework for determining the properties of the electron gas.

3.1. Discharge Voltage vs. Discharge Current for Various Pressures

The discharge voltage, V_0, was measured as a function of discharge current, I_d, for pressures varying from 15 to 30 torr in increments of 3 torr. The potential difference between probes 2 and 3 was measured, reporting values on the order of a few volts. However, since probes 2 and 3 were connected with a $10^8\Omega$ resistor, and probe 3 was in series with a $10^{10}\Omega$ resistor, the voltage divider formula was employed to measure the actual discharge voltage, V_0. Noting that the current is constant throughout the circuit, the voltages between V_0 and probe 2 can be equated with the current between probes 2 and 3:

$$\frac{V_0}{10^{10}\Omega + 10^8\Omega} = \frac{V_{measured}}{10^8\Omega}$$

So that the discharge voltage is given by:

$$V_0 \approx 100V_{measured}$$

The resulting measurements made for discharge currents ranging from $\frac{1}{2}$-2 mA are plotted in Figure 2, and the corresponding data are listed in Table 2 in the Appendix. Error bars in Figure 2 correspond to voltage oscillations and reading errors, however, the latter was the greater contributor.

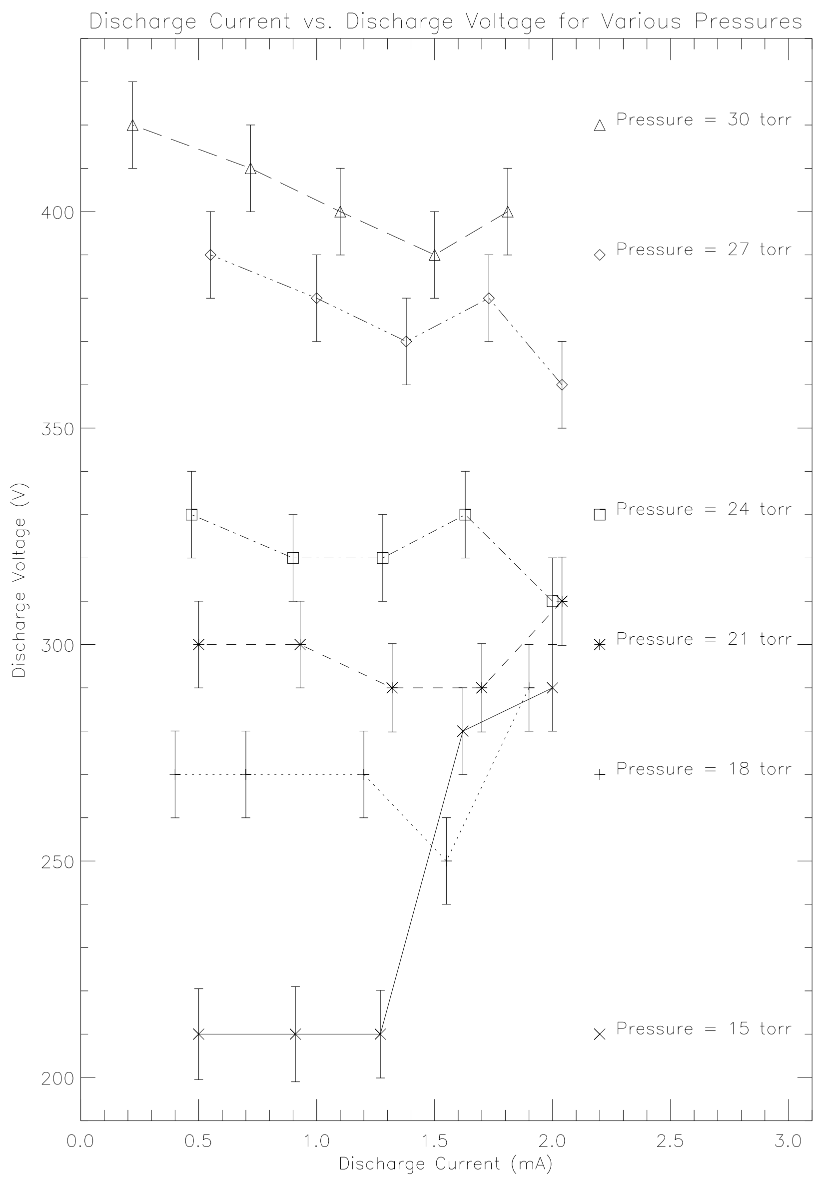

Fig. 2.— Discharge current vs. discharge voltage for ranging pressures.

