

Astro 13 Galaxies & Cosmology

LECTURE 9 Tues 27 April 2010 D. Koo

- 10m I Ask An Astronomer Period
- 20m II Finish up Black Holes
- 20m III Curvature, Density, & Evolution
- 10m V Break
- 45m VI Evolution of the Universe Geometry
- **LATE Homework 2 is Due at next class**
- ***Homework 3 DUE Next Week 4 May, no late accepted!***
- ***MIDTERM -- Next Week Thur 6 May 2010***

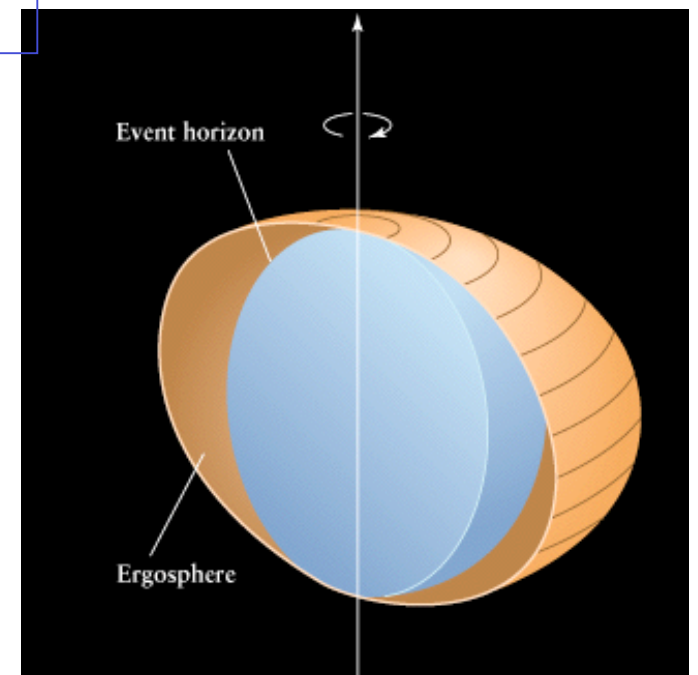
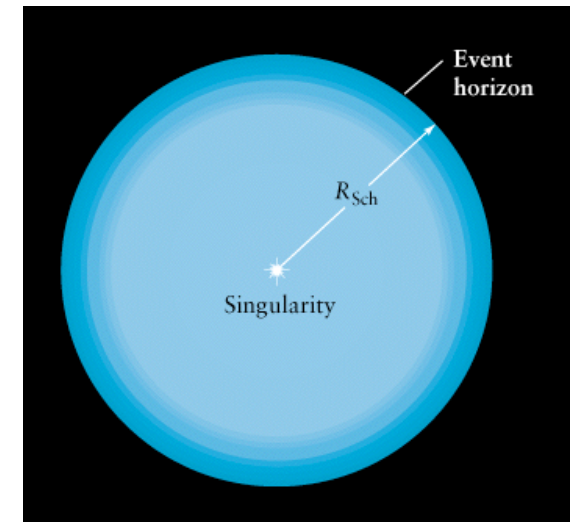
Structure of a Black Hole

A simple non-rotating BH is described by only its center (Singularity) and a surface (Event Horizon). The radius of the event horizon is known as the Schwarzschild Radius $R_{Sch}(m) = 2GM/c^2$
 $G = \text{const. gravity} = 6.67 \times 10^{-11} \text{ newton m}^2/\text{kg}^2$
 $c = \text{speed of light}$ $M = \text{mass of BH in kg}$
For sun, $R_{Sch} = 3\text{km}$ - smaller than Santa Cruz!

In principle, besides its **mass**, BH may also have **charge** and may be rotating (**angular momentum**). In the latter case, space-time appears to be dragged around so that objects grazing the ergosphere would be flung back out faster than it went in -- “FREE” KICK of ENERGY!

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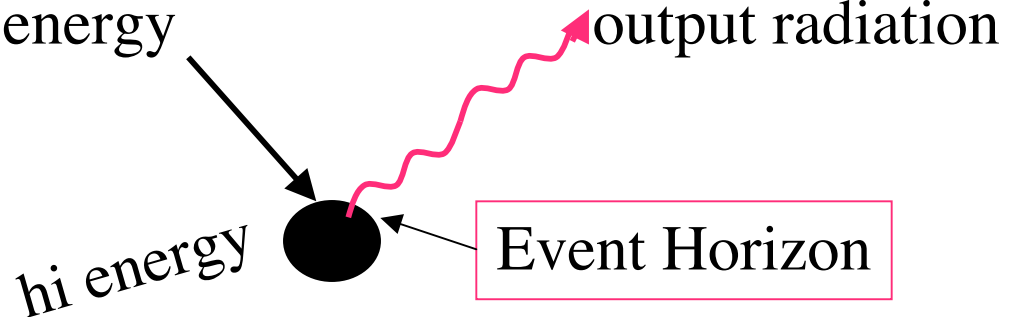


Properties of Black Holes

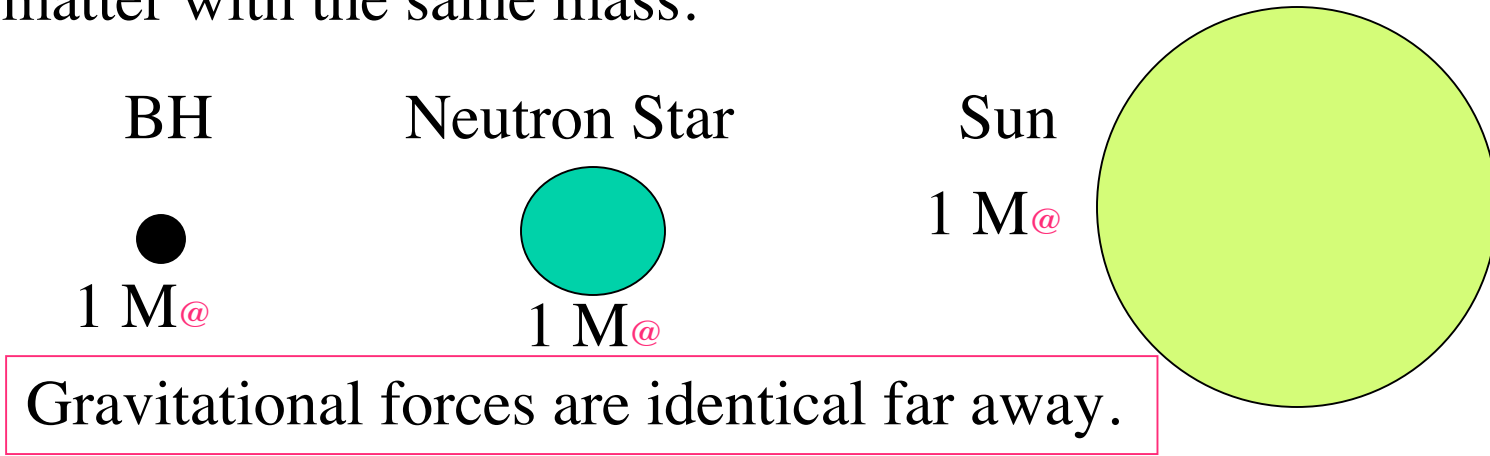
* True in general, but to lesser degree

1) Light cannot escape within Event Horizon (closer than Schwarzschild radius) -- reason for name sake of BLACK

