Astronomy 3 - Solution Set 2

1. **If you visited an asteroid 30 km in radius with a mass of 4x10^{17} kg, what would be the circular velocity at its surface? A major league fastball travels 90 mph. Could a good pitcher throw a baseball into orbit around the asteroid?**

   The circular velocity of an object in orbit is given on page 84 of the text:

   $V_c = \sqrt{GM/r} = \sqrt{\frac{(6.67x10^{-10} m^3/kg\ s^2)(4x10^{17} kg)}{3x10^4 m}} = 29.8\ m/s$

   A major league pitcher can throw a fastball at:

   $\frac{90\ miles}{hour} \left(\frac{90\ miles}{1\ hour}\right)\left(\frac{1\ hour}{60\ minute}\right)\left(\frac{1\ minute}{60\ seconds}\right)\left(\frac{1.6\ km}{1\ mile}\right)\left(\frac{1000\ m}{1\ km}\right) = 40\ m/s$

   So a major league pitcher *could* throw a baseball into orbit around the asteroid.

   In case you're curious, a planet's escape velocity is given by:

   $V_{esc} = \sqrt{2GM/r} = \sqrt{2}V_c = 42.1\ m/s$

   So the pitcher could throw the ball slightly faster than 90 mph, he could throw the ball so hard that it would never come back to the asteroid.

2. **Using the universal law of gravitation to answer the following question. a) How does quadrupling the distance between two objects affect the gravitational force between them? b) Compare the gravitational force between the Earth and the Sun to that between Jupiter and Sun. Jupiter's mass is 318 times that of the Earth and its distance from the Sun is 5.2 times that of the Earth's distance. c) Suppose the Sun were magically replaced by a star with twice as much mass. What would happen to the gravitational force between the Sun and the Earth?**

   The law of universal gravitation states that the gravitational force between two objects, with masses M_1 and M_2, separated by a distance d (p138) is

   $F_g = \frac{GM_1M_2}{d^2}$.

   a) If the distance between two objects is quadrupled, the force between them would be weakened by a factor of 16. b) Jupiter is 5.2 times further away from the Sun and has a mass 318 times that of the Earth. According to the above formula, the force between Jupiter and the Sun is 12 times stronger than that between the Earth and the Sun. c) Gravity between THAT STAR and the Earth would increase by a factor of 2 from gravity between the Sun and the Earth. (Gravity between the Sun and the Earth will be the same).

3. **a)Suppose around another star with the same mass as the Sun, there is a planet at 1 AU with twice the mass of the Earth, what would its orbit be? b) Suppose a solar system has a star that is four times as massive as our Sun. If that solar system has a planet the same size as Earth orbiting at a distance of 1AU what is the orbital period of the planet? Explain.**

   a) According to Newton's law of universal gravitation (p140), the orbital period of a planet with a mass M_2 around a star with a mass M_1 and a separation a is

   $P^2 = \frac{4\pi^2 a^3}{G(M_1+M_2)}$ where G is the universal gravitational constant. Since the mass of the Earth (M_2) is much smaller than that of the Sun (M_1), we generally neglect the contribution of M_2 in the above formula. Even if another planet has twice the mass as the Earth, it would not modify significantly its period around its own host star (with a similar mass as the Sun). Thus, the orbital period of this planet at 1AU has the same period as that of the Earth, namely one year.

   b) If the central stars mass is 4 times more massive, we would find from the above formula that P^2 would be 4 times smaller. Thus, the period of the planet at 1AU would be half that of the earth, ie 6 Earth month.

4. **Use the data in Table 4.1 in chapter 4 of the book, a) compare the energy of a 1-megaton hydrogen bomb to the energy released by a major earth quake. b) If the US obtained all**

its energy from oil, how much oil would be needed each year? compare the Sun' annual energy output to the energy released by a supernova?

a) The amount of energy released in a 1-megaton hydrogen bomb is 5×10^{15} joules. The energy released by a major (magnitude 8) earth quake is 2.5×10^{16} joules which is 5 times that of the hydrogen bomb. b) The amount of US energy consumption per year is 10^{20} joules. The energy released by burning 1 liter of oil is 1.2×10^{7} joules. A total of 8.3×10^{12} liter (or 2 trillion gallons) of oil is needed. That amount is also comparable to the energy release of 20,000 hydrogen bombs. c) The annual energy generation by the Sun is 10^{34} joules whereas the energy released by supernova is $10^{44} - 10^{46}$ joules. Thus the energy released at the end of a sun-like star's life is comparable to all that during it lifespan.

5. **Einstein's general theory of relativity predicts the curvature of space-time, but here on Earth we have little opportunity to observe such effects. Find an astronomical situation in which space-time curvature is evident from our observations, and describe the effect of the curvature on what we see when we view these objects.**

   There are many possible answers. A few are: Black holes, the apparent position of stars near the sun during a total solar eclipse, and the precession of the perihelion of Mercury's orbit.

6. **How does the light gathering power of the largest telescope in the world compare with that of the human eye? (Hint: Assume that the pupil of your eye can open to about 0.8 cm.)**

   See page 105 of the book. A telescope's light gathering power depends on the area that's collecting light. The largest telescope in the world is the Keck telescope in Hawaii, which is 10 meters in diameter.

   $A_{Keck}/A_{eye} = \frac{\pi R_{Keck}^2}{\pi R_{eye}^2} = \left(\frac{R_{Keck}}{R_{eye}}\right)^2 = \left(\frac{1000\ cm}{0.8\ cm}\right)^2 = 1.5$ million.

   So the Keck telescope gathers 1.5 million times as much light as the human eye.

7. **A spy satellite orbiting 400 km above Earth is supposedly capable of counting individual people in a crowd. What is the minimum diameter of the telescope that the satellite must carry? (Hint: Use the small-angle formula.)**

   There several steps to this problem. First you need to estimate how good your telescope's resolution has to be to be able to count people in a crowd. Let's say that a person is about 1 meter across. Use the small angle formula on page 42 of the text to find out how big a 1 meter object looks when you're 400 km away:

   $angular\ size = \frac{(206{,}265\ arcsec)(linear\ size)}{distance} = \frac{(206{,}265\ arcsec)(1m)}{4x10^8 m} = 0.5\ arcsec$

   Now we need to know how big a telescope is needed to resolve details 0.5 arcseconds across. Page 106 of the text has a formula for the resolving power of a telescope:

   $D = \frac{11.6cm\ arcsec}{\alpha} = \frac{11.6cm\ arcsec}{0.5\ arcsec} = 23.2\ cm.$

   So your telescope needs to be 23.2 cm in diameter to be able to see object 1 meter across from 400 km away.