As displayed in Figure 2, a near horizontal relationship exists between discharge current and voltage. The large deviations in discharge voltage at 15 torr of pressure for currents greater than 1.5 mA are a direct result of complications with referencing probe 2 to ground before taking measurements. For this reason, these two points ($I_d = 1.62, 2.00$ mA, $V_0 = 280, 290$ V, respectively) have been discarded in assessing the linear relationship between discharge voltage, discharge current and pressure. Using a least squares fit, the following linear relationships were established for the corresponding pressures:

$$\begin{aligned} V_0 &= 210\text{V} && (15 \text{ torr}) \\ V_0 &= 270\text{V} && (18 \text{ torr}) \\ V_0 &= (-12 I_d/mA + 308)\text{V} && (21 \text{ torr}) \\ V_0 &= (-13 I_d/mA + 334)\text{V} && (24 \text{ torr}) \\ V_0 &= (-24 I_d/mA + 403)\text{V} && (27 \text{ torr}) \\ V_0 &= (-23 I_d/mA + 425)\text{V} && (30 \text{ torr}) \end{aligned} \tag{4}$$

3.2. Magnetic Field Strength vs. Magnetic Current: Hysteresis Effect

The magnetic fields of ferromagnetic materials exhibit hysteresis behavior. Since ferromagnetic coils were used to generate the magnetic field in this experiment, it was necessary to measure the magnetic field for both increasing and decreasing currents in order to determine how much magnetization lags behind the applied field. Although the purpose of the Hall Effect experiment is to apply a magnetic field to a discharge current, the dangers of operating the Gauss meter and the high voltage supply simultaneously dictated that another approach be taken. Instead, the magnetic field strength, B, was measured as a function of magnetic current, I_m in the absence of a direct current, I_d. This was performed for both forward and reverse polarities. The results are plotted in Figure 3, and the corresponding data is listed in Table 3 in the Appendix.

The hysteresis behavior of the material was found to be minor and observed to be linear. Using a least squares fit, the following relationships between magnetic current and magnetic field strength were established:

$$\begin{aligned} B &= (279 I_m/A - 20) \text{ Gs} && (\text{Reverse Polarity, Increasing Current}) \\ B &= (268 I_m/A - 18) \text{ Gs} && (\text{Reverse Polarity, Decreasing Current}) \\ B &= (-272 I_m/A) \text{ Gs} && (\text{Forward Polarity, Increasing Current}) \\ B &= (-269 I_m/A - 20) \text{ Gs} && (\text{Forward Polarity, Decreasing Current}) \end{aligned} \tag{5}$$