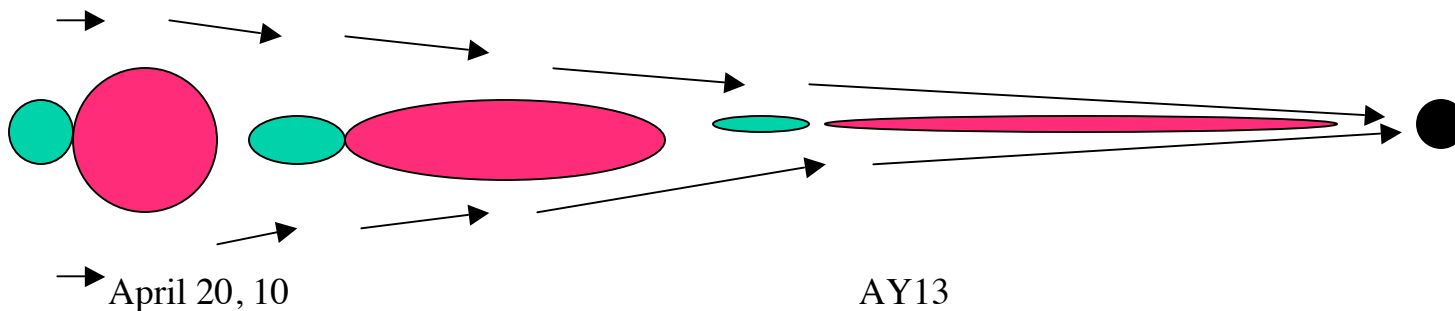
2*) Matter falling into the BH gains enormous kinetic energy, so regions outside the event horizon may give off enormous amounts of radiation and energy. low energy



3*) BH do NOT act like vacuum cleaners sucking up their surroundings. They act no differently than a much larger chunk of matter with the same mass.



4*) Objects falling into a BH would get stretched and squeezed to very high temperatures, and eventually split apart, even on the atomic level (from hot spaghetti to subatomic soup to nothing).



5*) Light from objects closer to a BH appear redshifted to a far away observer -- **Gravitational Redshift:**



Due to energy loss leaving the gravitational pull of the BH, similar to slowing down of a ball thrown up above the earth.

6*) Similarly, just as light frequency drops (wavelength increases) due to drop in energy, clocks appear to tick more slowly --

Time Dilation

BH



7) As observed object approaches the event horizon, **light redshifts to infinity** and **clocks appear to stop** to the outside world, but infalling object continues to the Singularity without noting any such peculiarities.

8) As stuff falls into a BH, outside world can only know:

MASS (independent of material -- rocks, iron, water or even light)

CHARGE (electric charge of + or -)

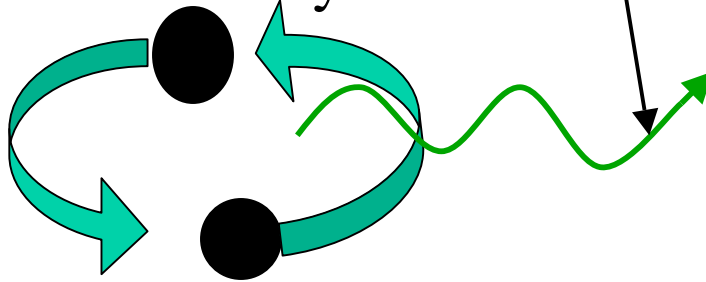
SPIN (angular momentum)

9*) Like E&M radiation, changing the mass distribution generates

GRAVITY WAVES

Prediction of **Gravity Waves** -

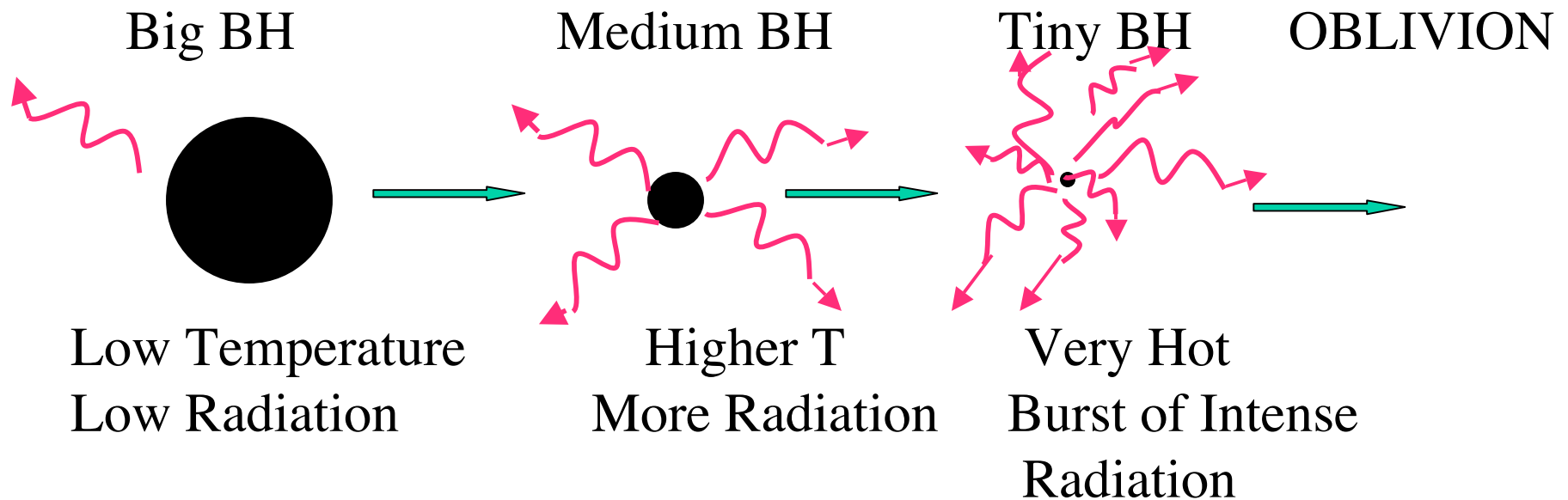
- observed in Binary Neutron
stars



Note objects can be BH or
more ordinary objects like stars.

10) SURPRISE!! Stephen Hawking has predicted that **UNFED Black Holes glow like blackbodies**, lose mass, and eventually explode with high temperature gamma radiation into oblivion.

