\( \ddot{r} - r \dot{\theta}^2 = -\frac{\mu}{r^2} \) → note \( \vec{h} = \vec{r} \times \dot{\vec{r}} \)

\( |\vec{h}| = r^2 \dot{\theta} \)

trick: \( u = \frac{1}{r} \), use \( h \) conservation

+ differentiation of \( r \) using the chain rule

\[
\dot{r} = -\frac{1}{u^2} \frac{du}{d\theta} \dot{\theta} = -h \frac{du}{d\theta}
\]

\[
\ddot{r} = -h \frac{d^2u}{d\theta^2} \dot{\theta} = -h^2 u^2 \frac{d^2u}{d\theta^2}
\]

Eqn (\( \ast \)) above can be re-written

\[
\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2}
\]

given two parameters \( \mu = 6M \) and \( h = r^2 \dot{\theta} \)

this 2nd order DE tells how \( r \) varies with \( \theta \)
gives the orbital figure

This Eqn. is clearly related
to simple harmonic motion -
an orbit is an oscillation!

solution

\[
\frac{\mu}{h^2} \left[ 1 + e \cos(\theta - \omega) \right]
\]

The constant \( p \) = "semilatus rectum" = \( \frac{h^2}{\mu} \)

The general solution to the relative motion in the two body problem is the equation of a conic.