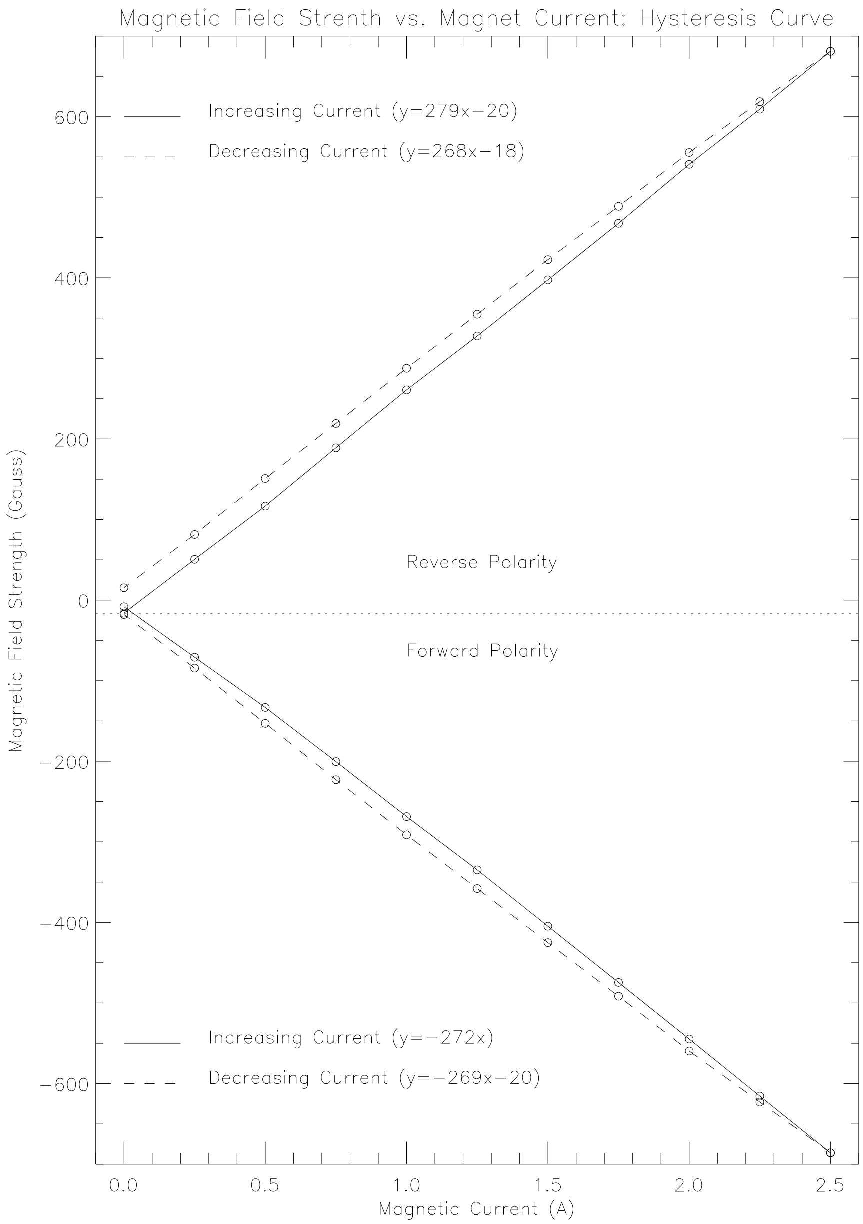

Fig. 3.— Discharge current vs. discharge voltage for ranging pressures.

3.3. Magnetic Field Strength vs. Hall Field for Various Pressures

For a range of pressures between 15 and 30 torr, the magnetic field was varied in order to determine its relationship to the Hall Field. Data was taken for a full range of current by increasing and decreasing current, respectively, and flipping polarities. The data acquired is plotted in Figure 4:

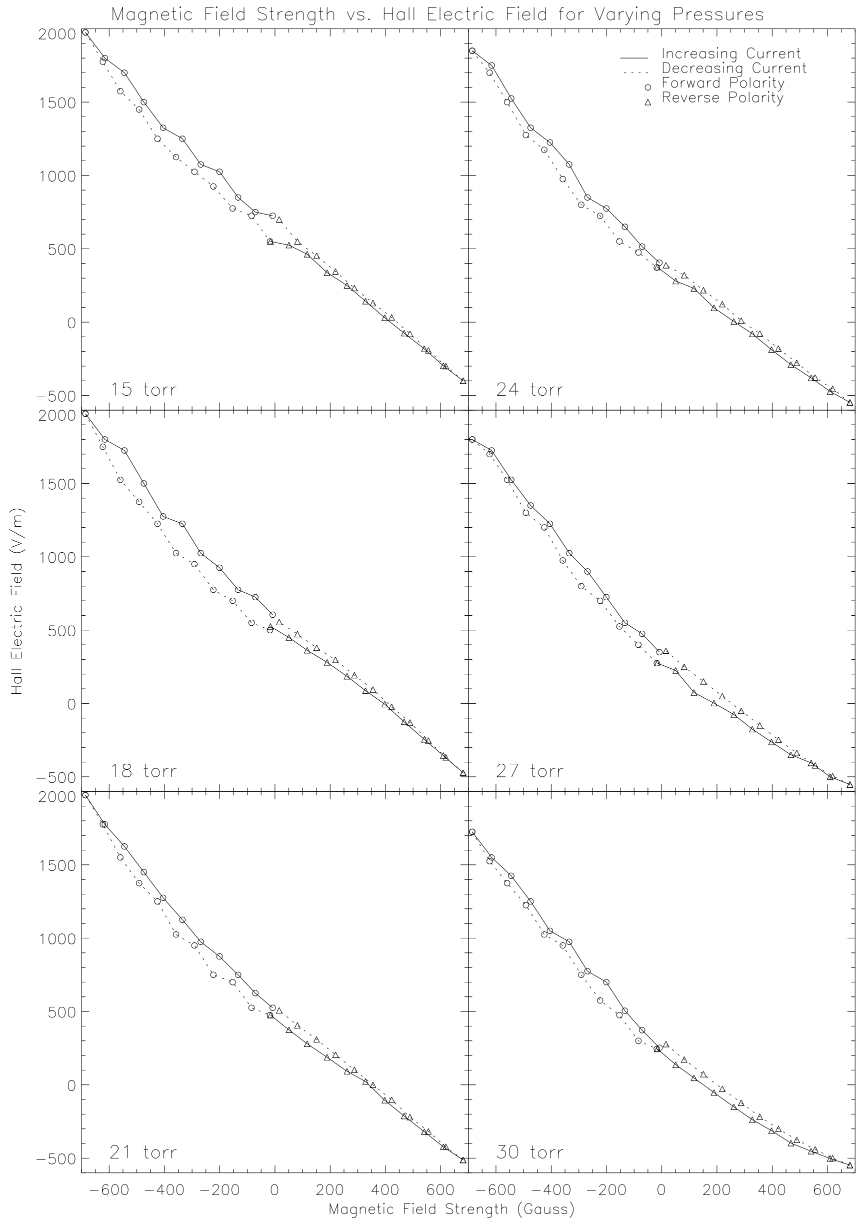

Fig. 4.— Magnetic field strength vs. Hall Electric Field for ranging pressures, current direction, and polarities.

As demonstrated in Figure 4, the Hall Field, E_H, is nearly linear with the magnetic field, B. The scatter seen is a result of reading errors, resulting in an error of $\sigma \approx 250$ V/m. Similarly, reading errors are associated with the magnetic current, and thus the magnetic field strength. However, these errors are significantly smaller than those produced by the voltage readings.

3.4. The Electron Gas

Because of the non-constant electric field (higher in center), the magnitude of the Hall voltage is one-half

$$\mid E_H \mid=\mid 2 \bigtriangledown V_H \mid$$

The electrons travel a distance of 8mm between probes 2 and 3, perpendicular to the electron velocity. Therefore, the Hall Field is given by:

$$E_H = \frac{2}{l} V_H = \frac{1}{4mm} V_H \qquad (6)$$

The ohmic field can be determined in a similar manner. However, in this case, the electric field is a constant over the 75mm length of the tube that the electrons travel, yielding:

$$E_0 = \frac{1}{l} V_0 = \frac{1}{75mm} V_0 \qquad (7)$$

Since the current (and thus the drift velocities of the electrons) and magnetic field are perpendicular, (2) becomes:

$$E_H = u_e B$$

Rearranging and applying (6), we acquire the relation for the magnitude of the electron drift velocity, u_e:

$$u_e = \frac{E_H}{B} = \frac{V_H}{B \times 4mm}$$

(3) relates the electron density, n_e, to the electron drift velocity, u_e. Noting that $j = I/A$ and $A = \pi(4mm)^2$, and employing (6), the following relation can be made:

$$n_e = \frac{j}{eu_e} = \frac{jB}{eE_H} = \frac{IB}{4\pi e V_H (mm)}$$

The collision frequency of electrons, ν_e, can be expressed in terms of the ohmic and Hall voltages by using (6) and (7):

$$\nu_e = \frac{eE_0 B}{m_e E_H} = \frac{4eBV_0}{75 m_e V_H}$$

where m_e is the mass of an electron. Another way of expressing the collision frequency is by relating it to the cross section, σ, for momentum transfer:

$$\nu_e \approx n_g \langle \sigma v \rangle_e \qquad (8)$$

where $\sigma = 3.8 \times 10^{-20} m^2$, n_g is the number density of the gas, and $< \sigma v >_e$ is the average thermal speed of electrons. The number density of the helium atoms in the gas can be determined by the ideal gas law:

$$P = n_g kT$$

Using a room temperature of 295K, the number density of the helium atoms in the gas is:

$$n_g = \frac{P}{k \times 295K} \tag{9}$$

Substituting (9) into (8) yields the following approximation for the average thermal speed of electrons:

$$\langle \sigma v \rangle_e \approx \frac{k\nu_e \times 295K}{P} \approx \frac{4ekBV_0 \times 295K}{15m_e PV_H} \tag{10}$$