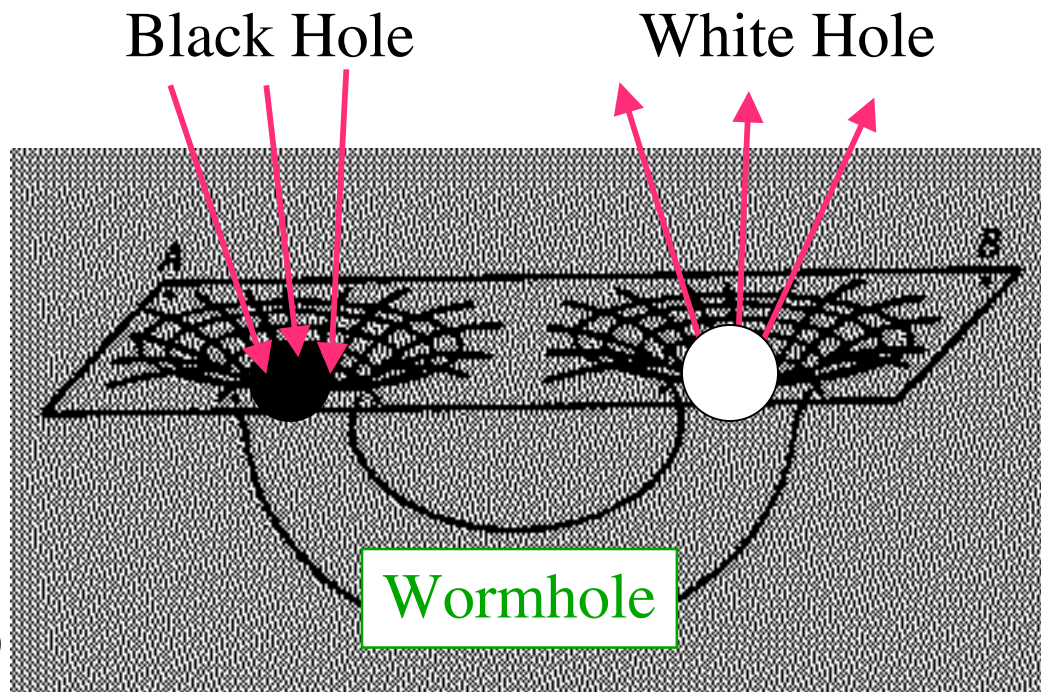
HAWKING RADIATION



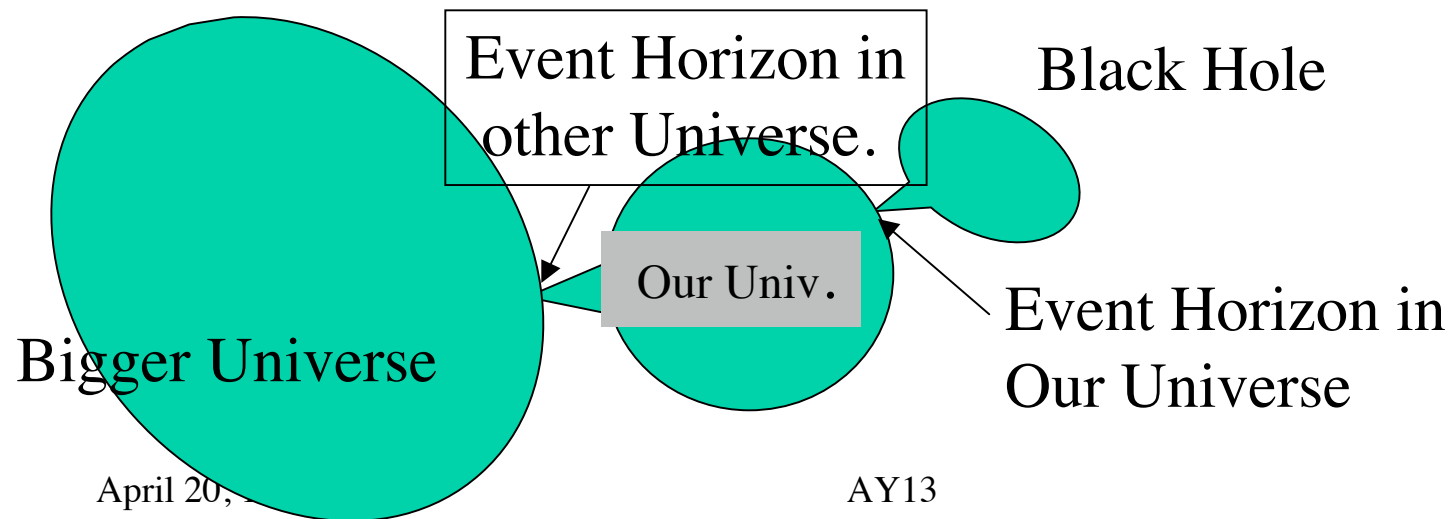
What Happens as One Enters within BH's Event Horizon into Singularity?

Physicists are unsure, since current physical laws do not apply.
Some unproven and unobserved proposals include:

- 1) BH connect through WORMHOLES into ours or other U or even White Holes, which spew out matter and energy (QSO)?



