UCSC PIE
Worksheet 9

1. One place that rational expressions come up a great deal is in calculating probabilities. Here we'll explore this a bit.

   (a) We often write the probability of an event occuring as an expression like $\frac{a}{b}$. For example, the probability of rolling a 6 on an orderinary 6-sided die is $\frac{1}{6}$. The probability that the event won't occur is $1 - \frac{a}{b}$. Write this as a single rational expression.

   (b) Use the expression you found in part (a) to compute the probability of *not* rolling a 6 on an ordinary 6-sided die. Does your answer make sense?

   (c) To find the probability of two independent events *both* occuring, you just multiply the probability of each one. For example, the probability of rolling 6 twice in a row is $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$. (Independent events are ones that don't affect each other, for example rolling a fair die twice in a row.) If the probability of an event is $\frac{a}{b}$, what is the probability that the event will occur twice in two tries?

   (d) What is the probability the event will occur the first time you try, but not occur the second time?

   (e) What is the probability that the event will not occur on the first try, but will occur on the second?

   (f) What is the probability the event will not occur in either of two tries?

   (g) Add up the probabilities you found in parts (c) - (f). Does your answer make sense?

2. Many questions in science, covering topics from physics to biology, depend on the idea of surface area to volume ratios. Here we'll explore an example of using surface area to volume ratios to understand the shapes of soap bubbles. Soap bubbles assume the shapes they do in order to minimize their surface area while still enclosing the same volume of air.

   (a) Consider a spherical soap bubble of radius r. The volume of a sphere is $(4/3)\pi r^3$, and the surface area is $4\pi r^2$. What is the surface area divided by the volume of the soap bubble? Write the answer as a simplified rational expression. We'll call this ratio x_0.

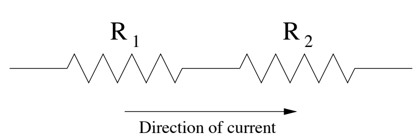
   (b) Now consider an alternate shape: a cylinder of radius r and height h. Write the surface area divided by volume for this shape? Again, write a fully simplified rational expression. We'll call this x_1.

   (c) We want to compare equal volumes. What height h does the cylindrical bubble have to have so that it contains the same volume as the spherical bubble? What is x_1 in terms of r in this case?

   (d) For equal volumes, which has a bigger surface area: a spherical bubble or a cylindrical bubble?

   (e) You should find that a sphere has a smaller surface area. By what factor is it smaller? In other words, what is x_0/x_1? An interesting side note is that it is possible to show that a sphere has the smallest surface area to volume ratio of *any* three-dimensional shape, which is why soap bubbles assume this shape.

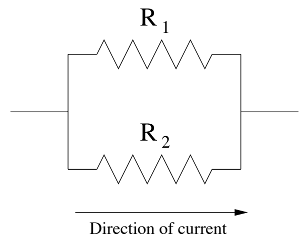
3. The resistance of an element of an electric circuit is a measure of how much effort it takes to make an electric current flow through that element. (Formally, it is defined as the voltage applied to the object divided by the current that flows through it when that voltage is applied.) When resistors are placed one after another in a circuit, their resistances simply add. For example, in the circuit below, the resistors (represented by the jagged lines) have resistances R_1 and R_2 by themselves, so the total resistance is $R_1 + R_2$. This arrangement, where the current must flow through one resistor after the other, is called placing the resistors *in series.*



(a) When resistors are placed *in parallel* as shown below, so that current can flow through one *or* the other, the resistances do not just add. Instead, the resistance is given by the expression

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

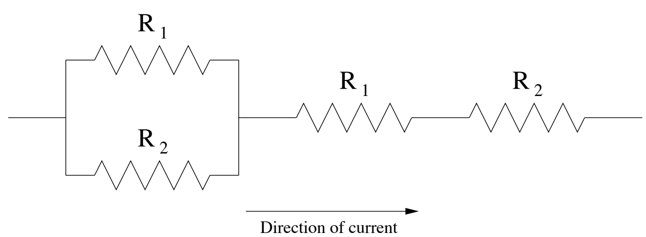
Write this sum as a single rational expression. (Hint: first try just rewriting the denominator as a single rational expression.)



(b) Consider two resistors of equal resistance R. What is the total resistance? What what factor is this larger or smaller than the resistance of a single resistor by itself?

(c) How about if the equal resistors are placed in parallel? By what factor is the resistance larger or smaller?

(d) One can use these rules for parallel and serial to compute the resistance of more complex circuits. Write a rational expression for the total resistance of the circuit shown below. (Hint: this is just the circuits from the example and the circuit from part (a) connected together in serial.)



(e) Write a rational expression for the total resistance of the circuit shown below?