Since σ is essentially a constant, we can express the average thermal speed of electrons as:

$$\langle \sigma v \rangle_e \approx \langle \sigma \rangle \langle | \ v \ | \rangle_e$$

so that:

$$\langle | \ v \ | \rangle_e \approx \frac{\langle \sigma v \rangle_e}{\langle \sigma \rangle} \tag{11}$$

In order to find the RMS speed of the electrons, we can assume the speed of electrons is described by a Maxwell Boltzmann distribution, and thus the RMS speed can be related to the average speed by:

$$\sqrt{\langle v^2 \rangle} \approx 1.085 \langle | \ v \ | \rangle_e \tag{12}$$

By relating the internal energy of the electrons to the kinetic energy, we can establish a relationship between the temperature of the electron gas and the RMS velocity:

$$\frac{3}{2}kT_e = \frac{1}{2}m_e \langle v^2 \rangle_e \tag{13}$$

Substituting (12) into (13) and rearranging yields:

$$T_e \approx \frac{m_e}{3k}(1.085 \langle | \ v \ | \rangle_e)^2$$

Using (10) and (11) we find the temperature of the electron gas is given by:

$$T_e \approx \frac{m_e}{3k}\left(1.085\frac{\langle \sigma v \rangle_e}{\langle \sigma \rangle}\right)^2 \approx \left(\frac{m_e 1.085^2}{43.32k \times 10^{-40} m^4}\right)\left(\frac{4ekBV_0(295K)}{15m_e PV_H}\right)^2 \tag{14}$$

We chose to compute the properties for an electron gas with a discharge current of approximately 1 mA and magnetic field strength of -269 Gauss (corresponding to a magnetic current of 1 mA, see Table 3 in the Appendix). The derived quantities were calculated for the range of pressures used and are reported in Table 1. The independent variables listed are the controlled variables, while the dependent variables are a combination of measured (i.e. E_H) and computed (i.e. V_0 from (4)) quantities.

Slight deviations in the discharge current, I_d, led to some difficulty in drawing conclusions about tight relations. However, an attempt will be made to draw some broad relations about independent and dependent quantities in the following section.

Table 1: Properties of Electron Gas for Fixed (Independent) Variables

Independent Variables						
P (torr)	15	18	21	24	27	30
I_d(mA)	1.09	1.02	1.11	1.05	1.11	1.11
j(A/m^2)	21.7	20.3	22.1	20.9	22.1	22.1
B(Gs)	-269	-269	-269	-269	-269	-269
Dependent Variables						
V_0(V)	230	275	330	339	423	450
E_0(V/m)	3052	3672	4393	4515	5651	5994
V_H(V)	4.3	4.1	3.9	3.4	3.6	3.1
E_H(V/m)	1075	1025	975	850	900	775
u_e(m/s)	4.0×10^4	3.8×10^4	3.6×10^4	3.2×10^4	3.4×10^4	2.9×10^4
n_e(1/m^3)	3.4×10^{15}	3.3×10^{15}	3.8×10^{15}	4.1×10^{15}	4.1×10^{15}	4.8×10^{15}
ν_e(1/s)	1.3×10^{10}	1.7×10^{10}	2.2×10^{10}	2.5×10^{10}	3.0×10^{10}	3.7×10^{10}
n_g(1/m^3)	4.9×10^{23}	5.8×10^{23}	6.9×10^{23}	7.9×10^{23}	8.8×10^{23}	9.8×10^{23}
$< \sigma v >$(1/m^3)	2.7×10^{-14}	2.8×10^{-14}	3.1×10^{-14}	3.2×10^{-14}	3.4×10^{-14}	3.7×10^{-14}
$<\mid v \mid>$(m/s)	7.2×10^5	7.6×10^5	8.2×10^5	8.4×10^5	8.8×10^5	9.8×10^5
$< v^2 >$(m^2/s^2)	6.1×10^{11}	6.7×10^{11}	7.8×10^{11}	8.3×10^{11}	9.2×10^{11}	1.1×10^{12}
T_e(K)	13,300	14,800	17,200	18,200	21,100	24,800