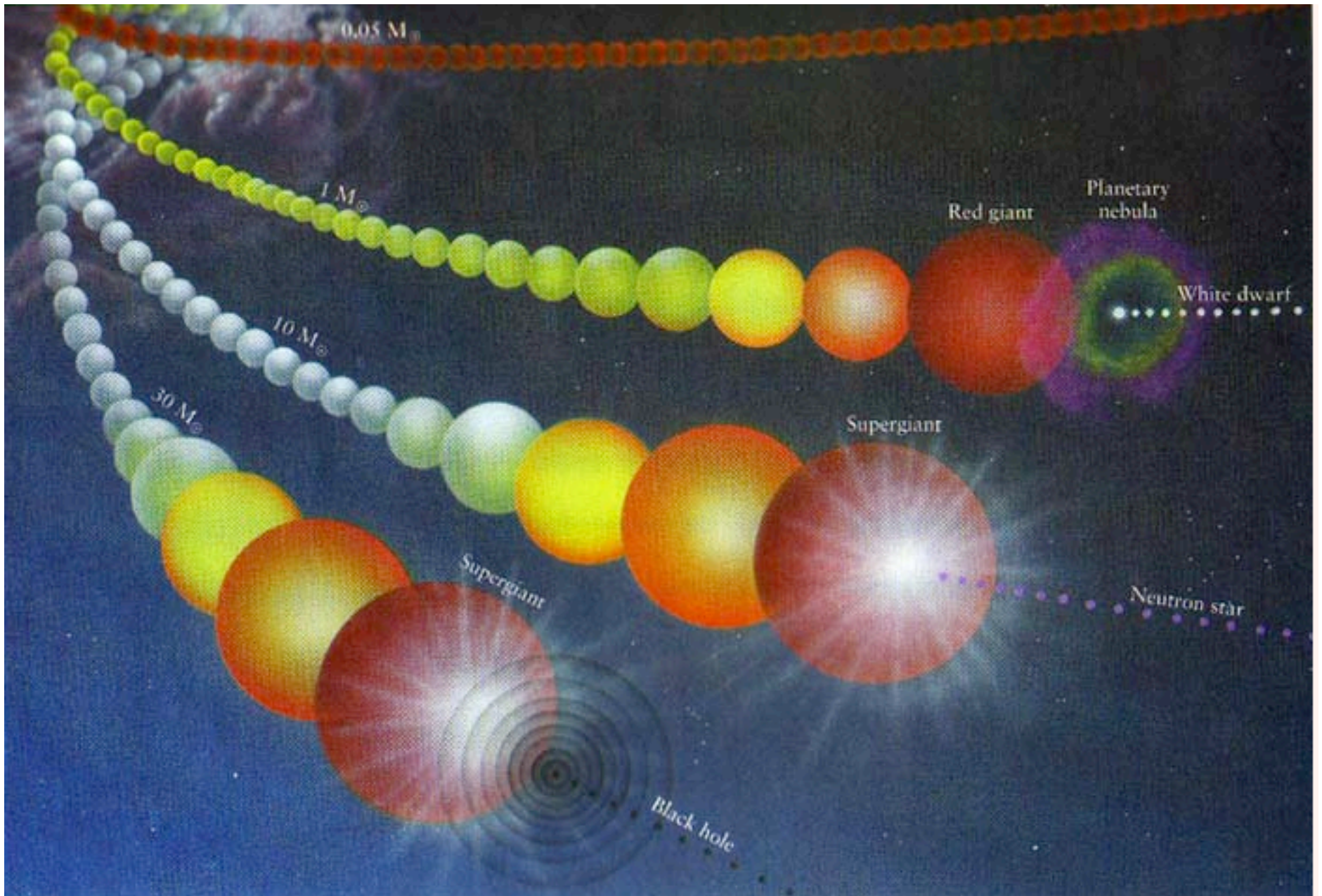
- 2) Creation of **new states of matter**.
- 3) Time travel within our U via **Einstein-Rosen bridges**, but then **causality**, meaning cause should occur before the effect, might breakdown.
- Some have proposed the new **PRINCIPLE of Cosmic Censorship**, so that TIME TRAVEL is not possible.
- 4) But maybe we **live in a huge Black Hole** -- the U itself. A BH is viewed as sealing itself from our U, but maybe we are sealed from an even larger U.



How can a Black Hole be Created?

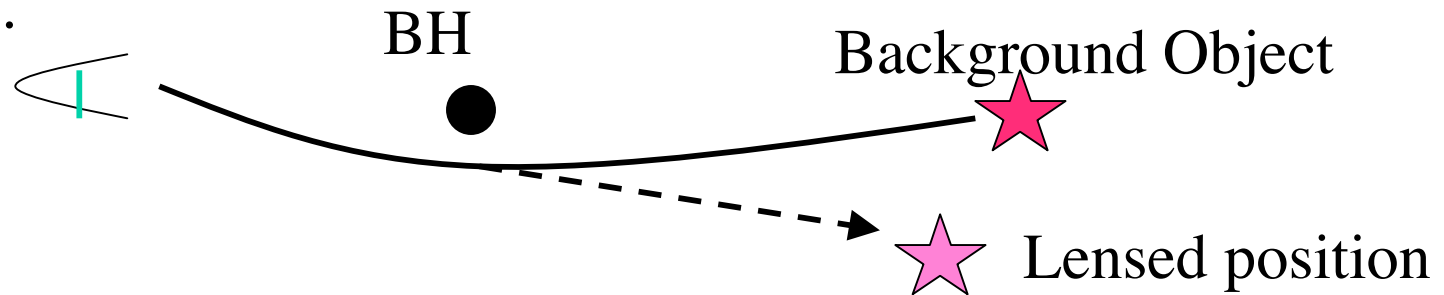
Simple Answer: squeeze a chunk of material to very small sizes: that is to the size of the Schwarzschild Radius.

Humans have not been able to do this in a laboratory, but exploding stars may squeeze their cores so much as to produce a BH. Theory predicts that if this core is more than 1.5-3.0 M_{\odot} , it will collapse into a blackhole. After the formation of a “seed” Black Hole, it can continue to grow by addition of mass from infalling gas and stars or perhaps other Black Holes.



How Can one Find a BH in Space?

1) See the effect of a BH via **bending of light**. E. g., if the Sun were a BH, we would see stars near it appear to move. But it is difficult to know where to look. So far, no candidates via this method.

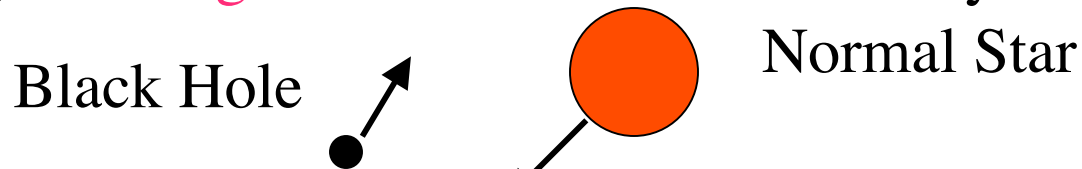






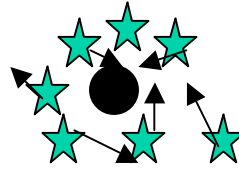
How Can one Find a BH in Space?

2) See its **gravitational effects** on nearby companion stars.

