

## Astronomy 3 Homework 3 Solution

January 21, 2009

1. **Human body temperature is about 310 K (98.6 F). At what wavelength do humans radiate the most energy? What kind of radiation do we emit?**

You can use Wien's radiation law on page 127 of the text:

$$\lambda_{max} = \frac{3,000,000^\circ Knm}{T} = \frac{3,000,000^\circ Knm}{310^\circ K} = 9680 \text{ nm}.$$

For comparison, visual light is about 500 nm.

2. **A UCSC graduate student/TA, Edmond Cheung, observes a star which is twice as hot as the Sun but much fainter than the Sun. How can he determine the star's temperature? Why does the star appear so much fainter even though it is much hotter?**

He determine the temperature of the star by its color. Hotter stars appear to be bluer. He can also examine the spectra and identify the emission and absorption lines. Some atoms are ionized at relatively hot temperatures.

Hotter stars should be intrinsically brighter than cooler stars if they are at the same distance. In this case, the hotter star is much further away than the Sun and that is why it appears to faint.

3. **Do all the photons come from this star have the same color? If so, how is this color determined? If not, how does the color spread out? Identify a natural phenomenon to support your answer.**

No. There is a distribution of photons with different wavelength and color. The color of a star is determined by the wavelength where the stellar brightness is at a maximum. If we use a prism to break up the light from the Sun, we can see its color spread

out over many color ranges. But the peak intensity of the Sun is centered on the yellow color.

4. **The same student observe a binary star. The peak wavelength of star A is 300 nm and that of star B is 600 nm. What are their frequencies? What about the speed at which these lights reach us?**

The frequency of the photons equal to the speed of light divided by the wavelength. The speed of light is 300,000 km/s. The frequency of star A is  $3 \times 10^8 \text{ m/s} / 3 \times 10^{-7} \text{ m} = 10^{15} / \text{s}$ . The frequency of star B is half that of star A.

5. **If one star has a temperature of 6000°K and another star has a temperature of 7000°K, how much more energy per second will the hotter star radiate from each square meter of its surface?**

Use the Stefan-Boltzmann law (p 162) to find out how much more energy the hot star radiates:

$$\frac{E_{hot}}{E_{cool}} = \frac{T_{hot}^4}{T_{cool}^4} = \frac{7000^4}{6000^4} = 1.85$$

So the hot star is emitting 1.85 times as much energy as the cool star.

6. **In the laboratory, the Balmer beta line has a wavelength of 486.1 nm. If the line appears in a star's spectrum at 486.3 nm, what is the star's radial velocity? Is it approaching or receding?**

Use the formula on page 166 of the text:

$$V = \frac{c\Delta\lambda}{\lambda_0} = \frac{(3 \times 10^8 \text{ m/s})(486.3 \text{ nm} - 486.1 \text{ nm})}{486.1 \text{ nm}} = 123,000 \text{ m/s}$$

The star is receding; this is called a redshift.

7. **Explain how studying a star's spectrum can allow us to detect: a) its surface chemical composition, b) its surface temperature, c) its speed moving relative to us, and d) its rotation.**

a) The energy levels of electrons in each chemical element are unique. As a result, each element produces its own distinct set of spectral lines, giving it a unique "spectral fingerprint."

b) Two simple rules describe how a thermal radiation spectrum depends on the temperature of the emitting object: 1) hotter

objects emit more total radiation per unit surface and 2) hotter objects emit photons with a higher average energy, which means a shorter average wavelength.

c) We can determine the radial motion (toward or away from us) of a distant object from changes in its spectrum caused by the Doppler effect.

d) As the object rotates, light from the part of the object rotating toward us will be blueshifted, away from us will be redshifted, and light from the center of the object will not be shifted at all. Consequently, each spectral line will appear to be wider than it would if the object is not rotating. The faster the object is rotating, the broader the line.

**8. Briefly explain two most important advantages of putting telescope in space.**

The earth atmosphere absorb photons in the infrared, ultraviolet, X-ray, and  $\gamma$  ray range. Space telescopes fly above the atmospheric absorption layers where “windows” to the universe are open. Turbulence in the earth atmosphere also distort any light which passes through it. Space telescope can resolve astronomical images down to the diffraction limit.

**9. Calculate the diffraction limit of the human eye, assuming a lens size of 0.8 cm for visible light of 500 nm wavelength photons. How does this limit compare to the diffraction limit of a 10-meter telescope?**

Using the formula (p181) on diffraction limit, we find

$$\text{diffraction limit} = 2.5 \times 10^5 \times \left( \frac{\text{wavelength}}{\text{telescope diameter}} \right) = 15.6 \text{arcsecond}$$

For a 10-meter telescope, we just need to increase the diameter from 0.8cm to 10meter. The ratio of 0.8cm:10meter = 1:1250. We divide the above result by a factor of 1,250 and obtain the diffraction limit to be 0.0125 arcsecond.