4. CONCLUSIONS

General conclusions can be drawn about the nature of the electron gas. For example, increasing the pressure of the plasma resulted in a significant increase in the electron temperature, T_e. This result is to be expected since an increase in pressure equates to an increase in collisions, and thus an increase in internal energy. This result could have also been anticipated upon examination of the inverse relationship between electron temperature and pressure in (14). Similarly, the increase in the average momentum transfer, $\langle \sigma v \rangle_e$, mean velocity, $\langle \mid v \mid \rangle_e$, and RMS velocity, $\langle v^2 \rangle_e$ are to be expected because of the pressure dependence of the average momentum transfer given in (10). However, since the collision frequency, ν_e, showed no pressure dependence, it is surprising that it consistently increases with pressure even with the fluctuations in the Hall Field. The explanation for this lies in the heavy pressure dependence of the discharge voltage, V_0. Obviously, the discharge voltage is a larger contributer to the collision frequency than the Hall Voltage.

At first, it is unclear if the pressure increase is also responsible for the Hall Field, E_H, decrease. Since the Hall Field is dependent on the electron drift velocities, u_e, the overall decline of the field is most likely attributable to the fluctuations in discharge current, I_d. However, the fact that the discharge currents for 21, 27, and 30 torr of pressure are equivalent, and yet there is still a decrease in the Hall Field, suggests that there is possibly some minor pressure dependence, that is E_H is a function of P. Likewise, the slight increase in electron number density, n_e, with pressure suggests some weak relation between these two quantities also.

The intent of this lab was to become familiar with the electro-magnetism in the realm of mediums such as plasmas. Better conclusions could have been drawn about the properties of the medium through a full analysis of the acquired data. The range of recorded values plotted in Figure 4 and discussed in sections 3.3 and 3.4 are not provided in a table due to overwhelming number of data points collected. However, a record of the measurements are available upon request.

5. ACKNOWLEDGMENTS

I would like to thank my lab partner, Deborah Hutchings, for her patience and persistence with this lab. I would also like to acknowledge Don Orlando for his infinite wisdom on tinkering with the equipment. Without him, we'd likely still be trying to figure out "what's wrong with this thing?!?".

6. APPENDIX

Table 2: Measure of Discharge Voltage as a Function of Discharge Current for Varying Pressures

Pressure (torr)	I_D (mA)	V_0 (V)	Oscillations p-p (mV)
15	2.00	290	16
15	1.62	280	14
15	1.27	210	12
15	0.91	210	100
15	0.50	210	50
18	1.90	290	16
18	1.55	250	12
18	1.20	270	12
18	0.70	270	12
18	0.40	270	25
21	2.04	310	20
21	1.70	290	20
21	1.32	290	20
21	0.93	300	18
21	0.50	300	16
24	2.00	310	16
24	1.63	330	16
24	1.28	320	16
24	0.90	320	12
24	0.47	330	12
27	2.04	360	16
27	1.73	380	16
27	1.38	370	12
27	1.00	380	12
27	0.55	390	12
30	1.81	400	16
30	1.50	390	12
30	1.10	400	12
30	0.72	410	12
30	0.22	420	8

Table 3: Measure of Magnetic Field Strength as a Function of Magnet Current: Hysteresis Curve

I_m (A)	B-forward polarity (Gauss)	B-reverse polarity (Gauss)	Current Direction
0	-8.26	-16.17	increasing
0.25	-70.9	50.58	increasing
0.5	-133.22	116.77	increasing
0.75	-200.42	189.01	increasing
1.0	-268.65	260.89	increasing
1.25	-334.8	327.8	increasing
1.5	-404.7	397.4	increasing
1.75	-474.5	467.6	increasing
2.0	-544.7	540.8	increasing
2.25	-615.5	609.3	increasing
2.5	-685.9	681.1	increasing
2.25	-622.9	618.6	decreasing
2.0	-559.6	555.5	decreasing
1.75	-491.6	488.7	decreasing
1.5	-425.1	422.5	decreasing
1.25	-358.0	354.8	decreasing
1.0	-291.3	287.7	decreasing
0.75	-222.9	519.2	decreasing
0.5	-153.0	150.9	decreasing
0.25	-84.4	81.46	decreasing
0	-17.9	15.36	decreasing