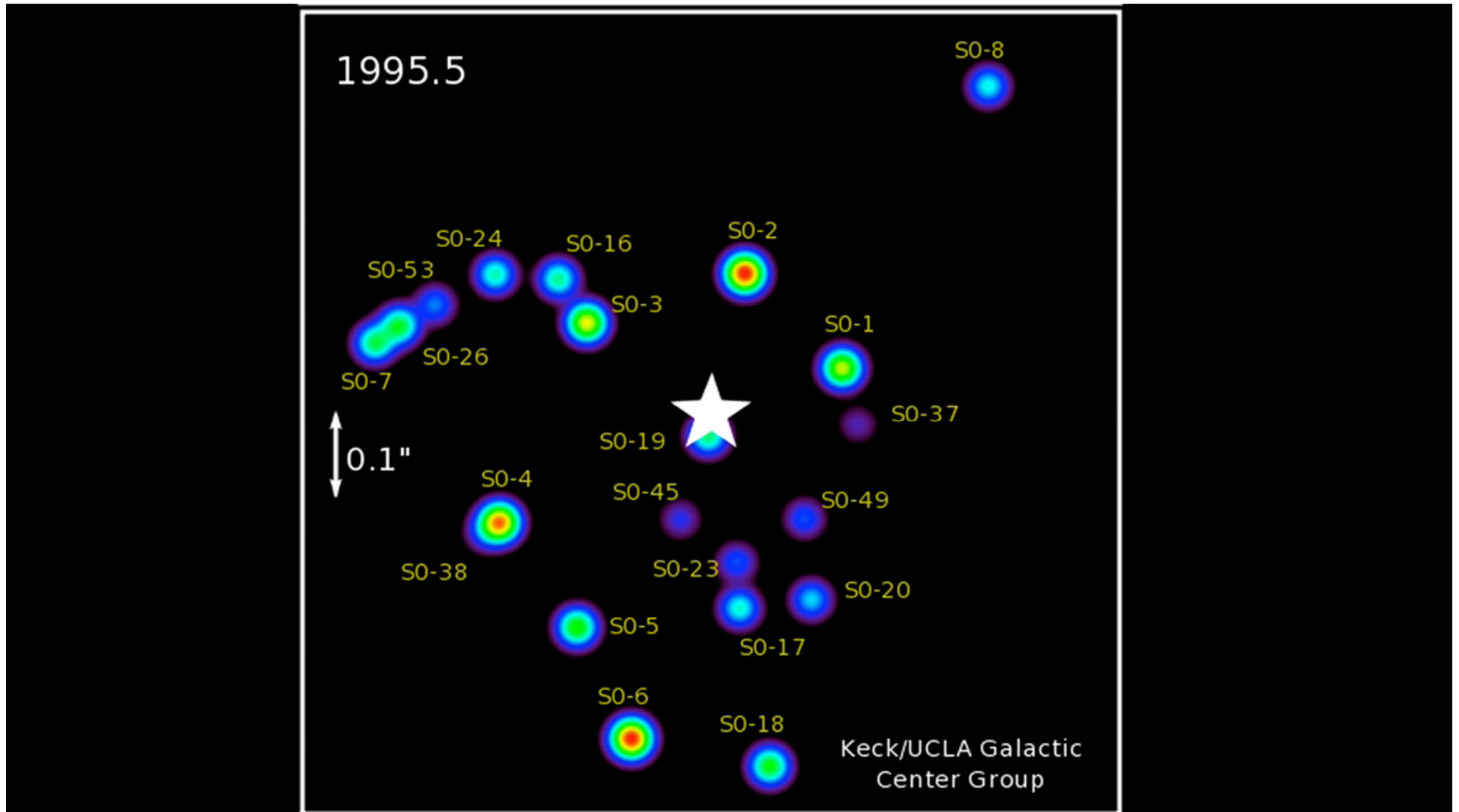


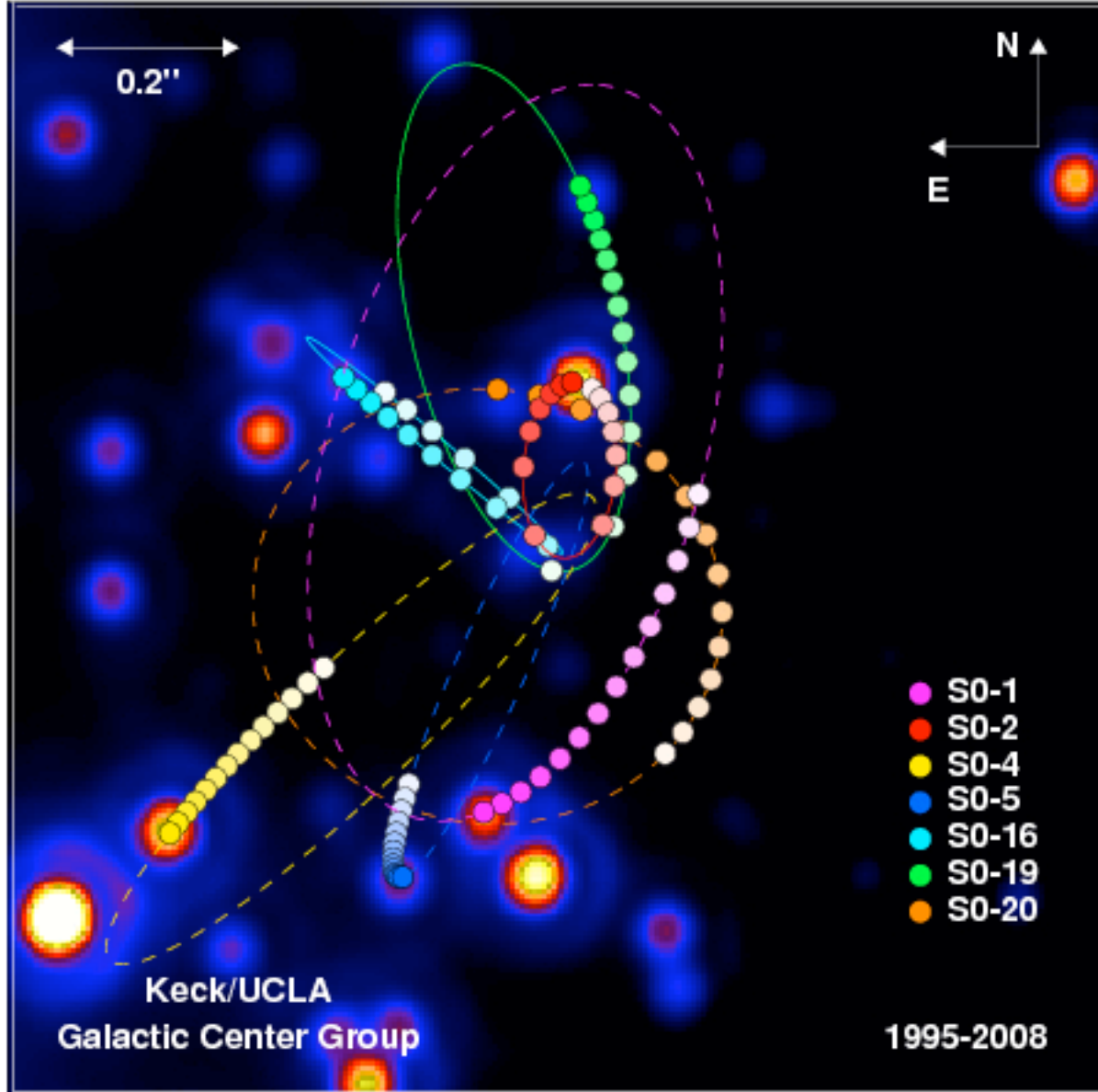
Astronomers have found two handfuls of good candidates, but for many, we cannot exclude other “Dark” objects, such as neutron stars.

3) Measure a very large mass in a small volume that is darker than expected.



Use the motions of surrounding stars to estimate masses.

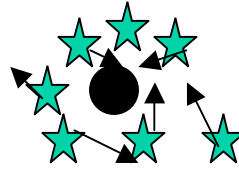




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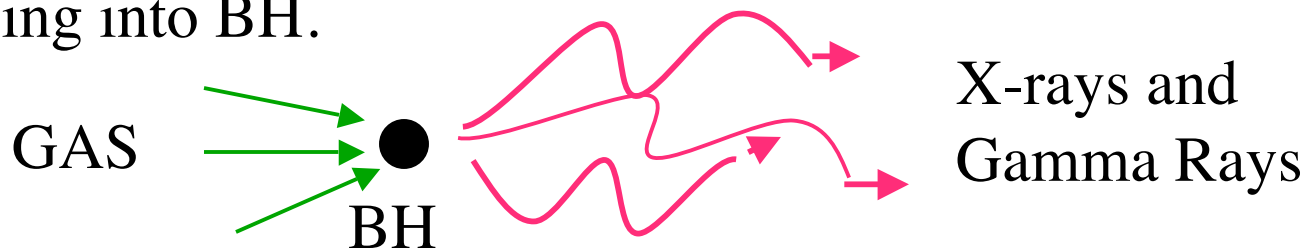
3) Measure a very large mass in a small volume that is darker than expected.



Use the motions of surrounding stars to estimate masses.

Using this method with Hubble Space Telescope, astronomers find the centers of many Galaxies to be massive and yet very small and darker than if made of ordinary stars.

4) find very high energies from tiny regions of space due to matter falling into BH.



5) Bursts of Gamma Rays from evaporating BH - Hawking Radiation.

Models of the Universe

Many possibilities exist, but we will focus on those which:

- Satisfy the Cosmological Principles
- Satisfy Einstein's Theory of General Relativity
- Satisfy simple geometries (curved spaces) - there are 3 basic types

We will also consider **INFLATION**, a process that occurred just after the Big Bang and is important in solving 3 major

Cosmological Mysteries:

- 1. Horizon Problem
- 2. Flatness Problem
- 3. Monopole Problem

Curvature of Space

There exists three basic curvatures:

POSITIVE

ZERO

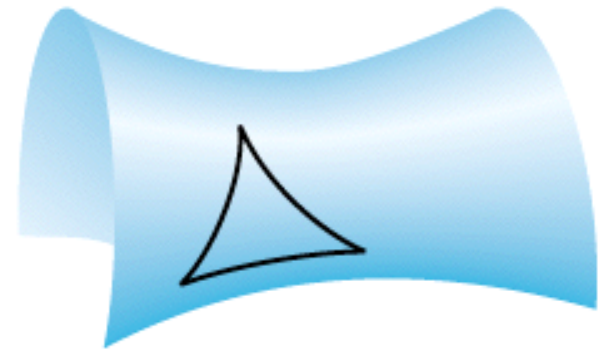
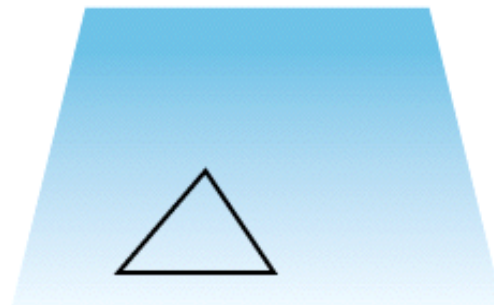
NEGATIVE

with 2-Dimensional equivalents that resemble the SURFACES:

SPHERE

FLAT

**HYPERBOLIC
(SADDLE)**



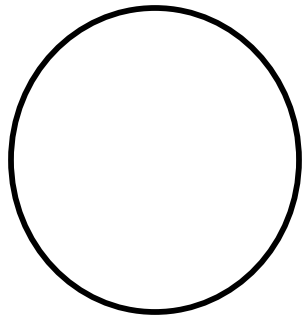
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Observable (Measurable) Properties of Curvature

- 1) Locally Flat (Euclidian) such that Special Relativity applies
- 2) Minimum path between two points (geodesic) appear as:

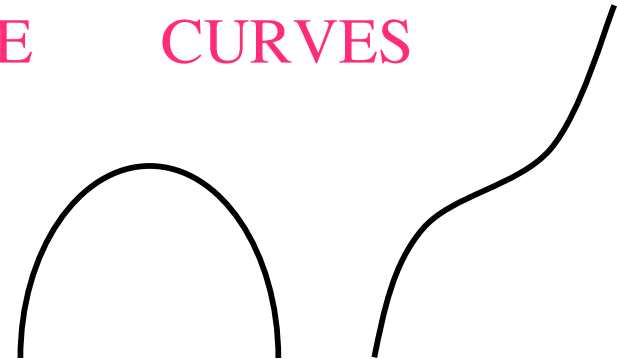
GREAT CIRCLE



STRAIGHT LINE



CURVES



Observable (Measurable) Properties of Curvature

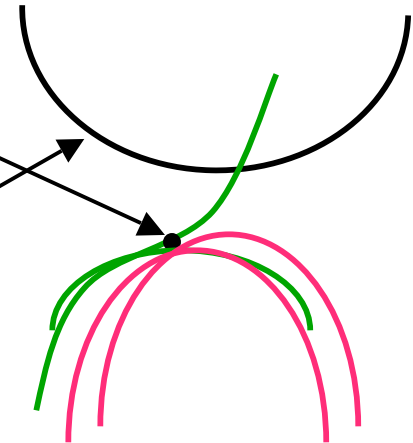
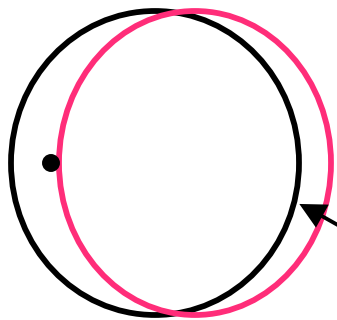
3) Number of Parallel Lines through a point away from
a

Geodesic:

All Geodesics cross,
So no parallel
Geodesics - 0

If geodesics are
Parallel, they
never cross & $\forall v$

An infinite number
of non-intersecting
lines are found in
Negative curvatures

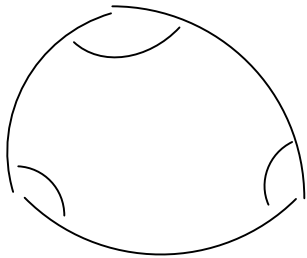


Geodesics

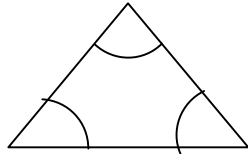
Observable (Measurable) Properties of Curvature

4) Sum of angles in triangles depend on geometry:

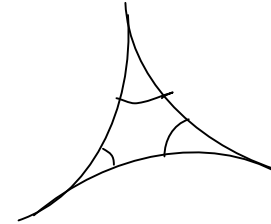
$> 180^\circ$



$= 180^\circ$



$< 180^\circ$



5) Finiteness

FINITE
“CLOSED”

INFINITE
“FLAT”

INFINITE
“OPEN”

Observable (Measurable) Properties of Curvature

6. Circumference of a Circle (equivalent to surface area in 3-D)

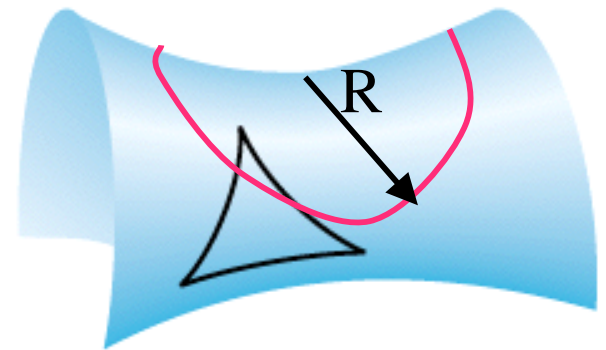
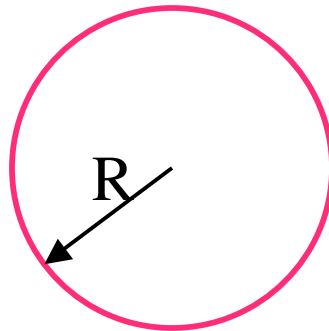
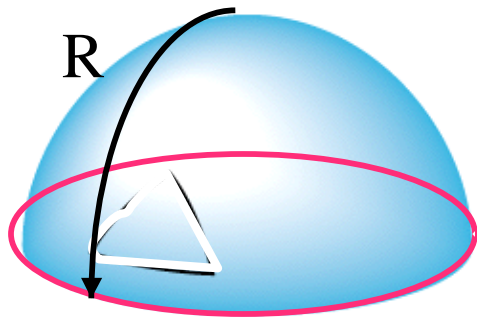
$< 2\pi R$

Pole to Equator

$= 2\pi R$

Normal Circle

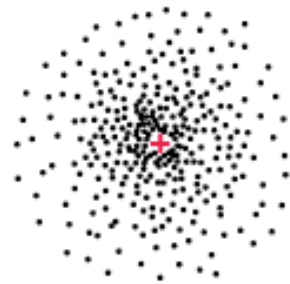
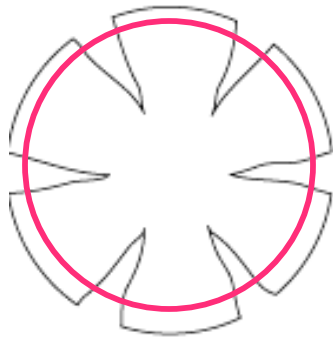
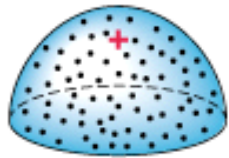
$> 2\pi R$



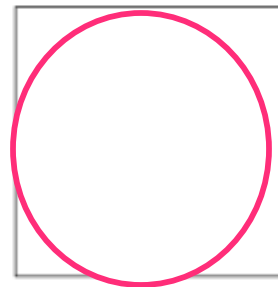
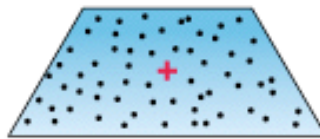
Observable (Measurable) Properties of Curvature

7. Area of a Circle (equivalent to Volume in 3-D)

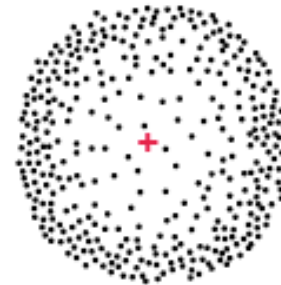
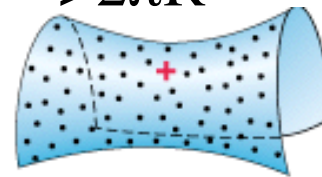
$$< \pi R^2$$



$$= \pi R^2$$



$$> 2\pi R^2$$



Curvature & Density of the Universe

Einstein's Theory of General Relativity plus adopting the Cosmological Principles of homogeneity and isotropy results in a variety of behaviors for the evolution of the Universe. By evolution, we mean how the SCALING SIZE of the universe, R , is changing with time, $R(t)$.

The key physical parameter that controls this evolution (assuming no **Cosmological Constant or Dark or Vacuum Energy**) is the **average density of the Universe** and how it compares to the **CRITICAL DENSITY (D_{crit})**.

Density is the Mass/Volume (kg/m^3)

CRITICAL DENSITY D_{crit}

This is the density of the Universe just enough to barely halt the expansion of the U after an infinite time and is directly equivalent to the idea of Escape Velocity $V_{\text{esc}} \propto \sqrt{\frac{\text{Mass}}{\text{Radius}}}$

A) Mass is just \propto Density(D) x Volume but Volume \propto Radius(R)³ and so Mass \propto D x R³ and so since $V_{\text{esc}}^2 \propto$ Mass/Radius this means that $V_{\text{esc}}^2 \propto D_{\text{crit}} \times R^2$ or $D_{\text{crit}} \propto V_{\text{esc}}^2/R^2$

B) But recall that Velocity = H_0 x Distance(R) or $V/R = H_0$ so we end up with $D_{\text{crit}} \propto H_0^2$ with actual value: $D_{\text{crit}} = \frac{3H_0^2}{8\pi G}$ where G is the Gravitation constant.

C) For $G = 6.7 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ and $H_0 = 70 \text{ km/s/Mpc}$
 $D_{\text{crit}} = 1.0 \times 10^{-26} \text{ kg/m}^3$ which is about 1 proton/m³ vs 10^{30} for water

Einstein's Equations for Evolution of the Universe

Using Newton's Laws, we can derive the equivalent of Einstein's Equations, by balancing energy (kinetic + potential = total) and adopting the gravitational force law ($F = ma = GMm/R^2$)

See page 413 and fig 5.7 in Silk for details.

A key theorem says that in a homogeneous universe, one can regard the force on a point as being the equivalent to that from an arbitrary sphere with the rest of the universe empty. Thus using spherical shells of matter surrounding a sphere of mass M with radius r , and assuming conservation of mass in this shell in an expanding universe, one obtains from the energy balance equation for a shell: $v^2/2 - GM/r = \text{constant}$

The constant in the Newtonian view is merely the total energy, which is conserved (constant) but in Einstein's viewpoint, the constant reflects the curvature of space.

To keep to conventional notation, the constant can be written as $-kC$, where k has the values of 1, -1, or 0 for negative, positive, and zero total energy cases, respectively.

Applying Hubble's Law so $v=H r$ and $M = \frac{4\pi r^3}{3} D$ where M = volume of sphere times density(D),

we get: $\frac{H^2 r^2}{2} - \frac{G4\pi r^3 D}{3r} = -kC$ and after dividing by r^2

We get: $\frac{H^2}{2} - \frac{4\pi G D}{3} = -\frac{kC}{r^2}$

Using instead the force equation $F = ma = -GMm/r^2$, we end up with:

$$a = -\frac{4\pi G D r}{3} \quad \text{Where } a = \text{acceleration of the shell}$$

Since any finite density D will result in acceleration, Einstein arbitrarily added his famous Cosmological Constant Λ to just balance the D term, so that one then has instead:

$$a = -\frac{4\pi G(D - \Lambda)r}{3}$$

In this case, one sees that a **positive Λ acts like negative density or negative gravity, that is a repulsive force**. But since it is a constant, unlike density which drops as the universe expands (r increases), it will dominate with time. The energy equation is also affected and one gets:

$$\frac{H^2}{2} - \frac{4\pi G}{3} D = -\frac{kC}{r^2} + \frac{\Lambda}{3}$$

Evolution of the Universe's Geometry

With Einstein's equations, we can now explore the various possible models of the Universe. Let us start with the energy equation:

$$\frac{H^2}{2} - \frac{4\pi G}{3} D = -\frac{kC}{r^2} + \frac{\Lambda}{3}$$

First note that if k and Λ are both 0 (zero total energy), then we achieve the critical density balance, so that:

$$D_{crit} = \frac{3H_o^2}{8\pi G}$$

In fact, when the Universe was young with a small r , notice that the D term (which rapidly increases as the inverse of the volume or $1/r^3$) will totally dominate over the $1/r^2$ term on the right hand side of the equation or the constant Λ term, so that the Universe acts as if it was close to a zero energy one.

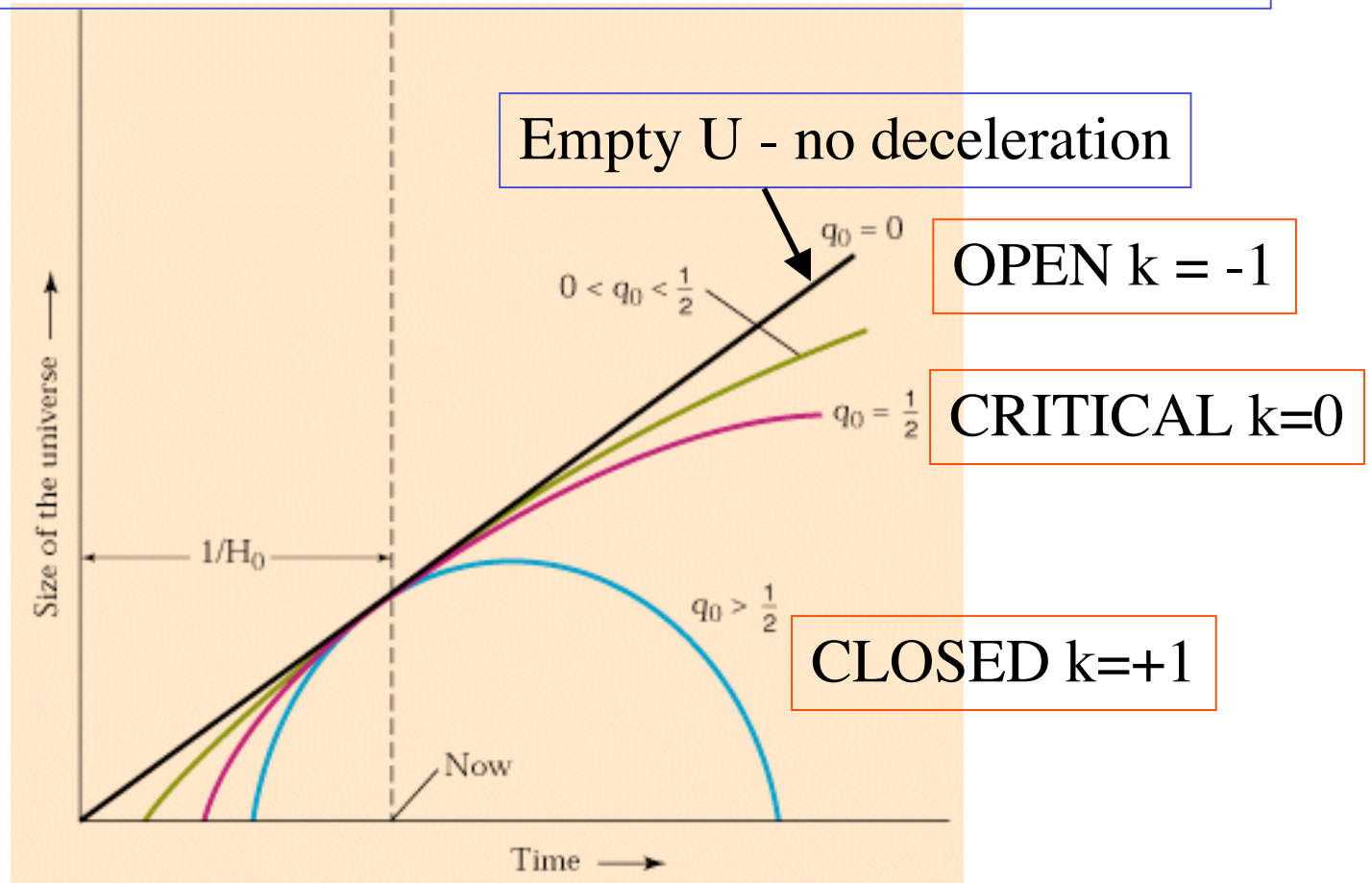
3 Basic Behaviors of Simple Universe

1) $D > D_{\text{crit}}$ Hubble expansion is not fast enough to escape and the U will fall back on itself. Like the **CLOSED** elliptical orbits of satellites, the U is considered **Bound**. General Relativity predicts that the U would have **positive curvature** ($k=1$), like the surface of a sphere in 2-D, and thus the U is **FINITE** in extent.

2) $D < D_{\text{crit}}$ Hubble expansion is fast enough so that the U will never stop expanding. Like the **OPEN** hyperbolic orbits of rockets with more than the escape velocity, the U is **UNBound**. General Relativity predicts that the U would have **negative curvature** ($k=-1$), like the surface of a saddle in 2-D, and thus the U is **INFINITE**.

3) $D = D_{\text{crit}}$ Hubble expansion just enough so the U will come to a halt after an infinite time. Like the **FLAT** coasting path of the parabolic orbits of rocket with exactly the escape velocity, the U is considered **CRITICAL between bound and unbound**. General Relativity predicts that space would have **zero curvature** ($k=0$), like the surface of a flat sheet in 2-D, and thus the U is **INFINITE**.

To succinctly express this dependence on density, astronomers use density parameter $\Omega = D/D_{\text{crit}}$ and also the deceleration parameter q_0 which is just $\Omega/2$, so $q_0 = 1/2$ is equivalent to the Critical Density case (when $\Lambda = 0$). In the 3 basic evolution models, we find the U to expand as follows:



Universe Models with Λ

With Λ not 0, one gets a much richer set of possible models of the U, since different amounts of repulsive force can lead to quite different evolution of the size of the U. Here are e.g.

